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# Appendix G: High School Math Course Recommendations

This appendix provides the recommended course codes and pathways for students' progression through the 9-12 experience. These recommendations were developed by the Math Standards Revision Task Force, informed by research conducted by the **Regional Education Laboratory (REL)** and other leading mathematics education organizations. The research highlights longstanding challenges with the traditional **Algebra I-Geometry-Algebra II** sequence, in particular, its limited flexibility and tendency to restrict access to rigorous mathematical learning for many students.

One of the key concerns addressed in <u>REL Handout C (2022)</u> is the use of tracking practices, which often result in classrooms where students receive varying levels of rigor based on perceived ability. In many cases, students on lower tracks are exposed to less challenging, more procedural instruction, which can hinder engagement, limit future opportunities, and expand disparities in learning outcomes. The research emphasizes the importance of creating course pathways that provide all students with four years of rich, relevant mathematics instruction, regardless of their intended postsecondary plans.

To support the application of this research, the task force has identified multiple course progression options, including both **traditional and integrated pathways**, to help schools structure 9-12 mathematics experiences. The appendix also includes recommendations for **upper-division mathematics courses**—those that extend beyond the minimum high school mathematics standards for graduation—to support students preparing for informed citizenship, advanced careers, college-level mathematics, or military pathways. These recommendations are intended to promote access to rigorous coursework, encourage flexible planning, and ensure that all students have opportunities to engage with mathematics in ways that align with their goals and aspirations and prepare them to be informed, productive citizens.

## **Traditional Pathway**

Students following the traditional pathway typically complete the sequence of Algebra I with Probability, Geometry with Data Analysis, and Algebra II with Statistics. This pathway reflects the conventional structure used in many high schools and is aligned to the Montana Mathematics Content Standards. While students may choose to pursue additional elective mathematics courses beyond this sequence, engaging in these three courses is the most direct way to ensure exposure to the full scope of the high school content standards.

	Algebra 1 with Probability	
Course Code: 02-052		
Montana Mathematics Content Standards Category	Domains within the Mathematics Content Standards Category Recommended for Coursework	Specific Standards within the Domain Recommended for Coursework
Core Numeric Reasoning Standards (NUM) ARM 10.53.518	The Real Number System (REAL)	All standards within this domain (3 total)
Core Algebraic and Functional Reasoning Standards (ALG) ARM 10.53.519	Understand Functions and Expressions (FUN) Linear Functions (LIN) Quadratic Functions and Expressions (QUAD) Exponential Functions and Expressions (EXP) Modeling with Functions (MOD)	All standards within these domains (27 total)
Core Data Reasoning and Probability	Visualizing, Summarizing, & Interpreting Data (INT)	Only standard CORE.DATA.INT.6 (1 total)
Standards (DATA) ARM 10.53.520	Probability (PROB)	All standards within the domain (2 total)
Total Number of Standards: 33		

	Geometry with Data Analysis	
Course Code: 02-072		
Montana Mathematics Content Standards Category	Domains within the Mathematics Content Standards Category Recommended for Coursework	Specific Standards within the Domain Recommended for Coursework
Core Data Reasoning and Probability	Quantitative Literacy (LIT)	All standards within this domain (3 total)
Standards (DATA) ARM 10.53.520	Visualizing, Summarizing, & Interpreting Data (INT)	All standards within this domain except CORE.DATA.INT.6 (6 total)
Core Geometric Reasoning Standards (GEOM)	Transformations (TRANS) Geometric Arguments, Reasoning, and Proof (ARG)	All standards within these domains
ARM 10.53.521	Measurement, Problem-Solving, and Geometric Modeling (MEAS)	- (8 total)
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	Algebra II With Statistics	
	Course Code: 02-056	
Montana Mathematics Content Standards Category	Domains within the Mathematics Content Standards Category Recommended for Coursework	Specific Standards within the Domain Recommended for Coursework
Core Plus Number and Quantity Standards (NUM) ARM 10.53.522	Numeric Reasoning (REAS)	All standards within this domain (2 total)
	Functions, Expressions, and Inequalities (FUN)	
Core Plus Algebraic and Functional	Polynomial Functions (POLY)	
Reasoning Standards (ALG)	Exponential and Logarithmic Functions (EXP)	All standards within these domains (20 total)
ARM 10.53.523	Trigonometric Functions (TRIG)	
	Modeling (MOD)	
Core Plus Data	Normal Distribution (NORM)	All standards within these domains
Reasoning Standards (DATA)	Experimental Design (DES)	(9 total)
ARM 10.54.524	Statistical Inference Using Simulation (INF)	
Total Number of Standards: 31		

#### **Integrated Pathway**

The Integrated Pathway weaves together content from algebra, geometry, probability, and statistics across a series of three courses—Integrated Mathematics I, Integrated Mathematics II, and Integrated Mathematics III. Rather than teaching mathematical domains in isolation, this pathway blends concepts throughout each year to highlight connections between topics and reflect how mathematics is applied in real-world contexts. This structure is also fully aligned with the Montana Mathematics Content Standards. Students on this pathway build a comprehensive understanding of high school mathematics over time and may also choose to extend their learning through advanced or specialized elective courses following Integrated III.



	Integrated Math I	
	Course Code: 02-062	
Montana Mathematics Content Standards Category	Domains within the Mathematics Content Standards Category Recommended for Coursework	Specific Standards within the Domain Recommended for Coursework
Core Numeric Reasoning Standards (NUM) ARM 10.53.518	The Real Number System (REAL)	One standard: CORE.NUM.REAL.3 (1 total)
	Understand Functions and Expressions (FUN)	All standards within this domain (8 total)
Core Algebraic and Functional Reasoning Standards	Linear Functions and Expressions (LIN)	All standards within this domain (6 total)
(ALG) ARM 10.53.519	Modeling with Functions (MOD)	All standards within this domain, as applied to linear functions (4 total)
Core Data Reasoning and	Visualizing, Summarizing & Interpreting Data (INT)	One standard: CORE.DATA.INT.6 (1 total)
Probability Standards (DATA) ARM 10.54.520	Probability (PROB)	All standards within the domain (2 total)
Core Geometric Reasoning Standards (GEOM) ARM 10.54.521	Transformations (TRANS) Measurement, Problem-Solving, and Geometric Modeling (MEAS)	- All standards within these domains (7 total)
Total Number of Standards: 29		



	Integrated Math II	
	Course Code: 02-063	
Montana Mathematics Content Standards Category	Domains within the Mathematics Content Standards Category Recommended for Coursework	Specific Standards within the Domain Recommended for Coursework
Core Numeric Reasoning Standards (NUM) ARM 10.53.518	The Real Number System (REAL)	Two standards: CORE.NUM.REAL.1 and CORE.NUM.REAL.2 (2 total)
	Quadratic Functions and Expressions (QUAD)	All standards within this domain (4 total)
Core Algebraic and Functional Reasoning Standards	Exponential Functions and Expressions (EXP)	All standards within this domain (5 total)
(ALG) ARM 10.53.519	Modeling with Functions (MOD)	All standards within this domain, as applied to quadratic and exponential functions (4 total)
Core Data Reasoning and	Quantitative Literacy (LIT)	All standards within this domain (3 total)
Probability Standards (DATA) ARM 10.54.520	Visualizing, Summarizing, & Interpreting Data (INT)	All standards within this domain except CORE.DATA.INT.6 (6 total)
Core Geometric Reasoning Standards (GEOM) ARM 10.54.521	Geometric Arguments, Reasoning, and Proof (ARG)	All standards within this domain (1 total)
Total Number of Standards: 25		



	Integrated Math III		
	Course Code: 02-063		
Montana Mathematics Content Standards Category	Domains within the Mathematics Content Standards Category Recommended for Coursework	Specific Standards within the Domain Recommended for Coursework	
Core Plus Number and Quantity Standards (NUM) ARM 10.53.522	Numeric Reasoning (REAS)	All standards within this domain (2 total)	
Core Plus Algebraic and Functional Reasoning Standards (ALG) ARM 10.53.523	Functions, Expressions, and Inequalities (FUN)		
	Polynomial Functions (POLY)		
	Exponential and Logarithmic Functions (EXP)	All standards within these domains (20 total)	
	Trigonometric Functions (TRIG)		
	Modeling (MOD)		
Core Plus Data	Normal Distribution (NORM)	All standards within these demoins	
Reasoning Standards (DATA)	Experimental Design (DES)	All standards within these domains (9 total)	
ARM 10.54.524	Statistical Inference Using Simulation (INF)		
Total Number of Standards: 31			

## Recommendations for Learning Outcomes Beyond the Core Plus Standards

This section provides guidance for elective mathematics courses that extend beyond the Core and Core Plus standards outlined in Montana's graduation requirements. Though these courses are **not required for graduation in most districts**, many schools **encourage a fourth year of mathematics** because it supports students' preparation for postsecondary experiences and **expands opportunities** for future academic, career, or military pathways.

Where possible, the recommended learning outcomes have been aligned with **dual credit**, **Montana University System (MUS)**, **and Advanced Placement (AP) offerings** to **encourage opportunities for students to earn college or trade school credit** while still in high school. These upperdivision courses are designed to align with diverse student goals—including higher education, workforce entry, military service, or active civic engagement—by providing mathematically rigorous and relevant mathematical experiences that reflect the range of possibilities available to Montana graduates.

To support implementation, the task force has included instructional resources where appropriate. Each course recommendation also includes the <u>SCED (School Courses for the Exchange of Data)</u> course code, developed by the Institute of Education Sciences (IES), to assist districts in identifying courses for credit delivery and enrollment reporting.

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	Data Science	
	Course Code: 30-7001	
Preamble (CIP. 2020)	Data Science is an interdisciplinary course that prepares students to analyze and interpret large- scale data in real-world contexts. Drawing from statistics, computer science, and mathematics, students explore topics such as data collection, visualization, modeling, and analysis. This course emphasizes trend identification, algorithmic thinking, and ethical data use to inform decision- making and problem-solving.	
Sample Learning Outcomes (IDS, 2022)	<ul> <li>Students will:</li> <li>Learn fundamental notions of data analysis—such as distribution and multivariate associations.</li> <li>Create and interpret visualizations of real-world data.</li> <li>Use numerical summaries to describe distributions.</li> <li>Learn about the various ways of collecting data and the effect that data collection has on conclusions and interpretations.</li> <li>Use computer simulations for informal inference.</li> <li>Make and use mathematical and statistical models to predict future observations and learn how data scientists measure the success of these predictions.</li> <li>Communicate methods and results to various audiences</li> </ul>	
Instructional Resources	<ul> <li>Sample Courses:</li> <li>Explorations in Data Science (YouCubed)</li> <li>Introduction to Data Science (UCLA)</li> <li>Additional Options for Sample Courses and Instructional Resources (Data Science 4 Everyone)</li> </ul>	



	Quantitative Reasoning
	Course Code: 02-158
Preamble	Quantitative Reasoning prepares students for informed citizenship and decision-making in daily life. The course emphasizes the application of mathematics to real-world problems across disciplines. Students develop skills in logical reasoning, data interpretation, proportional reasoning, and basic statistical analysis, equipping them to critically evaluate information and communicate their conclusions effectively.
Sample Learning Outcomes (Darnell, 2022 and Luebeck, 2024)	<ul> <li>Students will:</li> <li>Develop skills to think and reason mathematically in order to function more effectively in the modern world.</li> <li>Read mathematical material and write using mathematical notation correctly.</li> <li>Examine ways in which mathematics is used to solve applied quantitative problems. This includes learning to formulate a problem precisely, interpret solutions, and make critical judgments in the face of competing formulations and solutions.</li> <li>Follow and understand logical arguments.</li> <li>Apply elementary probability theory to construct models of random phenomena, including the use of simulations.</li> <li>Use elementary statistical tools such as measures of center and spread, graphical representations of data, and statistical estimation of population proportions.</li> </ul>
Instructional Resources	Teachers have broad latitude to choose focal topics, including, but not limited to: financial literacy (e.g., saving, investing, credit, managing risks and rewards), social choice and decision-making (e.g. elections, voting, fair division, Congress apportionment), geometry (e.g. symmetry, tiling), mathematical games, or management science (e.g. graph models for network problems).         Sample Courses:       M105 Syllabus at University of Montana         M105 Syllabus at Montana State University       M105 Syllabus at Montana Digital Academy



	Statistics
	Course Code: 02-201, 02-203, or 02-205
Preamble	Statistics is essential for careers in science, social science, health, and business. It involves the collection, organization, analysis, and interpretation of data. Students learn to draw meaningful conclusions from quantitative and categorical data through modeling, inference, and communication of findings.
Sample Learning Outcomes (Montana State University, 2025)	<ul> <li>Students will:</li> <li>Describe and explore sets of data both numerically and graphically.</li> <li>Use the normal model for the distribution of a single variable and the linear regression model for the relationship between two variables.</li> <li>Know the basic principles of good experimental design and good sampling design.</li> <li>Know the fundamental ideas of statistical inference for means and proportions including both hypothesis testing and confidence intervals.</li> <li>Interpret confidence intervals and P-values in the context of real problems.</li> <li>Be a critical consumer of statistical studies reported in the media.</li> </ul>
Instructional Resources	Sample course:         Advanced Placement Statistics Course Description         STAT 216 Introduction to Statistics Montana State University         Montana Digital Academy Statistics Introduction         Montana Digital Academy AP Statistics         Other Resources:         Common Online Data Analysis Platform (CODAP)         Gap Minder         Khan Academy         American Statistical Association         PBS High School Stats and Probability



Technical Math	
	Course Code: 02-153
Preamble	Technical Mathematics is recommended for students pursuing careers in industrial, technical, or applied fields. This course extends students' mathematical understanding by applying concepts to real-world technical and industrial contexts. Students develop skills in problem-solving, measurement, and modeling as they explore how mathematics supports processes and systems in the trades and technical industries.
Sample Learning Outcomes ( <u>Arends,</u> 2018)	<ul> <li>Students will:</li> <li>Utilize and apply mathematical operations, measurement (English and Metric Systems), introductory geometric principles, and applied algebra. to technical applications in academic and workplace situations.</li> <li>Read, interpret, and produce solutions to applications at the introductory technical mathematics level.</li> <li>Apply ratio and proportion concepts to introductory technical mathematical situations.</li> <li>Apply percent concepts to real-world applications.</li> <li>Apply appropriate technology in a mathematical situation.</li> <li>Convert the expression of a linear relationship between equation and graphical forms.</li> <li>Apply geometric formulas and concepts to solving technical applications.</li> <li>Analyze statistical data and determine the validity of results.</li> <li>Apply right triangle (trigonometric) relationships to problem-solving of technical applications.</li> <li>Apply formula manipulation and evaluation for problem-solving for unknown values.</li> </ul>
Instructional Resources	Sample courses: <u>The University of Montana M111</u> <i>Important comment about dual credit course offering: M111</i> <u>does not</u> count toward an Associate of Arts or Baccalaureate degree.



	Pre-Calculus
	Course Code: 02-110 or 02-114
Preamble	Pre-calculus is highly recommended for students aspiring to STEM careers requiring calculus. Pre-calculus completes the formal study of functions begun in the Core and Core Plus standards. Students focus on modeling, problem-solving, data analysis, translating mathematical information between representations, communicating with precise language, and providing rationales for conclusions.
Sample Learning Outcomes (Montana Digital Academy, 2025)	<ul> <li>Students will:</li> <li>Describe the concept of a function and explain its various properties.</li> <li>Define a function by ordered pairs, by a graph, and algebraically.</li> <li>Use transformations, symmetry, function operations, and inverses.</li> <li>Translate between verbal, numerical, graphical, and algebraic representations of functions.</li> <li>Apply a variety of techniques to find solutions to equations and inequalities.</li> <li>Simplify algebraic and transcendental expressions.</li> <li>Graph polynomial and rational functions, find the zeros of functions, and reconstruct a polynomial from its given zeros.</li> <li>Graph and analyze graphs of exponential and logarithmic functions, solve exponential and logarithmic equations.</li> <li>Find equations of populations that obey the law of exponential growth and decay.</li> <li>Analyze conic sections and solve problems with real-world applications.</li> <li>Perform matrix operations and find inverse matrices.</li> <li>Understand notation and applications of Sequences and Series.</li> <li>Correctly write and explain mathematics quantitatively and conceptually.</li> </ul>
Instructional Resources	Sample Courses: <u>AP Precalculus</u> <u>Montana Digital Academy Precalculus (Dual Credit M151)</u> <u>University of Montana - M151</u>

Calculus I
Course Code: 02-121 or 02-124
Calculus is required for most STEM careers. Calculus is the study of how things change. It provides a framework for modeling systems in which there is change and a way to deduce the predictions of such models.
<ul> <li>Students will:</li> <li>Explain the definition of limit, how to compute it in elementary cases, and how to determine the limits of transcendental, rational, and piecewise-defined functions.</li> <li>Define infinite limits, limits at infinity, asymptotes, indeterminate forms, and how to use L'Hôpital's Rule.</li> <li>Explain the limit definition of continuity.</li> <li>Explain the limit definition of the derivative of a function, how it relates to the function itself, and how to use it to compute derivatives.</li> <li>Use derivatives to find tangent lines to curves and velocity for particle motion.</li> <li>Apply the power, sum, product, quotient, and chain rules of differentiation.</li> <li>Use the derivatives of exponential, logarithmic, trigonometric, and hyperbolic functions.</li> <li>Explain implicit and logarithmic differentiation.</li> <li>Apply the Intermediate and Mean Value Theorems.</li> <li>Graphically analyze functions including using continuity and differentiation to determine local and global extrema, concavity, and inflection points.</li> <li>Use the derivative to solve challenging related rate and optimization word problems.</li> <li>Explain the Riemann integral, areas under graphs, antiderivatives the Fundamental Theorem of Calculus.</li> <li>Apply integration using the method of substitution.</li> </ul>
Sample Courses: AP Calculus AB

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