POW WOW CIRCUITS

Overview

This activity could be used within any class where different mathematical models are explored. It is an elementary introduction to non-algebraic graphs as models and to some of the algorithms used to analyze them. The context of planning a pow wow circuit is the basis for this introduction to Hamiltonian circuits and their analysis. Note that graph theory, which includes the concepts of vertex-edge graphs like Euler circuits and Hamiltonian circuits, is often found in a fourth-year course like Discrete Mathematics or as an extension in Algebra 1, Geometry, Algebra 2, or Integrated Mathematics courses.

Timeframe: Two to three 50-minute class periods

Math HS Conceptual Category: Modeling

Strategies & Practices

Critical Area (for instructional focus): Modeling to link classroom mathematics and statistics to everyday life, work, and decision-making.

Mathematical Practice (for student focus):

1. **Make sense of problems and persevere in solving them.** Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations.

3. **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.

4. **Model with mathematics.** Students are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions.

5. **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.

6. **Attend to Precision.** Mathematically proficient students try to communicate precisely to others.

Standards

Montana Content Standards for Mathematics

Math Clusters for Modeling Use a model to represent a real-world situation and analyze the situation appropriately.

Math Standards for Modeling *Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practices, and specific modeling standards appear throughout the high school standards and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).*

Mathematical Modeling *In addition, mathematical modeling occurs when students follow a multistep process of solving problems and represent the key ideas through a visual representation. These visual representations allow students multiple entry points for solving a problem, ensuring material that is both engaging and accessible. Examples of real world situations that could be modeled using a vertex-edge graph are 1) planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player or 2) engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.*

Montana Content Standards for Reading and Writing

College and Career Readiness Anchor Standards for Reading (Grades 6-12):

CCRA.R.1 Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.

CCRA.R.2 Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas**.**

Indian Education for All Essential Understandings Regarding Montana Indians

Essential Understanding 1 There is great diversity among the twelve sovereign tribes of Montana in their languages, cultures, histories, and governments. Each tribe has a distinct and unique cultural heritage that contributes to modern Montana.

Essential Understanding 3 The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs. Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories pre-date the “discovery” of North America.

Learning Objectives

In this lesson, students will . . .

- apply reasoning and problem solving, use algebraic concepts and procedures to understand processes involving number, operation, and variables, and use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts, including those of Montana American Indians.

- formulate a geometric model that describes a relationship between variables.
- analyze operations on these relationships to draw conclusions (using cultural contexts, including those of Montana American Indians).
- understand that there is great diversity among the tribal nations in Montana.
- recognize that pow wows are still in existence and that families use them as a time of gathering and sharing with other families and friends.
- learn that Native traditional beliefs and spirituality persist today and are still practiced by many American Indian people.
- acquire and use vocabulary associated with graph theory:

  algorithm
  weighted graph
  vertices
  edges
  nearest neighbor
  cheapest link
  Hamiltonian circuit

- create and use a weighted graph as a mathematical model.
- know two different algorithms for creating efficient Hamiltonian circuits, recognizing that using different algorithms on the same problem can result in different results.
- see the need for efficiency in analyzing models.
- evaluate the effectiveness of various algorithms.

Assessment

- Question 1 from student worksheet can be used as a pre-assessment to identify previous knowledge about possible effective algorithms.
- Questions 2, 3, and 4 from the student worksheet can be used as formative assessments by noting student level of understanding through their responses.
- Question 5 from the student worksheet can be used as a post-assessment or summative assessment.

Materials and Resources

For students

Pow wow Circuits *Worksheet* - Student pages in two parts (pages 1-4 and pages 5-7)

Montana State Highway Map (free from Montana Department of Transportation)

Outline of the state of Montana (optional)

For teachers

Pow wow Circuit – Teacher Notes

Your Guide to Understanding and Enjoying Pow Wows

Montana Content Standards and IEFA Essential Understandings Regarding Montana Indians

Montana Content Standards for Mathematics – Grades 9-12

Montana Content Standards for English language Arts and Literacy

Essential Understandings Regarding Montana Indians

Teacher Preparation

Read the background information provided (*Powwow Circuits – Teacher's Notes* and *Your Guide to Understanding and Enjoying Pow Wows*)

Instructional Plan

Learning Activity 1 (1 day)

Introduce students to pow wows. A brief introduction to the history of pow wows and other background information about pow wows is included in the student pages. Explain how some American Indian families travel from powwow to pow wow on what is called the "pow wow circuit" in Montana, using the time to camp and possibly compete. (For more background information, see *Your Guide to Understanding and Enjoying Pow Wows*).

Present the vocabulary that will be used in this lesson:

algorithm
weighted graph
vertices
edges
nearest neighbor
cheapest link
Hamiltonian circuit

Give every student a copy of the student pages through Question 1 ONLY (pages 1-4). Within these pages are a brief introduction to pow wows, a pow wow schedule, and a distance chart giving mileage between pow wow sites. You may want to include a blank outline of the state of Montana so students can locate sites appropriately. This will help students learn Montana geography while increasing their awareness of the presence of American Indians across the state. However, for the creation of the weighted graphs within this activity only the mileage is necessary.

Have the students do a "close reading" of pages 1-4 to build background information of pow wows in Montana. The intent of close reading is to have the students slow down, engage with the text in different ways, and reflect as they read. You may have the students read this in parts with highlighters to emphasize important ideas or underline things they already knew. See your English/Language Arts teacher for additional strategies for close reading.

Have the students work in pairs to answer question 1 on page 4 to determine the shortest distance between the five towns. Student pairs should be prepared to describe the route they determined, how many miles it contains, and explain the algorithm they used to find it. If additional time is needed for the student pairs to finalize their answers and explanations, assign the completion of question 1 for homework.

If time, have pairs share their algorithms (strategies) for determining the shortest distance between the five towns. As a class, discuss the different strategies used by partner pairs to determine their answers.

Learning Activity 2 (1 to 2 days)

If not completed in previous class, have students share their algorithms (strategies) for determining the shortest distance between the five towns. As a class discuss the different strategies used by partner pairs to determine their answers.

Give students the rest of the student pages (pages 5-7) and review the two algorithms, Nearest Neighbor and Cheapest Link, with the students to ensure understanding. These pages outline two algorithms mathematicians have developed for quickly finding circuits that have a minimal weight (in this case distance). Neither of these algorithms guarantees the least distance only an efficient way to find a relatively short distance.

Have students work in partner pairs to answer questions 2, 3, and 4. Student pairs should be prepared to justify their responses.

Discuss questions 2, 3, and 4 as a class. Remind the students there were various student algorithms attempted when first trying to find the shortest Hamiltonian circuit. Compare the student-generated algorithms to the Nearest Neighbor and the Cheapest Link algorithms examined in questions 2 and 3.

Have the students work individually (or in partner pairs) on question 5. This question could be used as a summative assessment question if desired or just as a culminating question for the learning experience. If used as a summative assessment, students will hand in responses to question 5 for scoring. If used as a culminating question, answers could be discussed in pairs or as a class.

Extension Activities and Additional Resources

The same activity could be extended to pow wows in other states. See the Pow wow schedules for other states at powwows.com. With some additional resources like maps of the reservations in another selected state (e.g., map of Indian Reservations in Arizona), a similar learning activity could be created.

Other circuits from graph theory could also be introduced to students, like Euler Circuits.

Pow Wow Circuits – Teacher's Notes

This activity could be used within any class where different mathematical models are explored. It is an elementary introduction to non-algebraic graphs as models and algorithms used to analyze them. The context of planning a powwow circuit is the basis for this introduction to Hamiltonian circuits and their analysis.

> A Pow Wow is a gathering where Native American dancing, singing, and celebration take place. There are several different pow wows that take place throughout the country....
>
> Some reports say the word pow wow has its origin from the Pawnee word pa-wa, meaning 'to eat.' Other sources say the word is of Algonquin origin and was originally pronounced pauau, which indicates a gathering of people for purpose of celebration or important event. In any case, it is a special time for people to gather and celebrate, meet old friends and create new friendships.
>
> *Your Guide to Understanding and Enjoying Pow Wows.*

General rules to pow wow:

- Stand during Grand Entry.
- Never call the dancer's regalia a "costume" (outfit is acceptable).
- Respect everyone; non-Native, Native, and especially the elders.
- Do not point at people.
- Ask permission to take photographs.
- Have fun.

Pow Wow Circuits Worksheet

Name: ____________________________

The following history of the pow wow is specific to the Salish people, who are the first recorded inhabitants of the Bitterroot Valley. The Bitterroot Valley became the primary wintering grounds for the Salish Tribe.

"In historic times, the Salish performed a dance called the war dance. The war dance was given before breaking camp to leave on the buffalo hunts and on the way east whenever there was danger of attack by the enemy. There was a war dance before starting on buffalo hunt because the Salish knew they would meet enemies before returning. A war dance was also given when a buffalo hide teepee cover had been finished and the teepee was erected. During this time all the camp would make a wish for the good luck and health for the household of the newly erected teepee."

"Around 1870s, a party of Salish was on its way to the Plains. Somewhere in the Musselshell country they met up with a party of Crow Indians. The Crow had a war dance similar to the Salish, but they told the Salish that they had a new dance. They had taken over this new dance from the Sioux. It was there that the Salish first saw the grass dance. The Salish did not participate in the dance that year, but watched and tried to learn the songs. After this, members of the Salish Tribe paid visits to the Crow people. . . and gradually learned more of the Grass Dance Songs. About ten years later, the Salish began to use the grass dance in the Bitterroot Valley."

"The Salish first held the grass dance during the winter months. It might be held at any time during the year, but in winter it was held in a home. Anyone could give this dance, but they must have food for guests, such as meat, roots, and berries. Four people were selected to be in charge of the food. At midnight the food was eaten. There was a leader for the grass dance. This was the same person who led the old Salish war dance. Only a warrior who captured a whip from an enemy was eligible to lead the Salish war dance. So the whip bearer or bearers, as there might be more than one, were considered the leader of the grass dance. "[1]

This is the origin of what would eventually become the modern day "pow wow" among the Salish people.

Powwows have continued to change over the years. They remain gatherings where Indian people can share part of their tribal traditions and culture. "Most of the original religious meanings and purposes for the dances performed at the pow-wows have been lost or forgotten. Due in part to federal policies in the late 1800s and early 1900s forbidding Indian dancing, whole generations [were not taught] the dances. Since the 1920s Indian people have revived some tribal dances and developed some new dances including the Grand Entry at the beginning of every session of most modern pow-wows."[2] They often include dancing and singing contests, "give-aways," encampments, feasting, and other cultural activities for all ages.

Today, powwow, or celebrations are still very much a part of the lives of many Native Americans. Competitive singing and dancing for prize money is a recent change to the traditional powwow. Prize money is awarded to the top point-getters at the culmination of the event for both dancing and singing/drumming competitions.

[1] Glenbow Institute, Calgary, Alberta. Claude Schaeffer collection, microfilm reels.

[2] Sherrole, B. Grand Entry: A New Ceremony Derived From The Old West, Tribal College 3 (Jan 31, 1997): 10.

While the prizes for dancing and singing/drumming are a part of today's pow wows, not everyone is there to compete. For many it is simply a time to celebrate and participate proudly in one's culture.

From as early as March to as late as September, pow wows take place in the northern Plains. They occur somewhere virtually every weekend. "Many families will pack up and go on the [pow wow] circuit, camping out and enjoying the celebration activities, singing, dancing, and seeing friends they may not have seen since the previous season."[3]

Table 1 shows the sites and approximate dates[4] of pow wows held on Montana Indian reservations.

Table 1: Powwow Sites and Approximate Dates

Powwow Name	Site	Approx. Date[4]	Host
North American Indian Days	Browning	July 11-13	Blackfeet
Heart Butte Celebration	Heart Butte	Aug 1-3	Blackfeet
Crow Fair Celebration	Crow Agency	Aug 15-18	Crow
Valley of the Chiefs	Lodge Grass	July 4-6	Crow
Chief Plenty Coups Day of Honor	Pryor	Aug 23	Crow
Arlee Powwow	Arlee	July 4-6	Salish-Kootenai-Pend d'Oreille
Standing Arrow Powwow	Elmo	July 18-20	Salish-Kootenai-Pend d'Oreille
Milk River Indian Days	Fort Belknap	July 25-27	Gros Vente-Assiniboine
Hays Powwow	Hays	July 11-13	Gros Vente-Assiniboine
Veteran's Powwow	Hays	Oct 31-Nov 2	Gros Vente-Assiniboine
Wadopana Celebration	Wolf Point	Aug1-3	Sioux-Assiniboine
Red Bottom	Frazer	June 20-22	Sioux-Assiniboine
Bad Lands Celebration	Brockton	June 27-29	Sioux-Assiniboine
Poplar Indian Days	Poplar	Aug 29-31	Sioux-Assiniboine
Wahicinca Dakota Oyate Celebration	Poplar	July 18-20	Sioux-Assiniboine
New Year's Powwow	Lame Deer	Jan 3-5	Northern Cheyenne
Fourth of July Powwow	Lame Deer	July 4-6	Northern Cheyenne
Northern Cheyenne Labor Day Powwow	Ashland	Aug 31-Sept 2	Northern Cheyenne
Rocky Boy Annual Powwow	Box Elder	Aug 1-3	Chippewa-Cree

[3] McCluskey, Murton, (January 2009). *Your Guide to Understanding and Enjoying Pow Wows*.

[4] These dates are only approximate and may not be an annual event. For more information regarding exact dates and locations contact tribal councils, cultural committees, school districts, or the Indian Education Division at the state Office of Public Instruction.

Table 2 indicates the mileage between the reservation towns hosting the powwows. For the purposes of this activity, it is being assumed you will not return home between pow wows, but instead continue on to the next site.

Table 2: Mileage between Reservation Towns

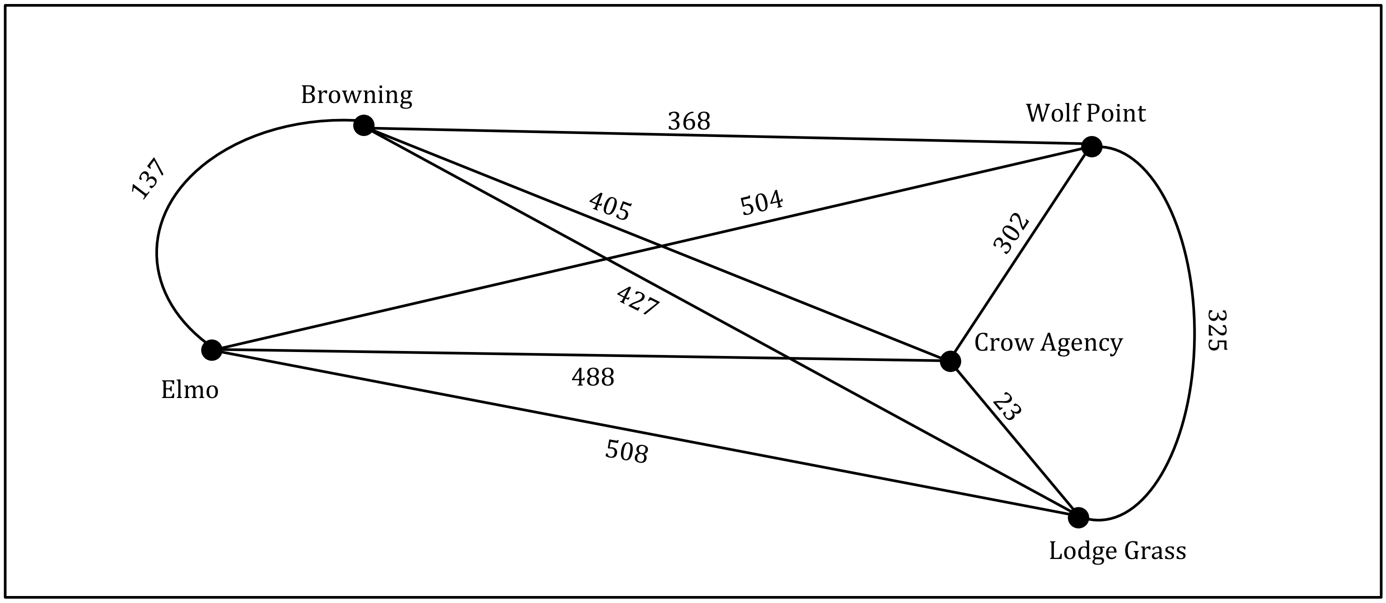
	Arlee	Ashland	Box Elder	Brockton	Browning	Crow Agency	Elmo	Frazer	Fort Belknap	Heart Butte	Hays	Lame Deer	Lodge Grass	Poplar	Pryor	Wolf Point
Arlee	– –	492	279	543	178	429	59	491	350	187	385	472	451	531	397	510
Ashland		– –	392	298	469	65	552	307	323	460	289	23	82	301	157	288
Box Elder			– –	266	178	328	315	212	70	185	105	372	350	252	308	231
Brockton				– –	403	312	539	54	197	409	231	284	334	14	359	36
Browning					– –	405	137	349	207	31	242	449	427	389	378	368
Crow Agency						– –	488	322	259	396	225	44	23	315	93	302
Elmo							– –	486	344	146	379	531	508	525	456	504
Frazer								– –	143	355	177	293	344	40	368	19
Fort Belknap									– –	213	36	302	281	183	238	162
Heart Butte										– –	374	439	418	395	369	374
Hays											– –	268	247	217	204	196
Lame Deer												– –	61	287	135	274
Lodge Grass													– –	337	114	325
Poplar														– –	362	21
Pryor															– –	349
Wolf Point																– –

Graph theory is used to model physical settings so they can be easily analyzed, often for decision making and planning. The situation is represented by a graph that consists of a series of points called **vertices** (plural of **vertex**) that are connected by lines or curves called **edges**. Once a situation is represented there are several different algorithms (step-by-step processes) that can be used to help analyze the graphs.

Because of the vast distances between pow wow sites, the distance and costs associate with traveling these distances should be considered.

Suppose you wanted to travel from Crow Agency to Browning, Elmo, Lodge Grass, Wolf Point, and back to Crow Agency, not necessarily in this order. You can create a graph on which the vertices represent the town and the edges are labeled with the distance between the towns represented by the vertices. Because the edges have a value, this type of graph is called a **weighted graph**. An example of a weighted graph representing all the possible routes between these five towns is shown in **Figure 1**. The graph shows that there are many possible ways to make a trip from Crow Agency to the other four towns and back again.

Figure 1: Weighted Graph of Distances between Reservation Towns[5]



Once the graph is created there are several different ways to analyze the graph and determine possible routes from the starting vertex to the various other vertices of the graph and back to the original starting vertex. This route is called a **Hamiltonian circuit** because the starting vertex and stopping vertex are the same and every other vertex is visited exactly once.

The more vertices in the graph, the more possible circuits must be considered. When the number of vertices gets large, there are mathematical ways to determine quickly one of the shorter (not always the shortest) routes. These step-by-step processes are called **algorithms**.

1. One circuit from Crow Agency to Wolf Point to Elmo to Lodge Grass to Browning and back to Crow Agency is 2146 miles long.
   a. Using the graph in Figure 1, see if you can find an algorithm that will result in route for starting at Crow Agency, visiting each other town only once and returning home that is shorter than 2146 miles?
   b. Describe the route and how many miles it contains.
   c. Explain the algorithm you used to find it.

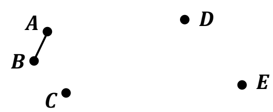
[5]**Note**: It is not important where you locate the vertices, only that the paths connecting them are labeled with the correct values. Also, there is no vertex where two paths cross unless there is a vertex shown at the intersection of the paths.

2. As the number of vertices increase, the difficulty in finding a reasonable route takes more and more time. Mathematicians have developed algorithms that guarantee a reasonably short route (not

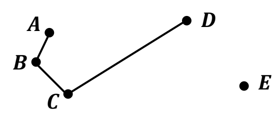
necessarily the shortest possible) quickly. One such algorithm is called the **Nearest Neighbor** algorithm. It is illustrated below.

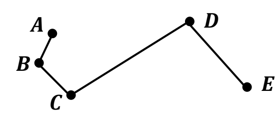
Draw only the vertices of the graph. For this example the vertices are ***A***, ***B***, ***C***, ***D***, and ***E***.

Then starting with any vertex draw the path to the vertex that is its "nearest neighbor." For this example vertex ***A*** is connected to vertex ***B*.**

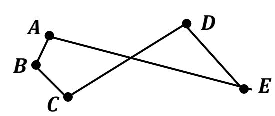


Continue this process from the second vertex, drawing an edge to the next nearest vertex not yet visited, and so on, until all the vertices have been visited. The three diagrams below show edges drawn from ***B*** to ***C*** then from ***C*** to ***D***. (Notice that the distance from ***C*** to ***A*** is shorter than ***C*** to ***D***, but drawing this edge would have completed the circuit without visiting all the vertices.) The next edge is drawn from ***D*** to ***E*** because this was the next closest vertex that had not yet been visited.





To complete the Hamiltonian circuit, return to the original starting vertex. In the example draw the edge from vertex ***E*** to vertex ***A***.



Use the nearest neighbor algorithm to create a Hamiltonian circuit to visit the powwows in the five reservation towns on the graph in Figure 1 that starts and ends in Crow Agency.

a. What is the distance traveled using the Nearest Neighbor algorithm?

b. How does the distance compare to the distance found in question 1?

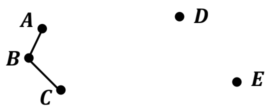
c. How is the algorithm used in question 1 different than the Nearest Neighbor algorithm?

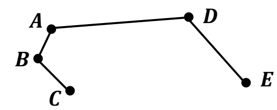
3. Another algorithm for finding a reasonably short Hamiltonian circuit quickly is the **Cheapest Link** algorithm. While the goal is the same, the Cheapest Link always uses the shortest edge no matter which two vertices it is connecting. It is explained below.

Draw only the vertices of the graph. For this example the vertices are ***A***, ***B***, ***C***, ***D***, and ***E***.

Then draw the shortest possible edge in the graph regardless of the vertices it connects. For this example vertex ***A*** is connected to vertex ***B.***

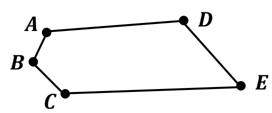
Next draw the next shortest possible edge that does not complete the circuit. Repeat this process using the shortest remaining edge each time until all the vertices have been visited once. In this example the next three edges drawn would be from ***B*** to ***C*** then from ***D*** to ***E*** then be from ***A*** to ***D***.





Notice that at times it is possible for individual disconnected edges to appear. In the example, the edge from ***A*** to ***C*** was the shortest, but that edge would have completed a circuit before all vertices had been visited. Therefore, the next shortest edge, from ***D*** to ***E***, was drawn. This left the edge from **D to E** disconnected until the next edge was drawn.

To complete the Hamiltonian circuit, return to the original starting vertex. In the example draw the edge from vertex **C** to vertex **E**.



a. Use the cheapest link algorithm to create a Hamiltonian circuit to visit the powwows in the five reservation towns on the graph in Figure 1 that starts and ends in Crow Agency.

b. What is the distance traveled using the Cheapest Link algorithm?

c. How does the distance compare to the distance found in question 1? In question 2?

d. How is the algorithm used in question 1 different than the Cheapest Link algorithm?

e. Is it possible that neither of the circuits found using the Nearest Neighbor algorithm and the Cheapest Link algorithm is the shortest?

f. What would have to be done to find the very shortest?

g. Would it be reasonable to always try to find the shortest possible circuit? Why or why not?

4. Create a weighted graph of *all* the different possible powwow sites.
    a. Using the graph, find the distance traveled using the Nearest Neighbor algorithm.
    b. Using the graph, find the distance traveled using the Cheapest Link algorithm.
    c. Were the distances the same? Do you believe you found the shortest possible distance? Why or why not?

5. Realistically, a graph like the one in question 4 would not be useful since the powwow dates are set. Those dates may not follow the same order as the sites found in question 4. Suppose, however, that you want to plan a summer trip attending as many different powwows as possible. You decide that if there is another powwow the following weekend, you will not return home, but continue on to the next powwow. If there is more than one week between powwows, you will return home before going to the next one.
    a. Use graphs to plan your summer powwow trip starting in June and ending on Labor Day, keeping the distance traveled in mind.
    b. Describe the route you will take and the total number of miles covered on the trip.
    c. Describe the algorithm you used to create your trip.