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This material has been prepared to provide access to information and to encourage discussion that can inform research, policy, and practice. The information contained in this document should not be used in isolation to reach definitive conclusions. REL Northwest staff are available to facilitate discussion, to provide further relevant information, and, in some cases, to partner on research to build an increasingly solid body of knowledge.

Handout A: Literature Scan–Foundational Math Content for PK–5

Review Summary

The Montana Office of Public Instruction (OPI) developed state-level content standards for mathematics in 2011, which they now seek to revise and update. OPI asked REL Northwest to summarize recent research about how to build a rigorous mathematics program incorporating foundational mathematical ideas for students in Grades PK–5. OPI was particularly interested in learning about the research evidence related to numeracy and number sense at these grade levels. To respond to this OPI request, this report presents three themes related to early mathematics instruction, drawing primarily on research studies from 2013 and later. Findings from the review are as follows:

Theme 1: What does research identify as the foundational mathematical content that Grade PK–5 students need to be successful? (p. 5-9)

- Children enter school with differing knowledge of number competency that varies with experience.
- Most young children come to formal schooling knowing how to count to 10, recognize basic shapes, and share small groups of objects between two people.
- Prior work emphasizes number sense and geometry as primary mathematical domains.
- Cross-cutting, "Big Ideas" in math are unitizing, composing and decomposing, relating and ordering, and recognizing patterns.
- Foundational content includes:
    - Counting with 1-to-1 correspondence
    - Subitizing
    - Using benchmarks to understand magnitude
    - Understanding place value
    - Algebraic thinking
    - Understanding mathematical equivalence
    - Building from knowledge of whole numbers to fractions
    - Reasoning related to geometry, space, patterning, and data

Theme 2: What instructional practices are recommended in Grades PK–5 for teachers to support their students' mathematical success? (p. 10-14)

- Use assessments to determine whether students have key knowledge and offer students interventions as appropriate.
- Use "developmental progressions" or "learning trajectories" that describe how mathematical knowledge builds to drive instruction, but acknowledge individual student knowledge and background.
- Motivate children with play-based activities.
- Organize learning activities so they are intentional and goal-driven.

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- Balance student-centered with teacher-directed learning activities to hold students' interest and support mathematical development.
- Nurture student mathematical discourse by using a "math talk learning community" during whole class and small group discussion.
- Facilitate students' use of formal mathematical language as they are conceptually ready.
- Use multiple representations and concrete or semiconcrete manipulatives to help students develop flexible and abstract thinking.
- Support problem-solving with rich tasks.

Theme 3: What is the research evidence related to incorporating cultural diversity in early childhood mathematics education? (p. 15-16)

- Culturally relevant pedagogy may benefit all learners.
- Incorporating cultural diversity may nurture early math students' acceptance of diversity and mathematical identity.
- More research is needed on the integration of culturally relevant pedagogies in early mathematics instruction.

Introduction

The development of mathematical skills and the teaching of emerging skills in a child's early years matter significantly for their later mathematics education, developmental progress, and school success in elementary through high school (Aunio & Niemivirta, 2010; Clements & Sarama, 2011; Duncan et al., 2007; Krajewski & Schneider, 2009).

Specific early numeracy skills measured in kindergarten and first grade predict later math and later reading achievement. For example, kindergarten skills such as counting, number knowledge, quantity comparison, story problems, and number–word linkages significantly predicted first grade (Desoete & Grégoire, 2006; Jordan et al., 2007), second grade (Aunola et al., 2004; Friso-van den Bos et al., 2015), third grade (Jordan et al., 2009; Missall et al., 2012), and fourth grade and beyond math achievement (Geary et al., 2012; Krajewski & Schneider, 2009; Nguyen et al., 2016).

Children who entered kindergarten with deficits in early numeracy skills such as counting and magnitude comparison continued to perform poorly in math throughout elementary school and showed slower growth in math compared to higher performing students (Geary et al., 2012; Jordan et al., 2009).

Children who fail to develop foundational numeracy skills before the end of kindergarten often struggle with higher level math skills in later grades. Development of rigorous math skills should be ongoing, as the impact of the early intervention fades with aging (Stipek & Valentino, 2014; Watts et al. 2018), and learners must continuously integrate their earlier learning with more complex mathematical ideas, such as algebraic thinking and fractions (e.g., Carpenter et al., 1989; Fennema et al.,1996; Lamon, 2020).

Why This Review?

The Montana OPI developed state-level content standards for mathematics in 2011, which they now seek to revise and update. OPI asked REL Northwest to summarize recent research about how to build a rigorous mathematics program incorporating foundational mathematical ideas for students in Grades PK–5. OPI was particularly interested in learning about the research evidence related to numeracy and number sense at these grade levels.

Organization of the Review

To respond to this OPI request, this report presents three themes related to early mathematics instruction, drawing primarily on research studies from 2013 and later.

These themes, stated as research questions that drove our literature review, are as follows:

1. What does research evidence identify as the foundational mathematical content that Grade PK–5 students need to be successful? (p. 5-9)
2. What instructional practices are recommended in Grades PK–5 for teachers to support their students' mathematical success? (p. 10-14)
3. What is the research evidence related to incorporating cultural diversity in early childhood mathematics education? (p. 15-16)

The following section "What Was Learned in the Review" summarizes information for each of these themes. Following these summaries, we provide a full bibliography for all works referenced within the document. In Appendix A, we provide information about the methodology used to identify the included, and Appendix B includes an annotated bibliography for resources from 2013 to the present.

Theme 1: What does research evidence identify as the foundational mathematical content that Grade PK–5 students need to be successful?

Background

Children enter school with differing knowledge of number competency that varies with experience (Okamoto, 2000, as cited in Gersten et al., 2005; Zmich et al., 2011).

- Number competency includes the following ideas:[1]
    - determination of set size
    - success carrying out simple calculations
    - understanding of and success in comparing quantities
    - shape recognition
    - measurement
    - mathematics language
    - nonverbal spatial sense
- Children's mathematics readiness is highly determined by their socioeconomic status and may impact their later school success.

Most young children come to formal schooling knowing how to count to 10, recognize basic shapes, and share small groups of objects between two people.

- Ninety-four percent of children can count to 10 and recognize basic shapes by school age (Clements, 2000; West et al., 2000).
- Most children starting kindergarten can count small sets of objects, solve problems involving small amounts, and share small groups of objects equally between two people (Baroody & Wilkins, 1999; Hunting, 1999).
- As most children can count and recognize shapes when they come to school, some advocate for more advanced early math instruction beyond counting and shape recognition (Sarama & Clements, 2008).

Prior work emphasizes number sense and geometry as primary mathematical domains.

- PK–5 instruction should emphasize number sense, with additional focus on geometry, spatial relations, and measurement:
- Number sense

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[1] Other definitions of number sense focus on four components (Kalchman et al., 2001): (a) fluency in estimating and judging magnitude, (b) ability to recognize unreasonable results, (c) flexibility when mentally computing, and (d) ability to move among different representations and to use the most appropriate representation.

    - number, number relations, and operations/algebraic thinking (including fractions for upper elementary students)
- Geometry, spatial relations, and measurement
    - Pattern recognition and data analysis are more strongly taken up in the CCSSM for students in later grades, beginning in Grade 4 (Common Core State Standards Initiative, n.d.; National Research Council [NRC], 2009)

Cross-cutting, "Big Ideas" in math are unitizing, composing and decomposing, relating and ordering, and recognizing patterns.

- Math "big ideas" "connect multiple concepts, procedures, or problems within or across domains or topics and are a particularly important aspect of the process of forming connections" (NRC, 2009, p. 44–47). "Big ideas" highlighted in the NRC report included the following:
    - unitizing in numerical, spatial, and geometric concepts
    - composing and decomposing
        - ("used throughout mathematics at every level and in all topics," p. 45)
    - relating and ordering
        - ("Relating and ordering allow one to decide which is more and which is less in various domains: number, length, area," p. 46);
    - looking for patterns
        - noticing structure and organizing information

Foundational Content

Counting with 1-to-1 correspondence

Counting with 1-to-1 correspondence means that children understand that each number in a counting sequence refers to a single item. Young children without 1-to-1 correspondence recite numbers while pointing, but they may say several numbers while pointing to a single item.

- Understanding of counting with 1-to-1 correspondence supports later skills like cardinality, set comparison, and simple addition and subtraction. Preschool interventions that included counting with 1-to-1 correspondence showed larger achievement treatment effects, and interventions that addressed counting and comparison also produced moderate-to-large or large effects on subsequent achievement (Nelson & McMaster, 2019).

Subitizing

Subitizing is the ability to understand a mathematical quantity without counting (e.g., recognizing there are four plates on a table without counting them individually).

- Early numeracy and number sense instruction should include a range of activities focused on subitizing and number relationships (Burchinal et al., 2022). Subitizing can be practiced with readily available materials like paper clips or having children roll a die and quickly say the number of dots on a side.

Using benchmarks to understand magnitude

- Young learners should learn to use benchmarks to approximate relative magnitude (e.g., 872 is closer to 1000 than 500) and recognize the absolute magnitude of or size of numbers in real-world situations (National Council of Teachers of Mathematics [NCTM], 2020).

Understanding place value

- Place value understanding "requires that children know more than which digit is in the tens place or which digit is in the thousandths place. ... It requires children to make connections among four mathematical properties:
  - **Additive property**. The quantity represented by the numeral is the sum of the values represented by the individual digits (9452 = 9000 + 400 + 50 + 2).
  - **Positional property**. Each position in the numeral is associated with a specific quantity (the 5 in 9452 is in the tens position).
  - **Base-ten property**. The value of each position increases in powers of ten from right to left (10 times larger to the left; 10 times smaller to the right).
  - **Multiplicative property**. The value of each digit relates multiplicatively to the value assigned to its position (the 3 is worth 3 × 10 = 30) (NCTM, 2020, p. 87)."

Algebraic thinking

- Developing algebraic thinking and reasoning helps young learners make mathematical generalizations. Additional instructional attention to three topics can help support students' algebraic reasoning:
  - the complexities involved with developing a deep understanding of the equal sign (see mathematical equivalence, below)
  - strategies for developing an understanding of the properties and behaviors of the operations
  - ways that varied representations support the understanding of change in functional relationships (NCTM, 2020)

Understanding mathematical equivalence

- Understanding mathematical equivalence involves knowing that numbers, measurements, and expressions can be represented in a variety of equivalent ways and that the equal sign signals the equivalence relation "is equal to."
- Formal understanding of mathematical equivalence in second grade predicts achievement on a standardized mathematics assessment in third grade (McNeil et al.,

2019), proficiency with algebraic procedures in fourth grade (Matthews & Fuchs, 2020), and algebra readiness in sixth grade (Hornburg et al., 2022).
- Understanding equivalence further predicts performance solving algebraic equations in middle school (Alibali et al., 2007) and college (Fyfe et al., 2020).

Building from knowledge of whole numbers to fractions

- Students typically do not start learning about fractions until third grade, and often have difficulty with fractions because they behave differently than whole numbers (Bharaj et al., 2020; Common Core State Standards Initiative, n.d.; Gabriel et al., 2013; Karp et al., 2014).
- Knowledge of how numerals and number words can be ordered and compared (i.e., symbolic whole number magnitude) in first grade predicts students' knowledge of fraction magnitudes and fraction arithmetic in middle school (Bailey et al., 2014).
- There are many interpretations of fractions (Behr et al., 1992; Lamon, 2020), and the part–whole interpretation frequently gets the most attention in early mathematics instruction.
- Instruction dominated by the part–whole interpretation may not provide the conceptual understanding students need to solve a problem involving fractions with a ratio meaning (Bharaj et al., 2020). Instruction should reinforce the conceptual meaning behind all fraction subconstructs to improve students' facility with fraction operations.
- Three topics in particular support deep understanding of fractions:
    - the role of unit fractions as building blocks for developing fractions knowledge
    - use of the number line for representing fraction magnitudes and operations
    - a focus on real-world contexts for understanding fraction operations conceptually (NCTM, 2020).

Reasoning related to geometry, space, patterning, and data

- Young learners' knowledge of geometry predicts later school achievement, especially the ability to solve nonroutine mathematical problems (see also Clements & Sarama, 2011; Dehaene, 1997; NRC, 2009; Verdine et al., 2014).
- Spatial reasoning[2] is important in many areas of mathematics and other school subjects like science, art, and geography. Spatial learning activities can be interesting for students and build an array of complex thinking abilities, such as perspective taking, visualizing, locating, orienting, dimension shifting, pathfinding, sliding, rotating, reflecting, diagramming, modeling, symmetrizing, composing, decomposing, scaling, mapmaking, and designing (see Bruce et al., 2015; Moss et al., 2014).

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[2] Spatial reasoning is defined as "our capacity to relate to and navigate the wider world around us; (involving) the ability to create and mentally manipulate 'representations of actual and imagined shapes, objects, and structures'" (Bruce et al., 2016, p. 19, citing Cohen & Hegarty, 2012, p. 868).

- Understanding of patterns positively effects students' numeracy and basic number concepts. Children should be encouraged to look for and identify patterns and then be taught how to extend, correct, and create patterns (Frye et al., 2013). For example, students could rearrange a simple pattern (boy, girl, boy, girl, etc.) to make a more complex one (a repeating pattern of two boys followed by two girls multiple times). This understanding can support additive (e.g., two boys repeated three times) or multiplicative thinking or other mathematical concepts like the distributive property.
- Students of all ages need to develop statistical skills that allow them to ask and answer investigative questions they encounter in the mathematics classroom and in their lives (Bargagliotti et al., 2020). Even at very young ages, children can begin exploring these mathematical ideas that will set them up well for subsequent study and use of data. For example, young students can describe and compare measurable attributes, represent and interpret data, and classify objects and count the number of objects in categories—all important skills that can deepen as students are exposed to increasingly more complex mathematical ideas (see also Frye et al., 2013).

Theme 2: What instructional practices are recommended in Grades PK–5 for teachers to support their students' mathematical success?

To effectively teach students for later mathematics success, teachers must know not only the mathematical content for their grade level, the preceding level, and the next grade level but also understand the most effective instructional strategies for supporting students' learning and believe that their young students can learn the material (for example, Ball & Bass, 2000; Clements & Sarama, 2020). In Theme 2, we present information on teachers' instructional practices for PK–5 students. Though most of these instructional practices are established, the resources described here include newer findings from research with young learners.

Use assessments to determine whether students have key knowledge and offer students interventions as appropriate.

- Educators must have the knowledge and resources they need to clearly identify what students do and do not know (Baroody et al., 2006).
- Early educational interventions with formal instruction in preschool and later can build on learners' informal knowledge and experiences to close gaps and support equitable opportunities (e.g., Starkey et al., 2022).

Use "developmental progressions" or "learning trajectories" that describe how mathematical knowledge builds to drive instruction but acknowledge individual student knowledge and background.

- Students can only understand more sophisticated mathematical ideas after they have attained knowledge of ideas at an earlier stage of development (see, for example, Clements & Sarama, 2020).
- For example, learners need to have a solid understanding of relating and ordering whole numbers before they can relate and order fractions effectively. Similarly, students must understand counting, cardinality, and number operations before they can build more complex mathematical knowledge such as understanding of the addition and subtraction operations (see Common Core Standards Writing Team, 2022).
- A birth to Grade 3 progression for number sense includes the following ideas: counting → subitizing → comparing numbers → adding/subtracting → composing numbers → multiplying/dividing → fractions → using patterns, structure, and algebraic thinking (Clements & Sarama, 2017/2019). Each of these primary stages of the progression also includes substages. For example, preschool and kindergarten students might work with small collections of objects (one to three items) and then move to progressively larger collections of objects (Frye et al., 2013) as their mathematical understanding develops.
- Research-based progressions documents are available for several mathematical domains for students at least from kindergarten through Grade 5:

  - number, operations and algebraic thinking, geometry, and measurement and data (Burchinal et al., 2022; Clements & Sarama, 2017/2019; Frye et al., 2013)
  - Operations and algebraic thinking (Carpenter et al., 2014; Fennema et al.,1996)
  - equi-partitioning (i.e., splitting a whole into equal-sized pieces) (Confrey et al., 2014)
  - length measurement (Maloney et al., 2014)
  - all K–5 (and beyond) mathematical domains (Common Core Standards Writing Team, 2022)
- Each students' developmental progress will be unique, based on their individual experiences (Schoenfeld & Stipek, 2011).
- Following a research-based developmental progression helps students more than skipping difficulty levels to expedite instruction. Following progressions helped students more effectively learn informal adding/subtracting (Clements et al., 2020) and develop greater fluency with mathematical basic facts, providing a stronger foundation for logical reasoning and problem-solving (McCray et al., 2019).
- Students should not be rushed to move along developmental progressions but allowed "weeks, months, and even years of mathematical work" to develop procedural fluency and conceptual understanding (Huinker & Bill, 2017, p. 81; see also NCTM, 2020, 2023).

Motivate children with play-based activities.

- Intentional instruction with engaging activities leverages children's natural curiosity and builds understanding of mathematical ideas and skills (Burchinal et al., 2022; Fuchs et al., 2021; Nakken et al., 2016 in Björklund et al., 2020; Vogler, 2019 in Björklund et al., 2020).
- Keeping students' interest and helping them find joy in their mathematical learning is especially important for young learners' mathematical interest and confidence: "[G]ood mathematics is about engagement and interest, not drudgery and drill" (Clements & Sarama, 2018, p. 2; see also Balala et al., 2021).
- Number games can help build number sense. For example, games that focus on complement numbers to make 10 (and then larger numbers) increase students' flexibility with making 10, support their understanding of place value, and support their use of the four operations (e.g., in Grade 1 knowing that addition may require composing a 10 and in Grade 2 knowing that subtraction may involve decomposing a 10). Educators should pay particular emphasis to difficult numbers—those that do not follow the standard number sequence (11, 12…) (Common Core Standards Writing Team, 2022).

Organize learning activities so they are intentional and goal-driven.

- Learning goals should be specified; mathematics learning cannot be assumed in play activities (e.g., Huinker & Bill, 2017; NRC, 2009).

- Learning goals should also be situated within a learning progression (Daro et al., 2011) and derived from content standards (Moss et al., 2014).
- Learning goals should emphasize multiple aspects of students' mathematical fluency, including their ability to
  - choose approaches appropriate to the numbers in a task,
  - understand and explain their approaches and strategies,
  - get the right answer and be able to judge the reasonableness of that answer (i.e., be accurate), and
  - see and understand a clear path to a solution (i.e., be efficient) (Huinker & Bill, 2017).

Balance student-centered with teacher-directed learning activities to hold students' interest and support mathematical development.

- For young children without mathematical difficulties, teacher-directed and student-centered instruction show approximately equal effects. Teachers may need to increase their use of teacher-directed instruction to raise the mathematics achievement of students with mathematical difficulties (Morgan et al., 2015).
- Early mathematics learning should combine teacher-directed instruction with guided play (Fisher et al., 2013) or flexibly use a range of teaching approaches (Baroody et al., 2006, shown below) to maximize student learning (Bruce & Flynn, 2012, as cited in Bruce et al., 2016).
- Differing educational strategies for early math learners—play versus learning academics, adult-directed versus child-directed, student-centered versus teacher-centered—may be "false dichotomies" that overly simplify the complexity of learning and may be limiting or damaging to children (Fuson et al., 2015). Educators should consider how each activity included in a class session supports students' mathematical development.

Nurture student mathematical discourse by using a "math talk learning community" during whole class and small group discussion.

- Engaging in mathematical discourse can help students solidify their own and their peers' mathematical learning, help teachers understand how students are progressing toward learning goals, and support students' achievement (Firmender et al., 2014; Hufferd-Ackles et al., 2015).
- Four developmental steps may help students engage in talk that moves them toward the intended mathematics learning goals: (a) helping individual students clarify and share their own thoughts, (b) helping students orient to the thinking of others, (c) helping students deepen their own reasoning, and (d) helping students engage with the reasoning of others (Chapin et al., 2013).
- Hearing clear and concise mathematical language from the teacher, hearing the mathematical language used by other students, and having the teacher connect words and phrases with something children can see (i.e., concrete items or motions) may especially help English learners enter into and participate in a classroom math talk learning community (Burchinal et al., 2022).

- Teachers' questioning and other interactions with students play an important role in the development of an equitable math talk learning community. Five types of questions are particularly important: (a) questions for gathering information, (b) questions for probing thinking, (c) questions for making the mathematics visible, (d) questions for encouraging reflection and justification, and (e) questions for engaging students with the reasoning of others (Huinker & Bill, 2017; NCTM, 2014b).
- Small group discussions can also help to build students' mathematical language and provide opportunities for students to engage with their peers mathematically (Burchinal et al., 2022; Gersten et al., 2015). For example, in small groups students might have the opportunity to engage in a brief conversation about the target mathematical idea or skill, learn how to carry out an interactive hands-on activity (chosen intentionally to help children apply that new idea or skill), and then carry out the activity themselves (Burchinal et al., 2022).

Facilitate students' use of formal mathematical language as they are conceptually ready.

- Students use informal and formal math talk to describe the world around them. Teachers should listen to how students are talking mathematically, with the goal of encouraging students' use of formal mathematical language (Frye et al., 2013; McCray et al., 2019). Building mathematical vocabulary from understandings of a concept rather than introducing a vocabulary term prior to students' conceptual understanding may support students' formal math talk development (Frye et al., 2013; Hundeland et al., 2020, as cited in Björklund et al., 2020).

Use multiple representations and concrete or semiconcrete manipulatives to help students develop flexible and abstract thinking.

- Mathematical representations (e.g., relating quantities, number words, and written numbers) can help students make mathematical connections and develop conceptual understanding (e.g., Björklund et al., 2020; Common Core Standards Writing Team, 2022; Fuchs et al., 2021; McCray et al., 2019; NRC, 2001, 2009;).
- Young learners' understanding of Arabic digits is a key foundation for arithmetic proficiency (Knudsen et al., 2015; Kolkman et al., 2013; Lyons et al., 2014; Nelson & McMaster, 2018). Childrens' understanding develops along a progression, from knowing the cardinal values of number words to recognizing Arabic digits to knowing their cardinal values and, concurrently, their ordinal position (Knudsen et al., 2015). Students need opportunities to make sense of symbolic representations of number by mapping onto quantity to build their foundational number sense.
- Early learners often use their fingers as a built-in mathematical manipulative before moving on to more abstract representations (Jordan & Levine, 2009). Using fingers to support their thinking can enable even 5-year-olds to attend to different quantities needed to solve an arithmetic task: the whole, the parts within the whole, and cardinality; their fingers support their ability to see a quantity in

multiple ways and, thus, their conceptual understanding (Kullberg & Björklund, 2020).

- Number lines can be helpful for young mathematics learners (e.g., Common Core Standards Writing Team, 2022; National Math Advisory Panel, 2008; NRC, 2009; Saxe et al, 2007). Using a sequence of number line learning activities can help students understand whole numbers, basic operations, and quantity at the early elementary grades and fractions and fraction operations at the upper elementary grades (Fuchs et al., 2021).
- Three types of representations might be included in core instructional programs: concrete, semiconcrete (or "representational"), and abstract representations (Fuchs et al., 2021). Students' use of concrete and semiconcrete representations to communicate mathematically over time can help build their understanding of abstract mathematics. Students at the same age or grade level may not all be ready to relinquish concrete or semiconcrete representations at the same time.
- As learning develops, students should be able to understand and translate between modes of representations and use a specific type of representation flexibly (e.g., understanding that $^5/_4$ is the same as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$; Lesh et al., 1987).
- Students who struggle to learn mathematics may need additional, focused instruction using representations to model mathematical ideas (Fuchs et al., 2021; Jitendra et al., 2016; REL Appalachia, 2021).
- Over time, students can add to their repertoire of new visual representations (e.g., tables, number lines, strip diagrams, percent bars, and schematic diagrams) that support more advanced problem-solving.

Support problem-solving with rich tasks.

- Only a few recent empirical studies focused on selecting and using rich tasks with early math learners were identified in the review.
- Even the youngest of learners can engage in mathematical problem-solving (Carpenter & Lehrer, 1999; Clements & Sarama, 2018); they can "put together what they know and invent ways to solve … problems" (Clements & Sarama, 2018, p. 2; Kullberg & Björklund, 2020).
- Rich tasks for early math learners are "activities that teachers use to help stimulate students' higher order thinking skills (Pianta et al., 2007), including their acquisition of knowledge and their ability to access and apply knowledge in new situations (Mayer, 2002)." (NRC, 2009, p. 244).
- In second-grade classrooms where rich tasks were used, students were asked more questions requiring them to describe and explain alternative strategies, talked more using longer responses, and showed higher levels of performance (Hiebert & Wearne, 1993).
- Studies of students not specifically in Grades PK–5 describe features of powerful tasks and examples (Krainer, 1993), the role of tasks in learning trajectories (Simon & Tzur, 2004) and features of high- and low-cognitive demand tasks (Henningsen & Stein, 1997).

Theme 3: Cultural Diversity: What is the research evidence related to incorporating cultural diversity in early childhood mathematics education?

Culturally relevant pedagogy may benefit all learners.

- Five equity-based practices that strengthen mathematical learning and cultivate positive student mathematical identities for K–8 students are shown in Figure 1 (Aguirre et al., 2013, as cited in Huinker & Bill, 2017).
- Teachers' selection of content and pedagogical approaches that recognize students' assets and funds of knowledge (i.e., their existing skills, experiences, and [cultural] practices; Moll et al., 1992, 2005) can help capture students' imaginations and interest and foster deeper understanding of domain knowledge (Lee, 2001; Rogoff, 2003).
- Attending to students' culture can also help new learning "stick" (Rebora, 2021), help students build bridges between informal and formal learning (and between the "school way and the home way" of doing math; Civil, 2018), increase student joy and motivation, and support more equitable student achievement (Boykin & Noguera, 2011; NCTM, 2014a).

Figure 1. Five Equity-Based Practices That Strengthen Mathematical Learning

Go deep with the mathematics. Develop students' conceptual understanding, procedural fluency, and problem-solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

Challenge spaces of marginality. Embrace student competencies, diminish status, value multiple mathematical contributions.

Draw on multiple resources of knowledge (math, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

Incorporating cultural diversity may nurture early math students' acceptance of diversity and mathematical identity.

- A focus on cultural diversity may be particularly important in early math education play to enable students to develop acceptance of diversity and differences from an early age (Chan, 2022; Djonko-Moore, 2020) and to build an early identity as a capable member of a mathematical community of practice (Abdulrahim & Orosco, 2020; Aguirre et al., 2013).

More research is needed on the integration of culturally relevant pedagogies in early mathematics instruction.

- There is limited empirical research available to support the use of culturally relevant pedagogies and their impact on early mathematics learners (Abdulrahim & Orosco, 2020).

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Appendix A: Methods

This scan was conducted for technical assistance purposes and, as such, was not intended to meet What Works Clearinghouse standards of review but rather to identify recent peer-reviewed research that supports the understanding of issues related to mathematics instruction for early math learners in Grades PK–5. Below we list information on how resources were identified.

Keywords and search strings used in the search

Theme 1: All possible configurations of "numeracy," "number sense," "math," and "early" were used.
Theme 2: "PME" AND "fractions" were searched separately to identify more recent conference presentations related to this Grades 3–5 topic (after other searches had identified limited new research on fractions).
Theme 3: Keywords used in Theme 1 were used in combination with "culture" or "disposition."

Search of Databases

ERIC, Education Source (i.e., EBSCO Host), Google, Google Scholar

Similar Prior REL Literature Review

A similar REL review on early mathematics conducted in 2015 provided some resources.

Similar Technical Assistance

Similar recent efforts to support another state Department of Education helped to identify some resources.

Search of Reference Lists of Relevant Reviews

(e.g., Björklund et al., 2020; Bruce et al., 2016; Frye et al., 2013)

Criteria for Inclusion

When reviewing resources, we considered four main factors:

- **date of the publication:** The most current information is included within the date range of 2013–present to capture publications within the last 10 years.
- **student age/grade range:** The most current information is included for students of the focal grade levels, Grades PK–5. Several resources that focus on mathematics teaching and learning across additional grade levels were also included.
- **source and funder of the report/study/brief/article:** Priority is given to IES sources, nationally funded sources, and certain other vetted sources known for strict attention to research protocols. However, in an effort to identify relevant studies, the search was not limited to peer-reviewed journals but instead also included gray literature, including conference presentations, technical reports, online research materials that are based on empirical and/or peer-reviewed research (e.g., Clements &

Sarama, 2017/2019), and professional development or practical resources based on empirical research (e.g., Huinker & Bill, 2017).

- **methodology:** Sources include randomized controlled trial studies, surveys, self-assessments, literature reviews, professional development resources, and policy briefs. Priority for inclusion generally is given to randomized controlled trial study findings, but the reader should note at least the following factors when basing decisions on these resources: numbers of participants, sample selection, and sample representation.

Appendix B: Annotated Bibliography of Empirical Research Contributing to the Body of Evidence, 2013–Present, Keyed to Primary Review Theme

3–Abdulrahim, N., & Orosco, M. J. (2020). Culturally responsive mathematics teaching: A research synthesis. *The Urban Review*, *52*(1),1–25. https://doi.org/10.1007/s11256-019-00509-2

The article synthesized empirical research conducted on culturally responsive mathematics teaching (CRMT) with culturally and linguistically diverse (CLD) learners. Thirty-five published studies between 1993 and 2018 met the criteria for inclusion in this review. Many of the studies considered for review were qualitative in nature and included small samples of teachers; fewer than half of the studies (14) included students in Grades PK–5. The article provides an informative summary of CRMT and illustrates the value of CRMT in fostering equitable and inclusive mathematics learning environments.

3–Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K–8 mathematics*. National Council of Teachers of Mathematics.

This book invites K–8 teachers to reflect on their own and their students' multiple identities and consider the rich possibilities for learning that may result when teachers draw on these identities. The authors hold that reflecting on identity and reenvisioning learning and teaching through this lens especially benefits students who have been marginalized by race, class, ethnicity, or gender. The authors encourage teachers to reframe instruction by using five equity-based mathematics teaching practices.

1–Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, *17*(5), 775–785.

Early knowledge of fractions is highly predictive of much later mathematics achievement, yet students often have difficulty with fractions because fractions behave differently than whole numbers. For example, while whole numbers have unique predecessors and successors, increase with multiplication, decrease with division, and can be represented by a single symbol, none of these properties holds true for fractions. The property that unites fractions, whole numbers, and indeed all real numbers is that they represent magnitudes that can be ordered on a number line. Although whole number knowledge can, and often does, interfere with fraction performance, the inverse finding may also hold true: Superior whole number knowledge might positively influence learning of fractions.

In this paper, Bailey, Siegler, and Geary (2014) examined whether it is possible to predict early in formal schooling which children will have difficulty learning fractions and to identify specific developmental antecedents of fraction difficulties. Participants performed the relevant tasks in first, seventh, and eighth grades. The results showed that (a) first graders' knowledge of whole number magnitudes predicted their knowledge of fraction magnitudes in eighth grade, even after

controlling for the first graders' whole number arithmetic knowledge, IQ, central executive functioning, parental education, household income, race, and gender; (b) first graders' knowledge of whole number arithmetic predicted their knowledge of fraction arithmetic in seventh grade, controlling for the same variables and whole number magnitude knowledge; and (c) the relation between first graders' knowledge of whole number magnitudes and their knowledge of fraction arithmetic in middle school was fully mediated by their knowledge of fraction magnitudes in middle school.

1, 2–Balala, M. M. A., Areepattamannil, S., & Cairns, D. (2021). Investigating the associations of early numeracy activities and skills with mathematics dispositions, engagement, and achievement among fourth graders in the United Arab Emirates. *Large-Scale Assessments in Education*, *9*, 13. https://doi.org/10.1186/s40536-021-00106-4.
The study examines the relations of early numeracy activities and skills to mathematics dispositions, engagement, and achievement among 26,859 fourth graders in the United Arab Emirates who took part in the sixth cycle of the Trends in International Mathematics and Science Study (TIMSS) in 2015. The study also explored the mediating effects of mathematics dispositions and engagement on the relations between early numeracy activities and skills and mathematics achievement among these fourth graders. Results of path analyses, after controlling for participants' demographic and socioeconomic characteristics, indicated that early numeracy activities and skills were significantly and positively related to mathematics dispositions, engagement, and achievement. Further, results of mediational analyses suggested that confidence in mathematics had a significant mediating effect on the relations between early numeracy activities and skills and mathematics achievement. The findings of the study highlight the crucial role that early numeracy activities and skills play in enhancing fourth graders' mathematics dispositions, engagement, and achievement in the United Arab Emirates.

1–Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L, & Spangler, D. A. (2020). Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education. National Council of Teachers of Mathematics. https://www.amstat.org/docs/default-source/amstat-documents/gaiseiiprek-12_full.pdf
The GAISE II is a professional report from the American Statistical Association (ASA) setting out guidelines for assessment and instruction in pre-K–12 in statistics and data science, an emerging area in K–12 mathematics instruction. The report argues that in an increasingly data-rich environment, students over all ages need to develop statistical skills. The report organizes statistical investigation around a four-component process, each of which involves exploring and addressing variability in data: I. Formulate Statistical Investigative Questions; II. Collect/Consider the Data; III. Analyze the Data; IV. Interpret the Results. The GAISE II includes guidance and examples for skills and concepts at the elementary, middle, and high school levels.

1–Bharaj, P. K., Jacobson, E., Lui, J., & Ahmad, F. (2020). Assessing students' understanding of fraction multiplication. In A. I. Sacristán, J. C. Cortés-Zavala, & P. M. Ruiz-Arias. (Eds.), *Mathematics education across cultures: Proceedings of the 42nd meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Mexico* (pp. 265–269). Cinvestav/AMIUTEM/PME-NA. https:/doi.org/10.51272/pmena.42.2020

The study investigated the range of strategies fifth graders used to solve a word problem involving fraction multiplication. The authors use qualitative analysis of elementary students' written work and the wide range of strategies for fraction multiplication that students used. The study suggested that students may be relying on their understanding of whole numbers, even when they are not appropriate, and that students may lack understanding of a fraction as a ratio. The authors speculate that elementary fractions instruction dominated by the part–whole interpretation of fractions may not provide the conceptual understanding necessary to solve a problem involving fractions with a ratio meaning. They argue that reinforcing the conceptual meaning behind all fraction subconstructs might improve students' facility with fraction operations.

1, 2–Björklund, C., van den Heuvel-Panhuizen, M., & Kullberg, A. (2020). Research on early childhood mathematics teaching and learning. *ZDM-Mathematics Education*, *52*, 607–619.

This paper reports an overview of contemporary research on early childhood mathematics teaching and learning presented at recent mathematics education research conferences and papers included in the special issue (2020–4) of *ZDM-Mathematics Education*. The summary highlighted research from recent conference presentations on factors that contribute to students' learning, such as the use of multiple representations (Bjørnebye, 2019), manipulatives (Lüken, 2019; Björklund & Runesson Kempe, 2019), or digital tools (Bakos & Sinclair, 2019); inquiry-based instruction and open-ended problem-solving (Breive, 2019); and being explicit about the content that should emerge from play-based (i.e., indirect) learning (Vogler, 2019; Nakken et al., 2016). Additionally, the authors highlighted studies describing the importance of children's awareness of mathematical structures for acquiring mathematical competence (Mulligan & Mitchelmore, 2018; Lüken & Kampmann, 2018) and the importance of teachers attending to the ways in which students engage with mathematics (i.e., with hand and body movements; Karsli, 2016; Rinvold, 2016). The summary also described lessons that can be drawn from early childhood teaching interventions (e.g., Paliwal and Baroody's 2020 study on various ways to see numbers' cardinality) and the effects of attentiveness to children's experiences and knowledge and the related choices of tasks (Clements et al., 2020; Grando & Lopes, 2020). Additional highlighted studies focused on mathematical concepts that are observed in young children, including a study by Hundeland et al. (2020) that describes how kindergarten-aged children learn to use and understand the language of mathematics through mathematical discourse. Studies highlighted how children's ability to perceive structure affects strategies for solving arithmetic tasks (Sprenger & Bentz, 2020), the use of finger patterns to structure number relations (Kullberg &

Björklund, 2020; Baccaglini-Frank et al., 2020), and the relationship of understanding of the concepts of cardinality and ordinality to awareness of additive and multiplicative number relations (Askew & Venkat, 2020).

1, 2–Bruce, C., Flynn, T., Moss, J., & the M4YC Research Team. (2016). *Early mathematics: Challenges, possibilities, and new directions in the research*. http://mkn-rcm.ca/wp-content/uploads/2016/11/M4YC_LiteratureReview_25June12_RevisedSept2016.pdf
The authors conducted a comprehensive review of over 500 articles to provide a synthesis of existing literature on mathematics for young children with a focus on the fields of education and educational research. Criteria for inclusion in the literature review included research involving treatment and control group studies with randomized field trials or quasi-experimental designs, longitudinal studies, a combination of quantitative and mixed-methods studies with large populations and highly descriptive studies involving smaller populations, research from academics who are well established in their field, and peer-reviewed articles from top tier journals (blind peer review with high rankings). The literature review was updated in 2016 to include the most up-to-date references and key developments in the research literature that occurred between 2012 and 2016, including the Math for Young Children research conducted in Ontario, Canada.

1, 2–Burchinal, M., Krowka, S., Newman-Gonchar, R., Jayanthi, M., Gersten, R., Wavell, S., Lyskawa, J., Haymond, K., Bierman, K., Gonzalez, J. E., McClelland, M. M., Nelson, K., Pentimonti, J., Purpura, D. J., Sachs, J., Sarama, J., Schlesinger-Devlin, E., Washington, J., & Rosen, E. (2022). *Preparing Young Children for School* (WWC 2022009). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from https://whatworks.ed.gov/
In this resource, the What Works Clearinghouse™ (WWC) and expert panel have distilled recent contemporary research into seven recommendations for preschool educators to use to help prepare children for school. Recommendations 3 and 4 focus on students' mathematical development. Specifically, recommendation 3.1 focuses on using intentional instruction with engaging activities and leveraging children's natural curiosity to build understanding of mathematical ideas and skills. Allowing children to share their thinking and engage in multiturn conversations during small group instruction is part of this recommendation. Recommendation 3.2 focuses on expanding preschool instruction beyond basic skills like counting and naming shapes to include more formal mathematical skills in numeracy, geometry, measurement, and patterning. Of these recommendations, those specifically related to supporting students' numeracy and number sense (the focus of this review) include activities focused on subitizing (instantly knowing how many items are in a group without counting) and number relationships (seeing a quantity in multiple ways). Recommendation 3.3 focuses on building children's mathematical knowledge and skills over time, following natural developmental progressions, and the Guide talks

about implications for curriculum selection and teachers' formative assessment during mathematics instruction.

Recommendation 4 focuses on engaging children in conversations about mathematical ideas and supporting them in using mathematical language. Recommendation 4.1 focuses on teachers introducing and explaining mathematical language during intentional, small group mathematics instruction starting with simpler language and introducing more complex words over time. Teachers should explain the meaning of words or phrases and use concrete examples and gestures to help explain ideas. Recommendation 4.2. suggests using shared math-focused book-reading activities, being careful to ensure that the book accurately depicts the mathematical idea or skill that is the learning focus. Teachers are encouraged to familiarize themselves with the book and story so they can identify words and phrases to emphasize or questions to support the conversation. With multiple readings of the same book, a teacher can increase the complexity of questions students are asked to consider. Recommendation 4.3 focuses on engaging children in natural opportunities to talk about mathematical ideas and use mathematical language.

1, 2–Carpenter, T., Fennema, E., Franke, M. L., Levi, L., & Empson, S. (2014). *Children's mathematics: Cognitively guided instruction*. Heinemann.
The first edition of *Children's Mathematics* helped teachers understand children's intuitive mathematical thinking and use that knowledge to help children learn mathematics with understanding. This edition provides new insights about Cognitively Guided Instruction (CGI) based on the authors' research and experience in CGI classrooms over the last 15 years. Highlights include the following:

- how children solve problems using their intuitive understanding of addition, subtraction, multiplication, and division
- the development of children's mathematical thinking throughout the primary grades
- instructional practices that promote children's active engagement in mathematics
- connections between children's strategies and powerful mathematical concepts

There is a new expanded collection of over 90 online video episodes illustrating children's mathematical thinking, interactions between students and teachers, and classroom instruction that builds on children's mathematical thinking. The Second Edition and videos provide a detailed, research-based account of the development of children's mathematical thinking and problem-solving and how teachers can promote this development in ways that honor children's thinking.

2, 3–Chan, A. (2022). *Cultural diversity in early childhood education*. Oxford Bibliographies. https://www.oxfordbibliographies.com/display/document/obo-9780199756810/obo-9780199756810-0289.xml
The author provides a brief literature scan focused on cultural diversity in early childhood education and highlights the importance of using language and discourse to create, develop, and explore worldviews.

2–Chapin, S. H., O'Connor, C., & Anderson, N. C. (2013). *Talk moves: A teacher's guide for using classroom discussions in math* (3rd ed.). Math Solutions.
This book examines the role that classroom discussions can play in teaching mathematics and deepening students' mathematical understanding and learning. Based on a 4-year research project funded by the U.S. Department of Education, this resource is divided into three sections focused on how to get started with classroom discussions, deciding on what to talk about, and implementing classroom discussions.

3–Civil, M. (2018). Intersections of culture, language, and mathematics education: Looking back and looking ahead. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, & B. Xu (Eds.), *Invited lectures from the 13th International Congress on Mathematics Education* (pp. 31–48). Springer.
The paper draws from a research agenda focused on the interplay of culture, language, and mathematics teaching and learning, particularly in working-class Mexican American communities in the United States. Drawing on data collected over several years, the author emphasizes the need for a coordinated effort in the mathematics education of nondominant students, an effort that involves teachers and other school personnel, the students' families, and the students themselves. Through the voices of parents, teachers, and students, the author illustrates the resources that nondominant students bring to school but often go untapped and the tensions that this may carry. The author argues for the need to develop stronger communication among the interested parties to develop learning experiences in mathematics that build on the knowledge, the language and cultural resources, and the forms of participation in the students' communities.

1, 2–Clements, D. H., & Sarama, J. (2018). Myths of early math. *Education Science*, *8*(2), 71. https://doi.org/10.3390/educsci8020071
In this article, the authors identify several myths about early education that have some aspects of truth to them but that the authors argue are largely myths that persist and may harm children. The authors identify nine common misconceptions teachers hold about early mathematics education and urge readers to rely on research findings and expert practitioners.

1, 2–Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach*. Routledge.
Clements, D. H., & Sarama, J. (2017/2019). *Learning and teaching with learning trajectories [LT]^2*. Retrieved from Marsico Institute, Morgridge College of Education, University of Denver. www.learningtrajectories.org
The third edition of the book summarizes current research into how young children learn mathematics and how best to develop foundational knowledge to realize more effective teaching. The authors show how *learning trajectories* help teachers understand children's level of mathematical understanding and lead to better teaching. The book emphasizes curiosity behind young children's mathematical reasoning and using learning trajectories to help teachers understand the varying levels of knowledge exhibited by individual students so that they can better meet the

learning needs of all children. This edition draws on numerous new research studies, offers expanded international examples, and includes updated illustrations throughout.

The book is also closely linked with *Learning and Teaching with Learning Trajectories–[LT]²*–an open-access, web-based tool for early childhood educators to learn about how children think and learn about mathematics. The linked website includes updates, interactive games, and practical tools that support classroom learning in four mathematical strands: number, operations and algebraic thinking, geometry, and measurement and data. For example, 20 learning trajectory levels are described for the mathematical idea of counting.

1, 2–Clements, D. H., Sarama, J., Baroody, A. J., & Joswick, C. (2020). Efficacy of a learning trajectory approach compared to a teach- to-target approach for addition and subtraction. *ZDM-Mathematics Education*, *52*, 637–648. https://doi.org/10.1007/s11858-019-01122-z
Clements et al. (2020) investigated the efficacy of implementing an intervention in which instructions and progression are grounded in a research-based learning trajectory for informal adding/subtracting. The authors define the three components of a learning trajectory: a goal, a developmental progression of levels of thinking, and instructional activities designed explicitly to promote the development of each level (Maloney et al., 2014; National Research Council, 2009). Even though the intervention had previously been found to have positive outcomes for preschool children's mathematics learning, the goal of the current study was to investigate how to teach in the most successful way. For this purpose, the authors used the same intervention but adapted the choices of the tasks' difficulty levels to the children's current knowledge levels. How to teach was then related to what to teach individual children. Results indicate that adapting the complexity of the content to the child's ability to learn best what is intended (i.e., skipping difficulty levels to shorten the steps to the learning goals) was not successful. Consistent with previous work, findings indicate that a learning trajectory–based approach to teaching early arithmetic will facilitate greater learning than will instruction that skips levels, at least for most children. The study supports child-centered approaches that are sensitive to the individual needs and potential of the child while aiming for the learning goals set by the curriculum.

1, 2–Common Core Standards Writing Team. (2022). *Progressions for the Common Core State Standards for Mathematics (February 28, 2023)*. Institute for Mathematics and Education, University of Arizona. Available at: https://mathematicalmusings.org/author/wgmccallum/
The 2010 Common Core State Standards in mathematics began with narrative documents describing the progression of a topic across a number of grade levels, informed both by educational research and the structure of mathematics. Those documents were then organized into grade-level standards, and subsequent work focused on refining and revising the grade-level standards rather than refining the

progressions documents. The 2023 Progressions for the Common Core State Standards are updated versions of the earlier progressions drafts, revised and edited to correspond with the Standards by members of the original Progressions work team together with other mathematicians, statisticians, and education researchers not involved in the initial writing. They note key connections among standards, point out cognitive difficulties and pedagogical solutions, and provide additional detail. The Progressions also provide additional resources for a curriculum that illustrates the range and types of mathematical work described by the standards, discussions of individual standards, classroom tasks, teacher professional development, and understanding of the importance modeling and language in students' mathematical development. The Progressions organize K–5 standards into domains. Counting and Cardinality (K) underline Operations and Algebraic Thinking and Number and Operations—Base Ten (both of which extend across all elementary grades). Four other mathematical domains are also highlighted for K–5 students: Number and Operations—Fractions (Grades 3–5), Measurement and Data (K–5), Geometric Measurement (K–5), and Geometry (K–5).

1–Confrey, J., Maloney, A. P., & Corley, A. K. (2014). Learning trajectories: A framework for connecting standards with curriculum. *ZDM-Mathematics Education*, *46*(5), 719–733.

This paper describes a body of work that associates the first nine grades of the Common Core State Standards (K–8) to 18 learning trajectories and, for each learning trajectory, unpacks, interprets, and fills in the relationships to standards with the goal of bringing the relevant research to teachers. The learning trajectories, previously available in an online interactive forum (TurnOnCCMath.net), describe the connections among the standards using a set of descriptor elements comprising conceptual principles, coherent structural links, student strategies, mathematical distinctions or models, and bridging standards. A more detailed description of the learning trajectory for equipartitioning (EQP) shows the detailed research base on student learning that underpins a particular learning trajectory. How curriculum materials for EQP are designed from the learning trajectory completes the analysis, illustrating the rich connections possible among standards, descriptors, an elaborated learning trajectory, and related curricular materials.

1–Daro, P., Mosher, F. A., & Corcoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction*. Consortium for Policy Research in Education.

This report provides an introduction to current work and thinking about learning trajectories for mathematics education, including why they are important and a strategy for how to think about what is being attempted in the field. The authors aim to clarify the varying ways in which the terms trajectory, progression, learning, teaching, and so on, are being used by the education community. Specifically, the report builds on arguments published elsewhere to offer a working definition of the concept of learning trajectories in mathematics and to reflect on the intellectual status of the concept and its usefulness for policy and practice. It considers the potential of

trajectories and progressions for informing the development of more useful assessments and supporting more effective formative assessment practices, for informing the ongoing redesign of mathematics content and performance standards, and for supporting teachers' understanding of students' learning in ways that can strengthen their capability for providing adaptive instruction.

3–Djonko-Moore, C. (2020). Culture in early childhood mathematics. *The Mathematics Teacher*, *113*(9), 702–707. https://doi.org/10.5951/mtlt.2019.0306
The author provides background on culturally responsive teaching and suggests strategies to innovate mathematics through cultural responsiveness and funds of knowledge.

2–Firmender, J. M., Gavin, M. K., & McCoach, D. B. (2014). Examining the relationship between teachers' instructional practices and students' mathematics achievement. *Journal of Advanced Academics*, *25*(3), 214–236.
The purpose of this study was to determine whether relationships existed between teachers' implementation of two specific discourse-related instructional practices and students' mathematics achievement in geometry and measurement as part of a research study on the effectiveness of an advanced mathematics curriculum for kindergarten and Grades 1 and 2. The mathematics units incorporated the following instructional practices: engaging students in verbal communication in mathematics and encouraging the use of appropriate mathematical vocabulary. Hierarchical linear modeling was used to determine the relationships between teachers' use of the instructional practices and the students' mathematics achievement. Results indicated that significant, positive relationships existed; the teachers' implementation scores for the verbal communication and mathematical language instructional practices were predictors of student mathematics achievement as measured by students' percentage gain scores on the Open-Response Assessments. Implications of these findings for mathematics instruction are discussed.

2–Fisher, K. R., Hirsh-Pasek, K., Newcombe, N., & Golinkoff, R. M. (2013). Taking shape: Supporting preschoolers' acquisition of geometric knowledge through guided play. *Child Development*, *84*(6), 1872–1878.
To understand variations in how children's exposure to shapes may affect the pace of their learning and the nature of their shape knowledge, researchers conducted a study of 70 four- and five-year-old children in which children were introduced to properties of different shapes, including regular and irregular triangles, rectangles, pentagons, and hexagons. The children were randomly assigned to one of three different groups. In one group, a free-play approach was used to introduce the concepts. The second group was a guided-play group, and in the third group, direct instruction was used to teach about the shapes. In the free-play group, the educator simply made the shapes available for children to use in their play. Results revealed that children taught shapes in the guided play condition showed improved shape knowledge compared to the other groups, an effect that was still evident after 1 week. Findings suggest that

scaffolding techniques that heighten engagement and direct exploration and that facilitate "sense-making," such as guided play, undergird shape learning.

Friso-van den Bos, I., Kroesbergen, E. H., Van Luit, J. E. H., Xenidou-Dervou, I., Jonkman, L. M., Van der Schoot, M., & Van Lieshout, E. C. D. M. (2015). Longitudinal development of number line estimation and mathematics performance in primary school children. *Journal of Experimental Child Psychology*, *134*, 12–29. http://dx.doi.org/10.1016/j.jecp.2015.02.002
Children's ability to relate number to a continuous quantity abstraction visualized as a number line is widely accepted to be predictive of mathematics achievement. However, a debate has emerged with respect to how children's placements are distributed on this number line across development. In the current study, different models were applied to children's longitudinal number placement data to get more insight into the development of number line representations in kindergarten and early primary school years. In addition, longitudinal developmental relations between number line placements and mathematical achievement, measured with a national test of mathematics, were investigated using cross-lagged panel modeling. A group of 442 children participated in a 3-year longitudinal study (ages 5–8 years) in which they completed a number-to-position task every 6 months. Individual number line placements were fitted to various models, of which a one-anchor power model provided the best fit for many of the placements at a younger age (5 or 6 years) and a two-anchor power model provided better fit for many of the children at an older age (7 or 8 years). The number of children who made linear placements also grew with age. Cross-lagged panel analyses indicated that the best fit was provided with a model in which number line acuity and mathematics performance were mutually predictive of each other rather than models in which one ability predicted the other in a nonreciprocal way. This indicates that number line acuity should not be seen as a predictor of math but that both skills influence each other during the developmental process.

1, 2–Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children: A practice guide* (NCEE 2014-4005). National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/early_math_pg_111313.pdf
The practice guide provides five recommendations for teaching math to children in preschool, prekindergarten, and kindergarten. Only one recommendation, to teach number and operations using a developmental progression, showed moderate evidence that early experience with number and operations supports students' acquisition of more complex math concepts and skills. With five specific subrecommendations related to the developmental progression (see Table 3), the guide describes how teachers first have students work with small collections of objects (one to three items) and then move to progressively larger collections of objects developing the following number and operations concepts:

- First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a

number word without needing to count them. [subitizing] Teachers might use similar and dissimilar items or examples contrasted with nonexamples to help students build fluency.

- Next, promote accurate 1-to-1counting as a means of identifying the total number of items in a collection. [1-to-1correspondence and cardinality] Predictable errors students have with counting are identified in Table 4 (p. 19).
- Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities. [compare quantities in collections larger than three, and number sequence (i.e., number-after knowledge), mental comparison of close or neighboring numbers]
- Encourage children to label collections with number words and numerals. [using numerals to represent quantity]
- Once children develop these fundamental number skills, encourage them to solve basic problems. [basic problem solving/addition and subtraction]

For Recommendation 2, there was only minimal evidence related to four other areas of mathematical development: geometry, patterns, measurement, and data analysis. While geometry and measurement are beyond the focus of this scan, studies on teaching patterns (10 studies) and data analysis (8 studies) showed some evidence of positive effects on general numeracy and basic number concepts. The patterning recommendation (2.2) focused on encouraging children to look for and identify patterns, then teaching them to extend, correct, and create patterns. Recommendation 2.4 focuses on helping children collect and organize information and teaching them to represent the information graphically. Ideas are provided in the Practice Guide for supporting English learners (e.g., linking visual representations of the most important vocabulary and concepts with terms in the child's home language as well as in English).

1, 2–Fuchs, L. S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K. S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., & Morgan, S. (2021). *Assisting students struggling with mathematics: Intervention in the elementary grades* (WWC 2021006). National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from http://whatworks.ed.gov/

This IES practice guide provides evidence-based practices that can help teachers tailor their instructional approaches and/or their mathematics intervention programs to meet the needs of their students. The guide features six recommendations, each based on strong evidence from the literature: (a) Systematic Instruction: Provide systematic instruction during intervention to develop student understanding of mathematical ideas (by systematic, the guide means that instructional elements intentionally build students' knowledge over time toward an identified learning outcome(s); topics are covered in an incremental and intentional way); (b) Mathematical Language: Teach clear and concise mathematical language, and support students' use of the language to help students effectively communicate their understanding of mathematical concepts; (c) Representations: Use a well-chosen set of concrete and semiconcrete representations to support students' learning of mathematical concepts and procedures; (d) Number Lines: Use the number line to

facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics; (e) Word Problems: Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas; and (f) Timed Activities: Regularly include timed activities as one way to build fluency in mathematics.

The recommendation most focused on number is recommendation 4, supporting the use of number lines. In the early grades the authors suggest a sequence of learning activities (shown on pp. 8–9 of the summary document) that help students understand whole numbers, basic operations, and quantity at the early elementary grades and fractions and fraction operations at the upper elementary grades.

2–Fuson, K. C., Clements, D. H., & Sarama, J. (2015). Making early math education work for all children. *Phi Delta Kappan*, *97*(3), 63–68.
This article is a follow-up to the 2009 National Research Council report on early math instruction. It summarizes some of the main issues of the report and describes some effective teaching–learning practices, including expecting and supporting children's ability to make meaning and mathematize the real world and creating a nurturing and helping math talk community. The article discusses "false dichotomies" in early education as well and encourages educators to consider learning goals of all activities.

1–Gabriel, F., Coché, F., Szucs, D., Carette, V., Rey, B., & Content, A. (2013). A componential view of children's difficulties in learning fractions. *Frontiers in Psychology*, *4*, 715. https://doi.org/10.3389/fpsyg.2013.00715
In this article, the authors designed a test aimed at assessing the main components of fraction knowledge, a mathematical concept known to be difficult for students to learn.

Multiple hypotheses have been proposed in order to explain learning difficulties: Fractions can denote different concepts, their understanding requires a conceptual reorganization with regard to natural numbers, and the use of fractions involves the articulation of conceptual knowledge with complex manipulation of procedures. In their test, conducted in Belgium with fourth-, fifth-, and sixth-grade students, the authors sought to distinguish between conceptual and procedural knowledge. Results showed that students seemed to master the part–whole concept of fractions, whereas numbers and operations posed problems. Students also seemed to apply procedures they did not fully understand. The study provides further directions to explain why fractions are among the most difficult mathematical topics in primary education and offers recommendations on how to teach fractions.

2–Gersten, R., Rolfuhs, E., Clarke, B., Decker, L. E., Wilkins, C., & Dimino, J. (2015). Intervention for first graders with limited number knowledge: Large-scale replication of a randomized controlled trial. *American Educational Research Journal*, *52*(3), 516–546.

Gersten et al. (2015) conducted a randomized controlled trial of a scale-up replication of Fuchs et al. (2005), which in a sample of 139 found a statistically significant positive impact for Number Rockets, a small group intervention for first-grade students at risk that focused on building understanding of number operations. The original study was a small-scale, highly controlled study. The study relied on a "Response to Intervention" framework calling for early intervention by providing small group instruction to students who, based on scores on a reliable and valid screening measure, appear likely to experience difficulty in learning. The replication study by Gersten et al. was implemented at a much larger scale—in 76 schools in four urban districts; 994 students at risk participated. Intervention students participated in approximately 30 hours of small group work in addition to classroom instruction; control students received typical instruction and whatever assistance the teacher would normally provide. Intervention students showed significantly superior performance on a broad measure of mathematics proficiency, suggesting support for data-informed small group instruction.

2–Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2015). Describing levels and components of a math talk learning community. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 1: Useful and usable research related to core mathematical practices* (pp. 125–134). National Council of Teachers of Mathematics.
This article addresses this question: How does a teacher, along with their students, go about establishing the sort of classroom community that can enact reform mathematics practices? An intensive yearlong case study of one teacher was undertaken in an urban elementary classroom with Latino children. Data analyses generate developmental trajectories for teacher and student learning that describe the building of a math-talk learning community—a community in which individuals assist one another's learning of mathematics by engaging in meaningful mathematical discourse. The developmental trajectories in the math-talk learning community framework are (a) questioning, (b) explaining mathematical thinking, (c) sources of mathematical ideas, and (d) responsibility for learning.

1, 2–Huinker, D., & Bill., V. (2017). *Taking action: Implementing effective mathematics teaching practices (K–5)*. National Council of Teachers of Mathematics.
This book offers a set of professional learning experiences designed to foster teachers' understanding of the effective mathematics teaching practices and their ability to apply those practices in their own classrooms. The book describes what each teaching practice would look like in an elementary school classroom, with narrative cases, classroom videos, and real student work, to bring the practices to life. Two types of activities run throughout the book: Analyzing Teaching and Learning, in which tasks or situations are presented to the reader to consider, work out, and reflect on, and Taking Action in Your Classroom, in which concrete suggestions are provided for exploring specific teaching practices in the classroom. Tools, such as a lesson plan template, a task analysis guide, and practices for orchestrating productive

discussions, are offered to assist teachers in applying the ideas discussed in the book to their own practices.

2–Jitendra, A. K., Nelson, G., Pulles, S. M., Kiss, A. J., & Houseworth, J. (2016). Is mathematical representation of problems an evidence-based strategy for students with mathematics difficulties? *Exceptional Children*, *83*(1), 8–25. https://eric.ed.gov/?id=EJ1116300
This review aimed to evaluate the quality of the research and evidence base for representation of problems as a strategy to enhance the mathematical performance of students with learning disabilities and those at risk for mathematics difficulties. The authors evaluated 25 experimental and quasi-experimental studies according to the Gersten et al. (2005) guidelines for group research studies. Results suggest that the representation of mathematical problems as a strategy is an evidence-based practice based on the criteria set by Gersten et al. Implications for research are discussed.

1–Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics*, *21*(1), 18–25.
The purpose of this article is to outline common rules and vocabulary that teachers share and elementary school students tend to overgeneralize—tips and tricks that do not promote conceptual understanding, rules that "expire" later in students' mathematics careers, or vocabulary that is not precise. In the article, the authors present 13 pervasive mathematics rules that "expire." These relate to overgeneralizing commonly accepted strategies, using imprecise vocabulary, and relying on tips and tricks that do not promote conceptual mathematical understanding, issues which can lead to misunderstanding later in students' math careers. The authors suggest that attending to these common instructional practices can support teachers to better prepare students and allow them to have smoother transitions from grade to grade.

1–Knudsen, B., Fischer, M., & Aschersleben, G. (2015). The development of Arabic digit knowledge in 4-to-7-year-old children. *Journal of Numerical Cognition*, *1*(1), 21–37.
In this study, the authors document the developmental trajectory of 4-to-7-year-olds' proficiency in accessing magnitude information from Arabic digits in five tasks differing in magnitude manipulation requirements. Results showed that children from 5 years onward accessed magnitude information implicitly and explicitly but that 5-year-olds failed to access magnitude information explicitly when numerical magnitude was contrasted with physical magnitude. Performance across tasks revealed a clear developmental trajectory: children traverse from first knowing the cardinal values of number words to recognizing Arabic digits to knowing their cardinal values and, concurrently, their ordinal position.

1–Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. (2013). Early numerical development and the role of non-symbolic and symbolic skills. *Learning and Instruction*, *25*, 95–103.

For learning math, nonsymbolic quantity skills, symbolic skills, and the mapping between number symbols and nonsymbolic quantities are all important precursors. Little is known, however, about the interrelated development of these skills. The current study focuses on numerical development by (a) investigating the structure of nonsymbolic, symbolic, and mapping skills and (b) examining the role of nonsymbolic versus symbolic numerical skills. Nonsymbolic, symbolic, and mapping skills of 69 children were assessed at ages 4, 5, and 6. Results provided evidence for (a) the developmental course of all numerical skills, showing distinguishable skills at a younger age versus an integration of skills in older children; and (b) the predominant role of symbolic skills versus the subordinate role of nonsymbolic skills in the development of mapping skills. Moreover, symbolic and mapping skills were found to be important predictors for math performance. These results provide new insights in early numerical development.

1–Kullberg, A., & Björklund, C. (2020). Preschoolers' different ways of structuring part-part-whole relations with finger patterns when solving an arithmetic task. *ZDM-Mathematics Education*, *52*, 767–778. https://doi.org/10.1007/s11858-019-01119-8
The authors studied 5-year-olds' use of finger patterns to structure number relations while solving an arithmetic problem. Children who solved the arithmetic task (3 + _ = 8) by creating a finger pattern of eight raised fingers and simultaneously identifying ("seeing") the missing part (5) on two hands (3 + [2 + 3] = 8) were more successful in solving arithmetic tasks, even in a later follow-up assessment. The authors suggest that there are three aspects that children need to discern in order to structure the task successfully in both the short and the long term: what constitutes the whole, the parts within the whole, and finger patterns as a representation of the cardinality of a set. The authors suggest that one possible reason for success in the children was due to their being able to see numbers as parts included in other numbers, which has been found in earlier research (Resnick, 1983) to be important for developing arithmetic skills. The pedagogical implications are that attention to the fact that children's ways of experiencing the number relations in arithmetic tasks provides clues as to why some children develop powerful strategies and how teachers can support children in their learning to solve arithmetic tasks.

1–Lamon, S. J. (2020). *Teaching fractions and ratios for understanding: Essential knowledge and instructional strategies for teachers, 4th Edition*. Routledge/Taylor & Francis Group.
This book helps preservice and in-service mathematics teachers build the comfort and confidence they need to begin talking to children about fractions and ratios, distilling complex ideas and translating research into usable ideas for the classroom. All of the material offered in the book has been used with students; each chapter includes children's strategies and samples of student work for teacher analysis as well as activities for practicing each thinking strategy, designed to be solved without rules or algorithms, using reasoning alone.

1–Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science*, *17*(5), 714–726.

Math relies on mastery and integration of a wide range of simpler numerical processes and concepts; numerical competencies predict variation in math ability. The authors examined the unique relations between eight basic numerical skills and early arithmetic ability in a large sample ($n = 1391$) of children across Grades 1–6. In Grades 1 and 2, children's ability to judge the relative magnitude of numerical symbols was most predictive of early arithmetic skills. The unique contribution of children's ability to assess ordinality in numerical symbols steadily increased across grades, overtaking all other predictors by Grade 6. The authors found no evidence that children's ability to judge the relative magnitude of approximate, nonsymbolic numbers was uniquely predictive of arithmetic ability at any grade. Overall, symbolic number processing was more predictive of arithmetic ability than was nonsymbolic number processing, though the relative importance of symbolic number ability appears to shift from cardinal to ordinal processing.

1, 2–Maloney, A. P., Confrey, J., & Nguyen, K. H. (2014). *Learning over time: Learning trajectories in mathematics education*. Information Age Publishing.

This book provides information on mathematics learning trajectories, which describe how students progress from prior knowledge through intermediate understandings to the mathematics target understandings. The book provides information about the research and methodology necessary for developing learning trajectories and describes their potential application in education. The book also discusses the potential of learning trajectories in contributing to coherence across classroom instruction, professional development, standards, and assessment by focusing squarely on conceptual understanding and reasoning instead of assessment-driven procedural knowledge. This book is an outgrowth of a conference on learning trajectories, hosted in 2009 at North Carolina State University, that examined research on learning trajectories. Among others, chapters of particular interest for the audience for which this scan was prepared focus on EQP, length measurement, the linkage of standards and learning trajectories, and the linkage of learning trajectories and curricula.

1, 2–McCray, J. S., Chen, J.-Q., & Sorkin, J. E. (2019) *Growing mathematical minds: Conversations between developmental psychologists and early childhood teachers*. Routledge.

This book aims to connect research and practice in early childhood mathematics (from birth to age 8) by translating research on early mathematics from developmental psychology into terms that are meaningful to teachers and readily applicable in early childhood classrooms. The book is organized to support a conversation between researchers and teachers, who each bring their expertise to bear to help address the question of how developmental psychology can improve math teaching and how math teaching can, in turn, inform developmental science. Chapters focus on using concrete objects, math anxiety, variability in children's thinking and learning, the role of gesture, math language, and pathways to basic combination fluency. The chapter "Pathways to Basic Combination Fluency in the Primary Grades," by Brownell, Hynes-Berry, and Baroody, may be particularly relevant, as it describes the shift in the meaning of basic facts mathematical fluency from fast and accurate recall to a rich understanding of numbers and numerical relationships that enable students to reason logically and solve problems.

2–Morgan, P. L., Farkas, G., & Maczuga, S. (2015). Which instructional practices most help students with and without mathematics difficulties? *Educational Evaluation and Policy Analysis*, *37*(2), 184–205.
The authors used population-based, longitudinal data to investigate the relation between mathematics instructional practices used by first-grade teachers in the United States and the mathematics achievement of their students. Factor analysis identified four types of instructional activities (teacher-directed, student-centered, manipulatives/calculators, movement/music) and eight types of specific skills taught (for example, adding two-digit numbers). First-grade students were then classified into five groups on the basis of their fall and/or spring kindergarten mathematics achievement—three groups with mathematics difficulties (MD) and two without MD. Regression analysis indicated that a higher percentage of MD students in the first-grade classrooms were associated with greater use by teachers of manipulatives/calculators and movement/music to teach mathematics. Yet follow-up analysis for each of the MD and non-MD groups indicated that only teacher-directed instruction was significantly associated with the achievement of students with MD (covariate-adjusted effect sizes [ESs] = .05–.07). The largest predicted effect for a specific instructional practice was for routine practice and drill. In contrast, for both groups of non-MD students, teacher-directed and student-centered instruction had approximately equal, statistically significant positive predicted effects (covariate-adjusted ESs = .03–.04). First-grade teachers in the United States may need to increase their use of teacher-directed instruction if they are to raise the mathematics achievement of students with MD.

1–Moss, J., Hawes, Z., Caswell, B., Naqvi, S., Bruce, C., & Flynn, T. (2014). Changing perceptions of young children's geometry and spatial reasoning: Lessons from the Math for Young Children Project (Research forum: Spatial reasoning for young learners, N. Sinclair & C. Bruce, Eds.) In P. Lillidalj (Ed.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education* (pp. 7–9). Vancouver, Canada: PME. Available at: www.pme38.com/wp-content/uploads/2014/05/RF-Sinclair-et-al.pdf
This paper presents results from a small study conducted to help demonstrate that young children—regardless of socioeconomic status (SES) background—are capable of exceeding current expectations in geometry and spatial reasoning given carefully crafted learning experiences. Researchers describe work with a team of eight teachers and their kindergarten and Grade 1 students from a large urban low-SES school, presenting information about the design, implementation and results of two lesson

study "Research Lessons," both of which involved knowledge and geometric reasoning well beyond curriculum expectations.

2–National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*.
This book sets forth a set of strongly recommended, research-informed actions to support the implementation of high-quality mathematics education. The book illustrates eight Mathematics Teaching Practices that research indicates need to be consistent components of every mathematics lesson and provides a set of productive and unproductive beliefs for each practice.

1, 2–National Council of Teachers of Mathematics. (2020). *Catalyzing change in early childhood and elementary mathematics: Initiating critical conversations*.
This book, part of a three-book series, focuses on policies, practices, and issues that impact mathematics education. It describes mathematical strengths and needs of young children to be considered when addressing the continuity and alignment of mathematics education for young learners. The book emphasizes four critical challenges:

- broadening the purpose of school mathematics to prioritize development of deep conceptual understanding so that children experience joy and confidence in themselves as emerging mathematicians
- dismantling structural obstacles that stand in the way of mathematics working for each and every student
- implementing equitable instructional practices to cultivate students' positive mathematical identities and a strong sense of agency
- organizing mathematics along a common shared pathway grounded in the use of mathematical practices and processes to coherently develop a strong foundation of deep mathematical understanding for each and every child

2–National Council of Teachers of Mathematics. (2023). *Procedural fluency: Reasoning and decision-making, not rote application of procedures position*.
https://www.nctm.org/uploadedFiles/Standards_and_Positions/Position_Statements/PROCEDURAL_FLUENCY.pdf
This position statement presents four declarations related to actions needed to ensure that every student has access to and develops procedural fluency, which the authors argue is an essential component of equitable teaching and is necessary to developing mathematical proficiency and mathematical agency. The declarations are that each and every student must have access to teaching that connects concepts to procedures, explicitly develops a reasonable repertoire of strategies and algorithms, provides substantial opportunities for students to learn to choose from among the strategies and algorithms in their repertoire, and implements assessment practices that attend to all components of fluency. These declarations apply to computational fluency across the K–12 curriculum, including basic facts, multidigit whole numbers, and rational

numbers, as well as to other procedures throughout the curriculum, such as comparing fractions, solving proportions or equations, and analyzing geometric transformations.

1, 2–Nelson, G., & McMaster, K. L. (2019). The effects of early numeracy interventions for students in preschool and early elementary: A meta-analysis. *Journal of Educational Psychology*, *111*(6), 1001–1022. https://doi.org/10.1037/edu0000334
This meta-analysis study examined the effectiveness of early numeracy interventions for young students, including students with disabilities or those at risk for MD. The study evaluated preschool, kindergarten, and first-grade interventions on early numeracy content, instructional features, and methodological components that improved students' math achievement. A total of 34 studies met inclusion criteria for the meta-analysis, based on the review of studies published from January 1980 through June 2016. The average weighted effect size for numeracy interventions with two outliers removed was moderate (*g* □ 0.64), and the 95% confidence interval did not include zero [0.52, 0.76]; in other words, the interventions were moderately effective. Results of the final metaregression model predicted larger treatment effects for interventions that included counting with 1-to-1 correspondence and were 8 weeks or shorter in duration. Metaregression results also showed that, on average, interventions were less effective for students with higher levels of risk for MD according to screening criteria and risk according to low socioeconomic status compared to typically achieving students. Interventions with students in preschool or kindergarten yielded larger study effects than did interventions with students in first grade, a finding which could be explained by the lack of exposure to regular math instruction. Interventions that featured explicit inclusion of math vocabulary produced large effects, while interventions that did not include vocabulary yielded slightly smaller and moderate effects. Additionally, studies that included treatment groups who received small group instruction (moderate-to-large treatment effects) or one-to-one instruction (moderate) produced larger effects than those including treatment groups who received peer-assisted interventions (small). Contrary to prior studies (Gersten et al., 2009; Jitendra et al., 2016; Mononen et al., 2015), the authors did not find a significant effect for the use of a concrete-representational-abstract (CRA) framework, although the results may be attributed to shorter duration preschool interventions that may not allow for teachers to incorporate a CRA approach to teaching math concepts. Similarly, in contrast to prior studies, the authors did not find significant effects for interventions that use explicit and systematic instruction (e.g., teacher modeling, guided practice, corrective feedback).

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly*, *36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003
In an effort to promote best practices regarding mathematics teaching and learning at the preschool level, national advisory panels and organizations have emphasized the importance of children's emergent counting and related competencies, such as the ability to verbally count, maintain 1-to-1 correspondence, count with cardinality,

subitize, and count forward or backward from a given number. However, little research has investigated whether the kind of mathematical knowledge promoted by the various standards documents actually predict later mathematics achievement. This study uses longitudinal data primarily of children from low-income and minority backgrounds to examine the extent to which preschool mathematical competencies, specifically basic and advanced counting, predict fifth-grade mathematics achievement. Using regression analyses, the authors find early numeracy abilities to be the strongest predictors of later mathematics achievement, with advanced counting competencies more predictive than basic counting competencies. The results highlight the significance of preschool mathematics knowledge for future academic achievement.

3–Rebora, A. (2021, December 6). Zaretta Hammond on equity and student engagement. *Educational Leadership*, *79*(4). https://www.ascd.org/el/articles/zaretta-hammond-on-equity-and-student-engagement
This author interview focuses on the role of student engagement in school equity efforts and the importance avoiding a "pedagogy of compliance" for struggling students.

2–REL Appalachia. (2021). *What are interventions or components of interventions that promote early numeracy skills for students performing below grade-level benchmarks?* Institute of Education Sciences, U.S. Department of Education. https://ies.ed.gov/ncee/rel/Products/Region/appalachia/Ask-A-REL/-89711+
This resource presents the results of a REL Reference Desk regarding evidence-based information about interventions or components of interventions that promote early numeracy skills for students performing below grade-level benchmarks. The resource provides references, referrals, and brief responses in the form of citations in response to questions about available education research. The authors searched for peer-reviewed articles and other research reports on interventions focused on early numeracy. We focused on identifying resources that specifically addressed the effects of various interventions on early numeracy skills for students performing below grade-level benchmarks. The sources included ERIC and other federally funded databases and organizations, research institutions, academic research databases, and general internet search engines.

1–Starkey, P., Klein, A., Clarke, B., Baker, S., & Thomas, J. (2022). Effects of early mathematics intervention for low-SES pre-kindergarten and kindergarten students: A replication study. *Educational Research and Evaluation*, *27*(1-2), 61–82.
An SES-related achievement gap in mathematics emerges prior to school entry and increases in elementary school. This gap makes implementation of demanding mathematics standards (e.g., the Common Core State Standards) an ongoing challenge. Early educational intervention is a strategy for addressing this challenge. A randomized controlled trial was conducted in public American preschools to (a) replicate the efficacy of an intervention, Pre-K Mathematics, for children of low SES and (b) test the combined impact of this intervention and a Common Core–aligned

kindergarten intervention, Early Learning in Mathematics. Forty-one clusters of prekindergarten and kindergarten classrooms, containing a sample of 389 children of low SES from an agricultural region, were randomly assigned to treatment and control conditions. The original impact findings were replicated: Child mathematics outcomes in prekindergarten were positive and significant. Gains were maintained in kindergarten. Thus, the gap can be reduced and gains maintained by sustained early intervention.

Stipek, D., & Valentino, R. A. (2014). Early childhood memory and attention as predictors of academic growth trajectories. *Journal of Educational Psychology*, 1–18.
Longitudinal data from the children of the National Longitudinal Survey of Youth (NLSY) were used to assess how well measures of short-term and working memory and attention in early childhood predicted longitudinal growth trajectories in mathematics and reading comprehension. Analyses also examined whether changes in memory and attention were more strongly predictive of changes in academic skills in early childhood than in later childhood. All predictors were significantly associated with academic achievement and years of schooling attained, although the latter was at least partially mediated by predictors' effect on academic achievement in adolescence. The relationship of working memory and attention with academic outcomes was also found to be strong and positive in early childhood but nonsignificant or small and negative in later years. The study results provide support for a "fade-out" hypothesis, which suggests that underlying cognitive capacities predict learning in the early elementary grades, but the relationship fades by late elementary school. The findings suggest that whereas efforts to develop attention and memory may improve academic achievement in the early grades, in the later grades interventions that focus directly on subject matter learning are more likely to improve achievement.

1–Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2014). Finding the missing piece: Blocks, puzzles, and shapes fuel school readiness. *Trends in Neuroscience and Education*, *7*(1), 7–13.
Experiences with spatial toys such as blocks, puzzles, and shape games and the spatial words and gestures they evoke from adults have a significant influence on the early development of spatial skills. Spatial skills are important for success in science, technology, engineering, and mathematics (STEM) fields and are related to early mathematics performance as early as age 3. This paper focuses on the effects of early spatial experiences and their impacts on school readiness, discusses factors that influence the amount and quality of spatial play, and suggests methods for providing a "spatial education" prior to school entry

Watts, T. W., Duncan, G. J., Clements, D. H., & Sarama, J. (2018). What is the long-run impact of learning mathematics during preschool? *Child Development*, *89*(2), 539–555. https://doi.org/10.1111/cdev.12713

The current study estimated the causal links between preschool mathematics learning and late elementary school mathematics achievement using variation in treatment assignment to an early mathematics intervention as an instrument for preschool mathematics change. Estimates indicate (*n* = 410) that a standard deviation of intervention-produced change at age 4 is associated with a 0.24-SD gain in achievement in late elementary school. This impact is approximately half the size of the association produced by correlational models relating later achievement to preschool math change and is approximately 35% smaller than the effect reported by highly controlled ordinary least squares (OLS) regression models using national data sets. Implications for developmental theory and practice are discussed.

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