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This material has been prepared to provide access to information and to encourage discussion that can inform research, policy, and practice. The information contained in this document should not be used in isolation to reach definitive conclusions. REL Northwest staff are available to facilitate discussion, to provide further relevant information, and, in some cases, to partner on research to build an increasingly solid body of knowledge.

Handout C: Literature Scan – High School Mathematics Pathways

Review Summary

The Montana Office of Public Instruction (OPI) developed state-level content standards for mathematics in 2011, which they now seek to revise and update. OPI asked REL Northwest to provide information from recent research about how to build a rigorous mathematics program incorporating effective, modern high school mathematics course pathways that prepare secondary students for postsecondary success. To respond to this OPI request, this report presents five themes related to high school mathematics pathways, drawing primarily on recent empirical research and policy documents (2013 and later) as well as relevant historical and policy/guidance documents. Findings from the review are as follows:

Theme 1: What are the features of the siloed Algebra I–Geometry–Algebra II high school mathematics model to which many states and districts default, and what does the research tell us regarding its impact and ability to serve a diverse student body and a wide range of postsecondary pursuits? (p. 7-12)

- The current default model is generally fewer than four years of high school mathematics. The required sequence typically includes Algebra II and student tracking.
- Under the current default model, most students are not reaching proficiency or having positive experiences with math. Many students leave high school underprepared to reach college and career goals.
- The current default model results in unintended and undesirable impacts for students, teachers, and schools.

Theme 2: To what extent are various U.S. stakeholder groups open to and engaging in reform to secondary and postsecondary mathematics pathways reform? (p. 13-16)

- Many professional organizations, math educational researchers, workplace advocates, and Montana leaders are calling for changes in mathematics requirements and pathways at the high school and postsecondary levels.

Theme 3: What are the key recommendations in the literature for creating high school mathematics pathways likely to serve a diverse student body effectively? (p. 17-26)

- Require four years of rich, rigorous mathematics for all students.
- Provide all students with a common grounding in essential mathematics that prepares them for a variety of majors and careers.

- Include opportunities for students to pursue mathematics that meet their unique needs, interests, and postsecondary goals.
- Offer flexibility and the ability to move between pathways if necessary.
- Include high-quality student supports to promote learning and academic success in mathematics for all students.
- Provide opportunities for highly motivated and enthusiastic students to satisfy their mathematical curiosity.
- Include appropriate ongoing support for teachers to successfully implement, improve, and sustain modern mathematics pathways.
- Include comprehensive student advising to guide students to the appropriate mathematics pathway for their postsecondary interests.
- Establish collaboration efforts between secondary and postsecondary institutions.
- Design and implement measures of success.

Theme 4: What common high school mathematics pathway models and structures are currently being implemented, in the United States or elsewhere? (p. 27-36)

- Siloed mathematics models
    - Geometry first
    - Geometry and algebra concurrent
    - Streamlined siloed
- Conceptually integrated mathematics models
- Creating Algebra II–equivalent courses and year three/four course sequences
- Courses/sequences for targeted "Beyond Essential Concepts"
- Transition courses
- Course substitutions

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Organization of the review

To respond to this OPI request, this report presents five themes related to high school mathematics pathways, drawing primarily on recent empirical research and policy documents (2013 and later) as well as relevant historical and policy/guidance documents.

These themes, stated as research questions that drove our literature review, are:

1. What are the features of the siloed Algebra I–Geometry–Algebra II high school mathematics model to which many states and districts default, and what does the research tell us regarding its impact and ability to serve a diverse student body with regard to a wide range of postsecondary pursuits? (p. 7-12)

2. To what extent are various U.S. stakeholder groups open to and engaging in reform to secondary and postsecondary mathematics pathways? (p. 13-16)

3. What are the key recommendations in the literature for creating high school mathematics pathways likely to serve a diverse student body effectively? (p. 17-26)

4. What common high school mathematics pathway models are currently being implemented, in the United States or elsewhere? (p. 27-36)

The following section, “What was learned in the review,” summarizes information for each of these themes. Following these summaries, we provide a full bibliography for all works referenced within the document. In Appendix A, we provide information about the methodology used to identify the included sources, and Appendix B includes an annotated bibliography for resources from 2013 to the present.

Why this review?

The Montana Office of Public Instruction (OPI) developed state-level content standards for mathematics in 2011, which they now seek to revise and update. OPI asked REL Northwest to summarize recent research about how to build a rigorous mathematics program incorporating effective, modern high school mathematics course pathways that prepare secondary students for postsecondary success.

Introduction

> *High school students come from very diverse backgrounds with respect to culture, experience, language, socioeconomic status, personal and professional goals, and postsecondary educational goals. A high school mathematics program must be designed to give this diverse set of students a mathematics education that will not only prepare them for the next steps in their future but also give them an appreciation of what mathematics is and how it can be useful in their lives, no matter what their current post–high school plans are. To maximize students' opportunities after high school and prepare them to actively engage in democratic society, high schools should ensure that all students enroll in a mathematics course every year in high school and complete four years of high school mathematics, including a mathematics or statistics course during their last year of high school.* (National Council of Teachers of Mathematics, 2018, p. 83)

In recent years, state and local educators, policymakers, and researchers have increasingly found themselves asking, "What mathematics should secondary students be learning? What ideas and topics in the current high school curriculum are relevant and support students' long-term success and goals? And what mathematical knowledge, skills, and ideas critical for personal and professional success and fulfillment in the twenty-first century are currently being neglected in most students' secondary experience?"

Many states and school districts have increased high school graduation requirements in mathematics in the last decade to prepare students for college and career (Dossey et al., 2016). Other purposes, such as teaching students to think critically, participate actively in civic life, and appreciate mathematics as a cultural and recreational pursuit, have frequently been given short shrift (Tate, 2013). Assumptions about the role and purpose of secondary mathematics are critical to surface and make explicit, as such assumptions can and do have significant implications for curriculum content and pedagogy (Ernest, 1991; Huckstep, 2000; Keitel, 2015), and can lead to different curricula for groups of students who are perceived to have different futures (Barton & Coley, 2011; Darling-Hammond, 2015; Knudson, 2019).

In its recent landmark publication *Catalyzing Change in High School Mathematics: Initiating Critical Conversations,* the National Council of Teachers of Mathematics (NCTM) (2018) has called on educators, school leaders, researchers, policymakers, parents, and other stakeholders to view the purpose of high school mathematics through a lens of *empowerment.* Specifically, NCTM recommends that high school mathematics should empower students to:

- **Expand professional opportunities.** A strong high school mathematics education opens doors to expanded professional opportunities. Students and parents expect rigorous high school math to lead to wider postsecondary options and expanded career options (D'Ambrosio, 2012; Jahn & Myers, 2014; Jiang et al., 2020). As careers in science, technology, engineering, and mathematics (STEM) require a strong mathematics background, policymakers and business leaders advocate for additional pathways in addition to educators, parents, and students (Gonzalez &

Kuenzi, 2012; National Research Council 2011a, 2011b, 2013a, 2013b; President's Council of Advisors on Science and Technology, 2012).

- **Understand and critique the world.** Students should leave high school with quantitative literacy and critical thinking skills necessary to determine the validity of claims made in scientific, economic, social, and political arenas (Berry & Lawson, 2019; Brelias, 2015). Math coursework that engages with meaningful topics such as analyzing public health problems, income inequality, and environmental sustainability can provide students with "access to rich, rigorous mathematics that offers opportunities and self-empowerment for them to understand and use mathematics in their world" (Stinson & Wager, 2012, p. 10).
- **Experience the wonder, joy, and beauty of mathematics.** In addition to understanding the utility and power of mathematics for solving problems, students should also have opportunities to appreciate the beauty of mathematics and experience the wonder of deep mathematical structure and symmetry. Like art and music, mathematics is a human cultural and recreational activity practiced in every civilization on earth with a history going back thousands of years, and students deserve rich access to it as part of their human heritage (Barta et al., 2014; Desai et al., 2021; Russell, 2019).

When school math emphasizes these multiple purposes, students are prepared to "flourish as human beings," no matter what paths they take in life or what profession they choose (Su, 2017, p. 483). In this review, we use these three purposes as a frame for exploring five major themes related to high school mathematics course pathways. To quote NCTM, a well-organized, multipurpose mathematics curriculum "plays a critical role in the cultivation of students who become fully engaged members of democratic society, who contribute to society in positive ways, and who become human beings capable of achieving their full potential, personally and professionally, through the intellectual experiences of their mathematics education" (2018, p. 13).

Theme 1: What are the features of the siloed Algebra I–Geometry–Algebra II high school mathematics model to which many states and districts default, and what does the research tell us regarding its impact and ability to serve a diverse student body and a wide range of postsecondary pursuits?

Background

Most other nations use a sequence of curriculum and instruction that emphasizes the connections between areas of mathematics (Hodgen et al., 2010). In contrast, the siloed Algebra I–Geometry–Algebra II sequence has remained the default course pathway at more than 90 percent of high schools in the United States for over a hundred years (Mackenzie, 1894; Dossey et al., 2016; Klein, 2002). The 1958 National Defense Education Act helped to cement a functions-based, calculus-driven mathematics curriculum as the default path for nearly all students (Klein, 2002; Waggener, 1996). The system has remained largely unchanged due to culture and tradition; postsecondary admission policies; placement and alignment mismatches between secondary and postsecondary institutions; and secondary teacher preparation curricula.

The current default model is generally fewer than four years of mathematics and frequently includes Algebra II and student tracking.

- As of spring 2023, 17 states and the District of Columbia now require four years of mathematics, 27 require three years, and three—California, Maine, and Montana—require only two years (California Department of Education, 2021; Maine Department of Education, 2019; Montana Secretary of State, 2013)[1],[2].
- Though states differ on specific requirements, Algebra II has become a de facto requirement for nearly all students due to real or perceived college admissions requirements (Daro & Asturias, 2019; National Science Board, 2018). College admission standards that require Algebra II signal to high schools, parents, and students that courses leading to calculus are the best, or only, mathematics options to pursue in preparation for college.
- Over the last four decades, students have increasingly enrolled in Algebra II (Loveless, 2013). However, over that time, students' mathematics knowledge and skills have remained stagnant as measured by National Assessment of Educational Progress (NAEP) mathematics scores.

[1] In the remaining states, either graduation requirements are set by Local Education Agencies (LEAs) or the state uses proficiency-based graduation requirements (Colorado Department of Education, 2023; Massachusetts Department of Elementary and Secondary Education, 2023; Pennsylvania Department of Education, 2018).

[2] Minimum requirements; some districts may set graduation requirements that exceed state minimums.

- Requirements for Algebra II have become unwieldy and incoherent for both teachers and students, with too much content and too little focus (Daro & Asturias, 2019). Students who struggle with algebra are often discouraged from postsecondary study and miss out on valuable and relevant aspects of mathematics (Daro & Asturias, 2019; Getz et al., 2016).
- Most high schools track students in mathematics (Antonovics et al., 2022; Loveless, 2013). Tracking places students into remedial or advanced classes, usually with little chance of shifting between such classes (Reis & Renzulli, 2010), and places students at the same grade into different courses with different content based on perceived ability (Schmidt, 2012).

The current default model does not reliably prepare students for college and career goals.

- Enrolling in Algebra II fails to predict postsecondary degree attainment for individuals (Kim et al., 2015), mathematics scores more broadly (National Center for Education Statistics, 2013, 2015; Organisation for Economic Co-operation and Development [OECD] 2016b, 2018a, 2018b), or preparation for credit-bearing college mathematics courses (Bailey et al., 2010; Complete College America [CCA], 2012; Chen, 2016; Dana Center, 2020).
- Though the calculus-oriented pathway aimed to create a STEM pipeline, only 25 percent of students who take Calculus in high school do so because they enjoy mathematics, want to learn more, or wish to take more advanced mathematics courses. The vast majority, 75 percent of students, only take Calculus for college applications or to fit in with their cohort peer groups (Bressoud, 2017).
- The default pathway fails to prepare most students who take high school Calculus to succeed in college-level mathematics. Only one in five students who takes Calculus in high school successfully skips Calculus I in college. Approximately 30 percent of high school Calculus students retake the course in college, and one in three gets a C or lower when they do. More than 35 percent of students who successfully complete high school Calculus are enrolled via placement exam in a precalculus, college algebra, or even a remedial mathematics course (Bressoud, 2017).
- Tracking may funnel students out of the Algebra I–Geometry–Algebra II pathway as early as 6th grade, with less than 4 percent of students completing the full course sequence by the end of high school (Daro & Asturias, 2019). Most students retake Calculus or a lower-level course once they reach college (Bressoud, 2017).
- Black and Hispanic students, students from low-income families, students at rural schools, and students with low educational expectations are less likely to take higher-level mathematics courses than their counterparts, and school socioeconomic level is negatively related to the likelihood of taking higher-level mathematics courses (Cha, 2015; Niederle & Vesterlund, 2010).

The current default model does not reliably equip students with the mathematics they need to understand and critique the world and participate meaningfully in civic life.

- U.S. young adults (ages 16–34) lack not only quantitative and problem-solving skills necessary for success in the workplace and postsecondary education but also the numeracy[3] and problem-solving skills necessary for "meaningful participation in our democratic institutions" (Goodman et al., 2015, p. 5).
- These barriers tend to disproportionately affect those holding marginalized identities, including Black and Indigenous people of color, women and girls, and those from low-income backgrounds (Aguirre et al. 2013, 2017; Ngo & Vazquez, 2020). The result of this differentiated access to postsecondary opportunities is to reproduce existing societal inequities along race, gender, and class lines (Carnevale & Strohl, 2010).

The current default model does not reliably provide opportunities to experience the beauty, joy, and wonder of mathematics.

- Math anxiety is rampant in the U.S. teen and adult populations (Beilock & Willingham, 2014; Boaler, 2008; Dowker et al., 2016; Geist, 2010), and most high school and college graduates see mathematics as difficult (Brown et al., 2008) and less accessible than other academic domains due to their beliefs about fixed mathematics ability levels (Meyer et al., 2015).
- Many elementary students in the United States enter school with positive attitudes toward mathematics but fail to maintain these attitudes as they progress through the grades (Cotton, 2004).
- A sense of belonging and positive math identity play a key role in students' mathematics success (Berry et al., 2014; Cheryan et al., 2015; Hyde & Mertz, 2009; Leyva et al., 2021a; Leyva et al., 2021b), yet women and girls (Cundiff et al, 2013; Gayles & Ampaw, 2014) and students of color (National Center for Science and Engineering Statistics, 2017; Palmer et al., 2011; Chen & Soldner, 2013), in particular, abandon mathematics coursework and career paths at higher rates than their peers due to stereotype threat[4] and lack of feelings of belonging in mathematics and related disciplines (Mansfield et al., 2014; Beasley & Fischer, 2012).

The current default model may create systemic inequities and opportunity gaps.

- School and district policies aim to place students in different mathematics tracks based on academic factors. However, in practice, students' perceived ability/readiness

[3] Goodman et al. defined numeracy as "the ability to access, use, interpret, and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life" (p. 46).

[4] Stereotype threat is defined as "being at risk of confirming, as self-characteristic, a negative stereotype about one's group" (Steele & Aronson, 1995, p. 797). According to stereotype threat, members of a marginalized group acknowledge that a negative stereotype exists in reference to their group, and they demonstrate apprehension about confirming the negative stereotype by engaging in particular activities. In mathematics, girls, women, and Black and Indigenous people of color are known to be impacted by stereotypes that people with these identities have less mathematics ability than other groups.

and placements often reflect nonacademic factors such as race, socioeconomic status, gender, or language rather than mathematics performance (Gutiérrez, 2002; Lawyers' Committee for Civil Rights of the San Francisco Bay Area, 2013; Mosqueda, 2010; Stiff et al., 2011; Thompson, 2017). As a result, students with marginalized identities have less access to "advanced" mathematics tracks and more rigorous coursework (Flores, 2007; Oakes, 2005; Riegle-Crumb & Grodsky, 2010; Siegle et al., 2016; Tyson, 2006).

- Differences in mathematical success between marginalized and non-marginalized populations (the "achievement gap") may result from systemic structures that offer students with marginalized identities fewer and lower-quality learning opportunities. The "achievement gap" may be more accurately described as an opportunity gap (Flores, 2007; Welner & Carter, 2013) created when students are placed into low-level courses and dead-end learning pathways with less effective instruction (e.g., Hallinan et al., 2003).
- Tracking *students* often results in the tracking of *teachers* as well. In one state, 70 percent of teachers were tracked by course level (e.g., Algebra I versus Algebra II versus Precalculus), course track (e.g., Honors or College Prep versus mainstream), or both (Nirode & Boyd, 2023). Teachers with the least experience were most likely to be assigned to low-track or entry-level courses, whereas the most experienced teachers were most likely to be assigned to higher-track or upper-level courses (Darling-Hammond, 2007; Nirode & Boyd, 2023; Strutchens et al., 2011). Key measures of teacher quality—experience, licensure exam scores, and value added—are inequitably distributed across numerous indicators of student disadvantage (free/reduced-price lunch status, underrepresented minority status, and low prior academic performance) (Goldhaber et al., 2015).

Tracking may create unproductive beliefs and dispositions toward mathematics.

- Tracking models implicitly assume a student's math ability is fixed rather than something they can develop. Students in the "top" track typically experience mathematics instruction that cultivates their mathematical identities, conceptual understanding, and critical problem-solving and thinking skills. Students placed in the "low" track tend to receive more deficit-based instruction focused on memorizing and reproducing rote procedures, with little or no attention to mathematical identities, beliefs and attitudes about mathematics, or their own mathematical abilities and potential (Boaler et al., 2000; Oakes, 2005; Thompson & O'Quinn, 2001).
- Otherwise capable students placed into low math tracks frequently show a decrease in their mathematics self-efficacy and an increase in mathematics anxiety (Akos et al., 2007; Boaler, 2011; Callahan 2005).
- Because moving from a lower track into a higher track is difficult if not impossible (O'Connor et al., 2007; Stiff & Johnson, 2011; Stiff et al., 2011), tracking ensures that students receive the same type of instruction year after year, producing long-term negative effects on both learning outcomes and students' mathematical identities and self-concept (Stiff & Johnson 2011; Stiff et al., 2011). Over time disparities in learning outcomes increase (Wheelock 1992; O'Connor et al., 2007*)*, and low

expectations placed on students in lower tracks often become self-fulfilling prophecies (Akos et al., 2007; Callahan, 2005; Flores, 2007; Montt, 2011; Soni & Kumari, 2015).

The current default model leads to fewer students enrolled in fourth-year mathematics courses.

- The number of mathematics course taken in high school predicts students' college enrollment generally, four-year college enrollment specifically (Aughinbaugh, 2012; Byun et al., 2015; Long et al., 2012), and adulthood earnings (Goodman, 2019; Joensen & Nielsen, 2009). However, many U.S. students opt out of high school mathematics courses as soon as possible (e.g., Asim et al., 2019).

The current default model leads to fewer students meeting graduation requirements and gaining access to educational and career opportunities of interest.

- Rather than increasing the number of college- and career-ready students, "Algebra II for All" policies in the 2000 became a barrier to graduation for many (Ohio Department of Education, 2022).
- Even for high school graduates, courses like Algebra II and Precalculus serve a gatekeeper function in terms of who can gain access to postsecondary education opportunities, particularly selective colleges and sought-after majors (Douglas & Attewell, 2017).
- As workplace skills rarely require Algebra II or Precalculus mathematics, students are unnecessarily being denied educational and career opportunities (Bromberg & Theokas, 2016; Douglas & Attewell, 2017).

The current default model leads to proliferation of non-rigorous mathematics courses presented as rigorous algebra courses.

- To address the "Algebra II for All" challenge, many schools began offering different "flavors" of Algebra II (e.g., Honors Algebra II, Algebra II Part A) covering a wide range of different algebra topics with different levels of rigor (Patchnowski & Cannelongo, 2021), rendering the course "Algebra II" on a student's high school transcript nearly meaningless (Ohio Department of Education, 2022).

Students currently may take Calculus in high school or "rushing to Calculus" regardless of whether the course aligned with their interests and goals.

- The default Algebra I–Geometry–Algebra II pathway is an inefficient use of resources in terms of time, teacher allocation, and student placement. Students frequently repeat courses without succeeding (Finkelstein et al., 2012). Technology replaces the need for skills from calculus-driven courses, limiting the relevance of typical advanced algebra and calculus courses for today's STEM students in terms of career preparation (Madison & Steen, 2009; Packer, 2003; Trefil, 2008).

- The perception of the default, Calculus-focused pathway as the most rigorous and the best preparation for college frequently pressures secondary students to accelerate through the course sequence to take Calculus as quickly as possible.
- For many students, this results in students failing to deeply master more basic mathematics such as algebra and trigonometry, and completing high school Calculus does not necessarily translate to success in university-level Calculus or even indicate readiness for entry-level college mathematics (Bressoud et al., 2015).

Theme 2: To what extent are various U.S. stakeholder groups open to and engaging in reform to secondary and postsecondary mathematics pathways reform?

There is general agreement about the need for changes in mathematics requirements and pathways at the postsecondary level.

- As of fall 2015, 58 percent of U.S. community colleges had reported implementing new mathematics course sequences (Blair et al., 2018), and as of 2018, more than 15 states were broadly implementing such novel pathways (Dana Center, 2019). We continue to see widespread implementation of multiple pathways that offer differentiated, rigorous options tailored to students' academic and career goals (Carnegie Math Pathways, 2023; Daro & Asturias, 2019; Dana Center, 2022; Smith & Funk, 2021; TPSEMath, 2021).
- Higher education mathematics professional associations[5] agree that most students are not well served by traditional college algebra and are calling for "mathematically substantive options for students who are not headed to calculus" focusing on problem solving, modeling, statistics, and applications aligned to students' intended fields of study (Saxe & Braddy, 2015).[6]
- Many states are also now working to remove Algebra II and College Algebra requirements in favor of other, more relevant options and to create pathways that align better with more students' chosen career paths (Barnett et al., 2022; Burdman, 2015; University System of Maryland, n.d.; Dana Center, 2018).
- More institutions of higher education are beginning to implement corequisite models that enable more students to enter immediately into college-credit-bearing courses with support (Boatman, 2021; Daugherty et al., 2021; Douglas et al., 2022; Ran & Lin, 2019).
- More and more colleges and universities are using multiple measures for college placement that consider factors other than placement tests, such as student performance in relevant high school coursework (Education Commission of the States, 2018). These modern approaches are drastically increasing student success across demographic groups (Henson et al., 2017; Long Beach Community College, n.d.).

[5] E.g., the Mathematical Association of America, the American Mathematical Association of Two-Year Colleges, the American Mathematical Society, the American Statistical Association, and the Society for Industrial and Applied Mathematics.

[6] This statement should not be misinterpreted as suggesting that only students who plan to take Calculus need algebra; the authors agree that algebraic reasoning and basic algebraic skills are critical foundations for many areas of mathematics, including data science and modeling. They instead suggest that educators need to consider which algebraic content and skills are necessary for different pathways and programs, rather than using mathematics as a sorting mechanism for college and program entry (Saxe & Braddy, 2015).

Professional organizations are calling for change in high school mathematics pathways.

- Nearly every major mathematical society and organization has stated that curriculum change in high school—and college—mathematics is desperately needed and long overdue.
    - **American Mathematical Association of Two-Year Colleges (AMATYC):** "The equivalent content in intermediate algebra courses is not required to master the content for most college-level mathematics courses that do not lead to calculus" (AMATYC, 2014, Statement 4).
    - **Mathematical Association of America:** "The current attention to big data and the demand for college graduates with data skills should prompt changes in our entry-level courses. Thus, there is a call to provide mathematically substantive options for students who are not headed to calculus. These entry courses should focus on problem solving, modeling, statistics and applications" (Mathematical Association of America, 2015, p. 13).
    - **National Council of Teachers of Mathematics:** "The evidence overall suggests that the status quo with respect to learning outcomes from high school mathematics is unacceptable. Everyone involved in the mathematics education of high school students must be committed to ensuring that each and every student has the opportunity to learn the mathematics necessary to be well prepared for whatever the future may hold for his or her educational, professional, and personal lives" (NCTM, 2018, p. 3).

In *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018), the authors outline several pressing problems with high school mathematics curricula and lay out a clear position regarding high school mathematics pathways, including a number of recommendations we discuss further in Theme 3.

Math education researchers are calling for change in high school and postsecondary mathematics pathways.

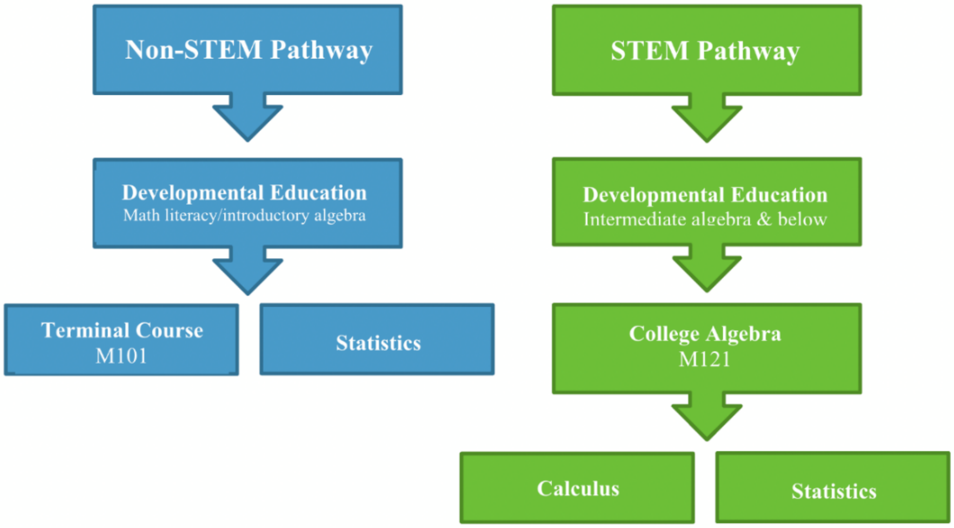
- Researchers at the Dana Center are calling on secondary education systems to collaborate with postsecondary partners to offer aligned mathematics pathways, and to develop and adopt course sequences for all third- and fourth-year high school students that situate statistics, quantitative reasoning, and/or computational thinking, alongside algebra-dominant sequences that prepare students for calculus, that are aligned to students' postsecondary and career goals (Dana Center, 2020).
- In their report *Branching Out: Designing High School Math Pathways for Equity,* researchers and highly respected leaders in the math education field Phil Daro and Harold Asturias (2019) describe how and why the current default mathematics pathways do not support most students' interests and career goals, particularly those pursuing careers in fields like law, politics, design, and media.
- In 2022, noted secondary mathematics researchers Eric Milou and Steve Leinwand (2022) called for similar changes in high school mathematics, stating that the default high school mathematics curriculum is not meeting the needs of most students and that "the need for change in high school mathematics has never been greater" (p. e1).

Workplace advocates and organizations are calling for change in high school and postsecondary mathematics pathways.

- A 2015 survey of manufacturing executives (Deloitte & the Manufacturing Institute) identified a growing talent gap that could result in millions of positions going unfilled; math and problem-solving skills were noted as one of the most serious skill deficiencies.
- Employer participants in the National Science Foundation–sponsored 2018 "Needed Math Conference" reported that those they hire for technical positions in STEM fields are frequently not prepared to problem-solve on the job (Hofstra University Center for STEM Research, 2018).
- A 2018 survey of human resources professionals reported that job candidates more often lack technical skills than soft skills, and of the top three missing technical skills named by survey respondents (human resources professionals), data science and data analysis skills were named second (Society for Human Resource Management, 2019).
- Many new jobs (Rieley, 2018) specialize in working with data, such as positions in the data sciences and data analytics. In many existing fields such as social work and nursing, data-driven analysis and mathematical modeling are becoming increasingly key in terms of informing research and decision making (Hughes et al., 2019a, 2019b).
- A recent analysis of more than 150 million unique U.S. job postings since 2007 yielded 14 skills that have become "foundational" in the new economy, including analyzing, managing, and communicating data—mathematical, quantitative skills that are frequently given short shrift in the typical algebra-to-calculus pathway, if they are taught at all (Business-Higher Education Forum & Burning Glass Technologies, 2018).
- Other reports suggest that in the next few years, the nation will see a significant increase of job openings requiring data and analytics skills, and that there will be a shortage of job seekers with these skills (Burning Glass, IBM, & BHEF, 2017; BHEF, 2019).

Montana leaders are calling for change in high school and postsecondary mathematics pathways

- In 2015, the Montana Math Pathways Task Force (MMPTF) (2015) concluded that "[f]or Montana to compete in an increasingly global economy, more emphasis must be placed on removing barriers to success in higher education" (p. 4). Governor Bullock then set a goal that by 2020, 60 percent of Montana's adults would have a postsecondary credential, a 20 percent increase (State of Montana, 2013). The report stressed that mathematics must play a key role in this shift and that "[i]t is imperative that issues such as math aversion, placement tests, developmental success rates, and course appropriateness are thoroughly investigated in Montana" (p. 4).



Proposed STEM and non-STEM mathematics pathways (Montana Math Pathways Task Force, 2015)

Theme 3: What are the key recommendations in the literature for creating high school mathematics pathways likely to serve a diverse student body effectively?

Pathways serve students to the extent that they empower students to expand their professional opportunities; understand and critique the world; and experience the wonder, joy, and beauty of mathematics (NCTM, 2018). This framework is echoed in the Dana Center's description of its vision for the Launch Years Initiative:

> *We envision a future in which every student is supported to learn mathematics, sees mathematics as relevant to his or her life and goals, and is able to transition smoothly from secondary to postsecondary education. . . . These pathways will be supported by structures, policies, and practices to propel students through junior- and senior-level high school mathematics courses and into—and through—the best option for each student from among the range of postsecondary paths. These paths include certification, apprenticeship, two- and four-year degrees, and education opportunities through the military.* (pp. 17–18)

The following recommendations for high school mathematics pathways are well supported by the literature:

1. Require four years of rich, rigorous mathematics for all students.
2. Provide all students with a common grounding in essential mathematics that prepares them for a variety of majors and careers.
3. Include opportunities for students to pursue mathematics that meet their unique needs, interests, and postsecondary goals.
4. Offer flexibility and the ability to move between pathways if necessary.
5. Include high-quality student supports to promote learning and academic success in mathematics for all students.
6. Provide opportunities for highly motivated and enthusiastic students to satisfy their mathematical curiosity.
7. Include appropriate ongoing support for teachers to successfully implement, improve, and sustain modern mathematics pathways.
8. Include comprehensive student advising to guide students to the appropriate mathematics pathway for their postsecondary interests.
9. Establish collaboration efforts between secondary and postsecondary institutions.
10. Design and implement measures of success.

In addition to these recommendations, one organization recommends that discussions about mathematics pathways have "the right people . . . in the room" when key decisions are being made (Dana Center, 2022, p. 41), such as representatives of indigenous communities or other historically marginalized populations. Switching to virtual meetings was another common strategy for ensuring that the right people were able to attend key meetings. The presence of key representatives in such conversations may both help to respond to the needs of historically underserved populations and allay opposition to equity-driven strategies that characterize mathematics pathway efforts as teaching about race, racism, or critical race theory.

Require four years of rich, rigorous mathematics for all students.

- To be properly prepared for postsecondary coursework, all students should be enrolled in a mathematics course in every semester of high school, especially their senior year. Students without sufficient math frequently arrive unprepared for college-level mathematics and other quantitative courses (Academic Senate of the California State University [CSU] Quantitative Reasoning Task Force, 2016; NCTM, 2018). Students with four years of high school mathematics tend to score significantly higher on college entrance exams and require less remediation (Achieve, 2020).
- Taking four years of high school mathematics improves standardized mathematics scores, high school graduation rates, rates of entry into postsecondary institutions, and postsecondary performance, often with larger effects for disadvantaged students and those attending disadvantaged schools (Atewell & Domina, 2008; Long et al., 2012).
- In high-need and high-poverty schools, schools with majority minority populations, rural and urban schools, and schools designated for comprehensive support and improvement, math graduation requirements have been associated with a 12–17 percent increase in the likelihood of students taking advanced math courses (Gao, 2021).
- In California, higher math graduation requirements have also been associated with better economic outcomes, particularly in high-need, high-poverty, and high-minority schools, and do not lead to higher dropout rates (Gao, 2021). This can help close the earnings gap between those students and their middle-income peers (Goodman, 2019; Joensen & Nielsen, 2009; Rose & Betts, 2004).
- No pathway should be considered terminal with respect to a student's mathematical future, and schools should carefully guard against the "ranking" of pathways, where some are seen as the pathways for "math people" or "smart people," while others are stigmatized as for "non-math people" or people who are "bad at math," as this can lead to unproductive attitudes that undermine the purpose of offering multiple rich and rigorous pathways designed to open doors for students (Celedón-Pattichis et al., 2017).

Defining Rigor

Though the word "rigor" is used in a variety of ways in the fields of mathematics and education as well as in policy discussions (Dana Center, 2019), it is rarely understood or defined (Barnett et al., 2022) and often merely used as code for "better" (Gojak, 2013). Perhaps most problematically, "rigor" and "college algebra" are sometimes used interchangeably (Dana Center, 2019).

In recent years, stakeholders have been explicitly defining rigor for the purposes of establishing shared understanding of what it means for a mathematics course or mathematics instruction to be "rigorous." The Common Core State Standards for Mathematics (Achieve the Core, n.d.) discusses rigor as a "shift" in instruction that many schools and teachers would need to make in order to align teaching with the new standards, defining rigor as conceptual understanding, procedural skill and fluency, and application with equal intensity. Similarly, Hull, Balka, & Miles (2013, 2014) define rigorous mathematics content as "the depth of interconnecting concepts and the breadth of supporting skills students are expected to know and understand," and rigorous mathematics instruction as "effective, ongoing interaction between teacher instruction and student reasoning and thinking about concepts, skills, and challenging tasks that results in a conscious, connected, and transferable body of valuable knowledge for every student" (p. 22).

In *Catalyzing Change in High School: Initiating Critical Conversations* (2018), NCTM defines courses as "mathematically demanding" if they:

- Require clarity and precision in reasoning;
- Have focused and significant mathematics learning standards;
- Maintain the integrity of the mathematical standards;
- Are part of a coherent mathematical learning progression (i.e., they are courses that prepare students to continue their study of mathematics; they are not dead-end courses); and
- Approach mathematics in an instructionally balanced way that includes attention to conceptual understanding, procedural fluency, problem solving, and mathematical reasoning and critical thinking practices.

In 2017, NCTM posited that rigor pertains to students' ability to "use mathematical language to communicate effectively and to describe their work with clarity and precision. Students demonstrate that what they have done works, when it works, and why the procedure they selected is appropriate" (Dana Center, 2019, p. 7). This definition does not pertain to a specific course but instead positions rigor as one component in a broader group of elements that constitute an effective course, including procedural fluency and skills, conceptual understanding, productive persistence, and application. This definition argues that rigor can and should be found in all mathematics courses in both K–12 and higher education.

Discussion of recent efforts to define and understand mathematical rigor.

Provide all students with a common grounding in essential mathematics that prepares them for a variety of majors and careers.

- In the first two to three years of high school, students should participate in a common, rigorous mathematics experience in which they cover "Essential Concepts"—the critical mathematics of which all students should gain deep understanding by the end of their secondary education. (See NCTM, 2018, pp. 37–80.)
- These core topics and ideas should be selected because they "open up professional and personal opportunities for students, as well as cultivate a rich set of mathematical tools that they can use to meaningfully understand and critique the world that they inhabit" (NCTM, 2018, p. 5).
- "Essential" high school mathematics should include topics directly applicable to a much wider range of fields and careers beyond those that typically come to mind as "mathematical," such as engineering, physics, economics, and finance (Academic Senate of the CSU Quantitative Reasoning Task Force, 2016: National Academies Press, 2016).
- Oregon's redesigned high school mathematics sequence includes a common two-year core to provide the algebra, geometry, and statistics necessary for all students to be college and career ready (Oregon Math Project, n.d.).

- Alabama chose to adopt the Essential Concepts as laid out in NCTM's *Catalyzing Change* (Mackey, 2019).

Include opportunities for students to pursue mathematics that meet their unique needs, interests, and postsecondary goals.

- Students should have opportunities to pursue additional mathematical knowledge and skills "Beyond Essential Concepts" (BEC) that support their interests and postsecondary goals (NCTM, 2018; Daro & Asturias, 2019). States should (1) create rigorous pathways that articulate with postsecondary policies and practices and align with a range of student aspirations, and (2) give more weight to student aspirations and less to students' perceived preparation levels.
- In the final one to two years of high school mathematics, schools can offer courses such as precalculus, calculus, statistics, quantitative literacy, financial mathematics/financial algebra, discrete/finite mathematics, mathematical modeling, or discrete mathematics. Other possibilities include dual enrollment or high school–to-college transition courses (described in Theme 4).
- Coursework that goes "beyond essential topics" should be organized into clearly articulated pathways that lead, in turn, to meaningful postsecondary pathways. Courses that address unique students' interests such as mathematics for computer science or other STEM careers may also be considered, as long as they provide students with nonterminal, meaningful mathematics learning (NCTM, 2018).
    - For example, in Oregon, a two-credit core sequence is followed by a choice of Algebra II/Precalculus, Statistics/Modeling, or an Applied Math/Modeling course designed to prepare students for technical careers (Oregon Department of Education, n.d).
- Daro and Asturias (2019) use the term "BRANCH" to describe exciting non-STEM careers that students might pursue such as journalist, elected official, high school principal, marketing executive, attorney, game designer, first responder, movie producer, or stockbroker.[7] They suggest that most high schools should be able to offer a STEM pathway and one or two BRANCH pathways focusing on relevant mathematics for students in areas such as the humanities and social sciences that lead to worthwhile postsecondary opportunities. These courses should be open to STEM students to take in addition to any required STEM courses.

Identifying Essential Concepts
In deciding which mathematics topics should be considered "Essential Concepts" and critical for all students, mathematics leaders have found it helpful to begin with the following questions: • Which mathematics is good for a particular pathway [after high school] versus which mathematics is needed for all high school students regardless of their specific future goals?

[7] Though BRANCH is not an acronym, Daro and Asturias (2019) use all capitals "to indicate that these pathways should be comparable and equally rigorous to STEM pathways" (p. 8).

- Should there be multiple versions of Algebra II, or should we reimagine and update Algebra II for all students (National Center on Education and the Economy [NCEE], 2013)?
- How much mathematics beyond grade 8 do students need?
- How do we leverage the guidance and Essential Concepts in the National Council of Teachers of Mathematics book *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* to support content selections and to help students discover the wonder, joy, and beauty of mathematics?
- What would you be willing to withhold a high school diploma for? (This question was posed by one state to get a content committee to be extremely selective in the choices they were making.)

Leaders reported that these questions led to a different set of non-negotiables in terms of content and academic standards than what was currently in place because they helped pathways designers to think critically about the mathematics content students might need across a wide range of postsecondary opportunities. They also helped leaders come to agreement on more optional content that did not seem critical for the vast majority of students and might best be opted into (for example, polynomial long division and conic sections) (Dana Center, 2022).

As a result, some states arrived at the conclusion that not all STEM majors and careers require a calculus-focused pathway; for example, one representative pointed out that "If I want calculus-based STEM, I need a traditional path. If I want to go into cyber security, I may be able to take a non-calculus-based pathway" (Dana Center, 2022, p. 38).

Discussion of questions to consider in identifying "Essential Concepts" for all high school students.

Offer flexibility and the ability to move between pathways if necessary.

- Whereas *tracks* lock students into a predetermined course sequence and mathematical future based on perceived ability, *pathways* should give students the agency to pursue mathematics that suits their interests and future plans and also to shift into a different pathway if they change their minds (NCTM, 2018).
- For example, if a student opts into a data-and-statistics-focused pathway for grade 11 and completes a third-year course in that pathway but then decides that they would like to pursue Calculus later on in their education, the "Essential Concepts" coursework that they completed in 9th and 10th grade should be designed such that they are still prepared to take a rigorous third-year course pointed toward Calculus (for example, Precalculus) in grade 12.
- One way to do this is to provide summer or semester courses that can serve as bridges for students who choose to switch pathways (Daro and Asturias, 2019).

Include high-quality student supports to promote learning and academic success in mathematics for all students.

- Practices such as removing students from grade-level instruction, drilling them on low-level procedural skills, or placing them in a less rigorous track not only are ineffective but have significant long-term negative effects on students' mathematics learning outcomes, achievement, attitudes, and postsecondary opportunities (Balfanz et al., 2006; Burris et al., 2006).

- More effective strategies include designing targeted instructional and support strategies that address uneven or missed learning opportunities and damaged math student identities (Daro & Asturias, 2019).
- Not all students learn at the same pace, and some students simply need additional instructional time focused on building deep conceptual understanding and developing procedural fluency that builds on that understanding (NCTM, 2014, 2018). At the secondary level, one effective way this additional time is often structured as a second period of mathematics instruction. Which students need this extra support and when should be determined by the results of frequent classroom-based formative assessments (Gresham & Little, 2012; Kanold et al., 2018).
- Additional learning time is often most effective when the time is structured in a fluid way, providing students with opportunities to move in and out of extra support as needed and multiple opportunities to demonstrate improved learning (Cortes et al., 2013; Gresham & Little, 2012; Kanold et al., 2018).
- Extra time can also be structured in the first period of a double-period course[8] in which students receive just-in-time support on prerequisites, a preview of key concepts to be covered in upcoming lessons, or continued work with concepts from previous lessons. In this way, they are prepared to fully participate in the second, core period of the double-period course.
- Other effective strategies include proactive supports around building the study skills, critical thinking skills, and resourcefulness that students may not have yet had sufficient opportunities to develop (Dana Center, 2020; NCTM, 2018). This can take the form of tutoring, academic coaching, or supplemental instruction, and can be provided in person, virtually, or via computer application.

Provide opportunities for highly motivated and enthusiastic students to satisfy their mathematical curiosity.

- Students who desire a greater challenge or wish to extend their learning in mathematics can benefit from opportunities to deepen their mathematical understanding in ways that do not rely on the harmful practices of tracking.
- In-class strategies for supporting these students include differentiation strategies where they work on the same grade-level content and standards at deeper levels or with extension tasks or prompts (Sullivan, 2006); they may also be well-served through structured peer-to-peer programs or mathematics learning communities, in or outside of the classroom (Dana Center, 2020).
- Leaders must understand that acceleration is strongly tied up with issues of equity and privilege (Grissom & Redding, 2016) and must carefully guard against re-creating systematic inequities through acceleration. NCTM (2018) recommends that acceleration be an exceptional practice, that it be along a common shared single

[8] Double-period versions of a course do not represent lower-level versions of the grade-level course and are different from tracking in that the double-period version of the course should have the same instructional objectives and expectations and use the same core instructional materials and assessments (NCTM, 2018).

pathway (versus re-creating qualitatively different "tracks" such as Algebra I versus Honors Algebra I), and that it be reserved for those students who will benefit from acceleration because of the complete and deep understanding beyond grade-level learning standards that they have demonstrated.

- In Alabama, pathways were carefully and intentionally designed to serve students with exceptional interest and preparation in mathematics while avoiding re-creating inequitable tracking systems.
    - All students take a common grade 6 course and leave grade 8 prepared for Geometry with Data Analysis in grade 9.
    - Most students will complete the standard, rigorous middle school pathway, while those who are especially interested and strongly motivated to study mathematics have the option of choosing an accelerated pathway that combines standards from three courses (Grade 7 Mathematics, Grade 8 Mathematics, and Algebra I with Probability) over two years (grades 7 and 8).
    - In high school, all students still begin 9th grade in Geometry with Data Analysis. Most students then proceed to Algebra I with Probability in grade 10, while those who followed an accelerated course in middle school are able to move on to Algebra II with Statistics.
- Schools and districts should develop and implement measures of success that systematically monitor the extent to which acceleration is being implemented appropriately and equitably. If education agencies find that the demographics of students accelerated in mathematics in a school or district are not evenly distributed across racial, linguistic, cultural, and economic lines, then actions should be taken to remove whatever biases or structural barriers led to this inequitable outcome.

Include appropriate ongoing support for teachers to successfully implement, improve, and sustain modern mathematics pathways.

- A lack of intensive, ongoing, sufficiently resourced support is one of the most common barriers to lasting and successful school change (Cohen, 2022; Fullan, 2020; Senge, 2006; Senge et al., 2012). State and local agencies have a duty to ensure that all students receive instruction from qualified teachers. Thus, sufficient opportunities for professional development (PD), reflection, and ongoing support are critical to the success of such efforts.
- PD can take many forms, including leveraging technology to make professional learning more easily accessible to more teachers, pairing content experts with instructional efforts, connecting teachers to experts in private and nonprofit sectors, and developing regional partnerships to train and support teachers (Dana Center, 2020).
- Changes to curriculum and instruction at the secondary level may have implications for K–8 mathematics. For example, some leaders felt that elementary school teachers in their state were frequently not well trained in number sense and how to teach it, generating ripple effects through high school. For these reasons, states may find it

useful to incorporate the perspectives of K–8 teachers as they reform secondary pathways (Dana Center, 2022).

- A key element of professional development supporting successful change management is explicit, transparent discussion of the purposes behind and goals of a given change. Attempts to change curriculum, course structure, or instructional style or strategies have to be accommodated to teachers' beliefs as to what good teaching is and what it is reasonable to expect of students (Howson et al., 1981), and unless goals underlying the innovation are shared by those who are to implement it, success is unlikely (Burkhardt et al., 1990). When teachers do not understand why they are being asked to learn and do something different, they are less likely to implement the change with fidelity (Cohen, 2011; Cohen, 2022; Fullan, 2020).

Include comprehensive student advising to guide students to the appropriate mathematics pathway for their postsecondary interests.

Education agencies should design systems and processes that ensure that pathway options are communicated early, publicly, and clearly to all stakeholders (Daro & Asturias, 2019).

- Starting in middle school, students and parents need to understand the mathematics pathways options available to them, how those pathways align with postsecondary options, and how to prepare for the coursework they will need to complete in order to achieve their education and career goals (Dana Center, 2020). Students should be educated about the benefits and limitations of different pathways and how they can "bridge" to other pathways should their interests or goals change.
- Successful pathways require thorough training of counseling staff regarding available pathways, their goals, requirements, benefits, and limitations. Collaboration with partners in postsecondary institutions and in business and industry can help provide students with relevant, up-to-date advising, career exploration, college counseling, and advice regarding how different pathways may support their college and career interests (Dana Center, 2020).
- All stakeholders must actively guard against allowing biases to prejudice the direction in which students are advised regarding their secondary mathematics pathways, as student entry into and progress through college-level mathematics has often been influenced by biases about who "can" or "should" do certain mathematics courses or who belongs in academic or career fields (Dana Center, 2019). Mathematics pathways and the structures and practices associated with them should be carefully and intentionally designed to eliminate tracking and implicit and explicit bias and to take actions to address problems quickly when they arise.

Establish collaboration efforts between secondary and postsecondary institutions.

- Partnerships with postsecondary counterparts help ensure that high school counselors and teachers have accurate, up-to-date information about institutional processes and requirements, and also help high schools to promote policies and strategies that ensure alignment between the last two years of high school, college readiness

placement and first college-level course requirements, students' major/program plans or career pathways, and labor market demands (Dana Center, 2020).
- Collaboration helps ensure that mathematics courses create clearly articulated and aligned pathways that lead to appropriate postsecondary education and ultimately high-wage, high-skill, in-demand jobs, and helps ensure that pathways do not become obsolete (Dana Center, 2019).
- Collaboration can also look like partnering with postsecondary institutions and business/industry to offer programs and networks designed to help students explore career and education options, such as campus and workplace visits, mentorships, summer college or work experience programs, or new student orientations (Dana Center, 2020).
- California's Early Assessment Program (EAP) is another example; students who pass the EAP test automatically qualify to enroll in credit-bearing courses at a California State University (CSU) campus, and those who do not have a clearer idea of how to strengthen their skills in mathematics and English language arts (ELA) during their last year of high school. Those who participate in EAP lessen their probability of needing a developmental mathematics course by 4.1 percentage points (Howell et al., 2010).

Design and implement measures of success.

- Leaders need to define, communicate, and implement both qualitative and quantitative success metrics to drive policy and practice alignment across K–12, higher education, and workforce sectors. These success metrics should be based on common definitions of goals and aimed at improving equitable access and outcomes in mathematics education (Dana Center, 2019).

<table>
<tr><td>Quantitative success metrics. Short-, intermediate-, and long-term milestones are used as quantitative metrics to assess progress and impact of each full mathematics pathway to and through postsecondary education and into the workforce.</td></tr>
<tr><td>Examples of quantitative success metrics include:
• Student learning outcomes, such as grades on early and final course assessments, measures of postsecondary readiness, and completion of college-level mathematics course(s) in year one aligned to a program.
• Student mindset development metrics, such as self-efficacy, sense of belonging, self-regulation, productive struggle, and growth mindsets, especially as they relate to goal-setting, help-seeking behaviors, and learning mathematics.
• Student progression measures, such as high school graduation, postsecondary matriculation, time to completion of a degree or credential, entry into the workforce, and wages at first job.
• Pathways implementation indicators, such as alignment of secondary pathways to postsecondary program requirements; access to/success in different mathematics pathways; and representation of demographic groups.</td></tr>
<tr><td>Qualitative success metrics. Qualitative data capture the student experience in and through mathematics pathways. These data reveal how students understand the mathematics pathways in their education system and how students access information. Qualitative data also show how students access—and are provided—opportunities and how they are considered successful.</td></tr>
<tr><td>Examples of qualitative metrics include:</td></tr>
</table>

- Students' experiences within the pathways, such as engagement in learning, identities as mathematical learners, and sense of belonging.
- Students' experiences navigating across the pathways, such as experiences with advising and student access to relevant, timely, and actionable information about mathematics pathways; how students make decisions about selecting or switching pathways.

Examples of quantitative and qualitative success measures (Dana Center, 2020, pp. 54–55).

Theme 4: What common high school mathematics pathway models and structures are currently being implemented, in the United States or elsewhere?

Siloed mathematics models

The Algebra I–Geometry–Algebra II–Precalculus–Calculus course progression is frequently described as the "traditional model" or "traditional pathway." To avoid confusion, this review uses the phrase "siloed model/pathway" to refer to models where branches of mathematics such as algebra or geometry are separated out into isolated courses.

Geometry-first models

- In these models, students encounter a geometry-focused course first, then proceed to a more algebra-focused course. This model offers several advantages:
  - Fluency around transformations-based geometry concepts like similarity, congruence, scaling, invariance, and proportionality are essential preparation for higher-level mathematical topics, particularly algebra (e.g., Makgakga & Sepeng, 2013; Teuscher et al., 2015; Wiles et al., 2019; Wu, 2017). For example, understanding properties of similar triangles supports understanding of slope of a straight line and of linear functions (National Mathematics Advisory Panel, 2008; Wu, 2011).
  - Geometry promotes spatial awareness, dynamic visualization skills, and an overall deeper conceptualization of mathematical structure critical to accessing more advanced mathematics, particularly the more abstract concepts students encounter in algebra-focused courses (Cox, 2013; Seago et al., 2013).
  - Identifying and using mathematical structure in a proportional situation is central to problem solving in both geometry and algebra (Lamon, 2007; Tanton et al., 2020), yet students (and teachers) often struggle (Berk et al., 2009; Sowder et al., 1998; Slovin, 2000; Weiland et al., 2021). Transformations-based geometry offers opportunities to connect the mathematical structure behind multiplicative reasoning with visual representations (Cox, 2013), and supports the transition from additive to multiplicative thinking.
  - Visual representations within geometry may enable more equitable access and opportunities for students (Driscoll et al., 2007; Goldenberg and Cuoco, 1998), particularly for multilingual learners and those with special needs around working memory or language processing (Hord et al., 2018; Wiles et al., 2019).
  - New and creative instructional approaches (Jones, 2002) used within geometry may be engaging for students (Leonard & Bannister, 2018; Shaghaghian et al., 2021) and help students to identify geometric properties and relationships (DePiper et al., 2018; Noble et al., 2004; Steketee & Scher, 2016).

- Two recommended geometry-first pathways have been described (NCTM, 2018):

Pathway A	Pathway B
Grade 9—Essential Concepts of Geometry in semester 1 followed by Essential Concepts of Statistics and Probability in semester 2. **Grade 10 + Grade 11 semester 1**—Continuation of the study of algebra begun in middle school, moving from previous work with coordinate geometry and linearity to quadratics and exponential equations.	**Grade 9**—Geometry and measurement, descriptive statistics, and graphing in the coordinate plane, using the role of linearity to link all three of those areas; introduction to quadratic equations in one variable. **Grade 10**—Functions, function notation, and exponential equations. **Grade 11**—Analysis of function families related to quadratic and exponential functions; probability; statistical inference and its connection to probability. **Grade 11 with precalculus focus**—The course above, but expanded to include logarithmic and trigonometric functions, composition of functions, inverse functions, and their graphs; this course targets students who wish to take Calculus. **Grade 11 with modeling focus**—The course above, but expanded to include modeling experiences that use various function families in a variety of contexts and applied problems; this course targets students who wish to take Statistics or other applied mathematics.
Advantages to these pathways: • The common 9th-grade mathematics experience allows all students to start at the same point, giving everyone access to the same mathematics. • The 9th-grade concepts leverage prior grade-level or middle school work. For example, 9th-grade geometry and measurement concepts build on students' recent work with congruence, similarity, and proportionality in grade 8. • More abstract algebraic concepts are postponed, offering students opportunities to hone their reasoning and sense-making skills and experience the wider applicability of mathematics. • Algebraic skills and concepts are not broken up over two years and separated by an entire other course. Paring down algebra-focused topics to "essential topics" can give students time to continue their study of mathematics in courses suited to their future needs and interests. • Pathway B leverages the connections between rate of change, linearity, and linear functions (typically addressed in Algebra I) and similarity, scale, dilation, ratio, and proportion (typically explored in Geometry).	

- Alabama's geometry-first high school math pathway has all 9th-grade students begin their high school career with a Geometry with Data Analysis course. Those students who have not completed the state's accelerated middle grades curriculum then proceed to Algebra I with Probability in grade 10, while those who completed the accelerated middle grades curriculum move on to Algebra II with Statistics. Students who take Algebra I with Probability in grade 10 take Algebra II with Statistics in grade 11 (Mackey, 2019, p. 108 shows these course sequences).

Geometry and algebra concurrent models

- These models enable students to take an algebra and geometry course concurrently in grade 9. For example, Alabama students who have not completed the accelerated middle grades curriculum can choose to take Geometry with Data Analysis and

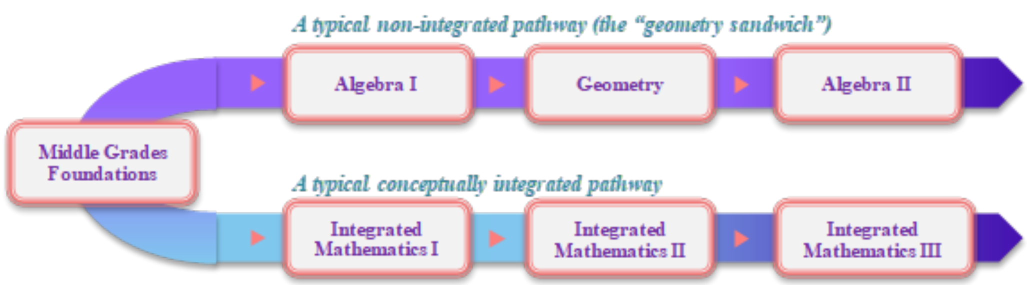
Algebra 1 with Probability concurrently, as neither course is designed as prerequisite to the other.

Streamlined siloed models

- These models retain the split between algebra and geometry but streamline the existing content, eliminating outdated and irrelevant topics so that coursework that may have required three to four full years in the past can be completed more quickly, leaving additional space in students' schedules to take additional mathematics courses that support their needs, interests, and postsecondary goals.
- In Oregon's streamlined siloed system, students take a year of Algebra, Function, and Number; a year of Geometry (semester 1) + Data Science and Statistics (semester 2); a third-year "specialized" option of advanced algebra (Algebra II or Precalculus), data science (a non-Advanced Placement [AP] statistics course or another applied data option), or quantitative math (Constructive Geometry, Financial Algebra, or a similar course); and an optional fourth-year course with flexibility between paths (AP Calculus or another advanced algebra course; AP Statistics or another data science course; or an additional quantitative option) (Oregon Department of Education, 2020).

Conceptually integrated mathematics models

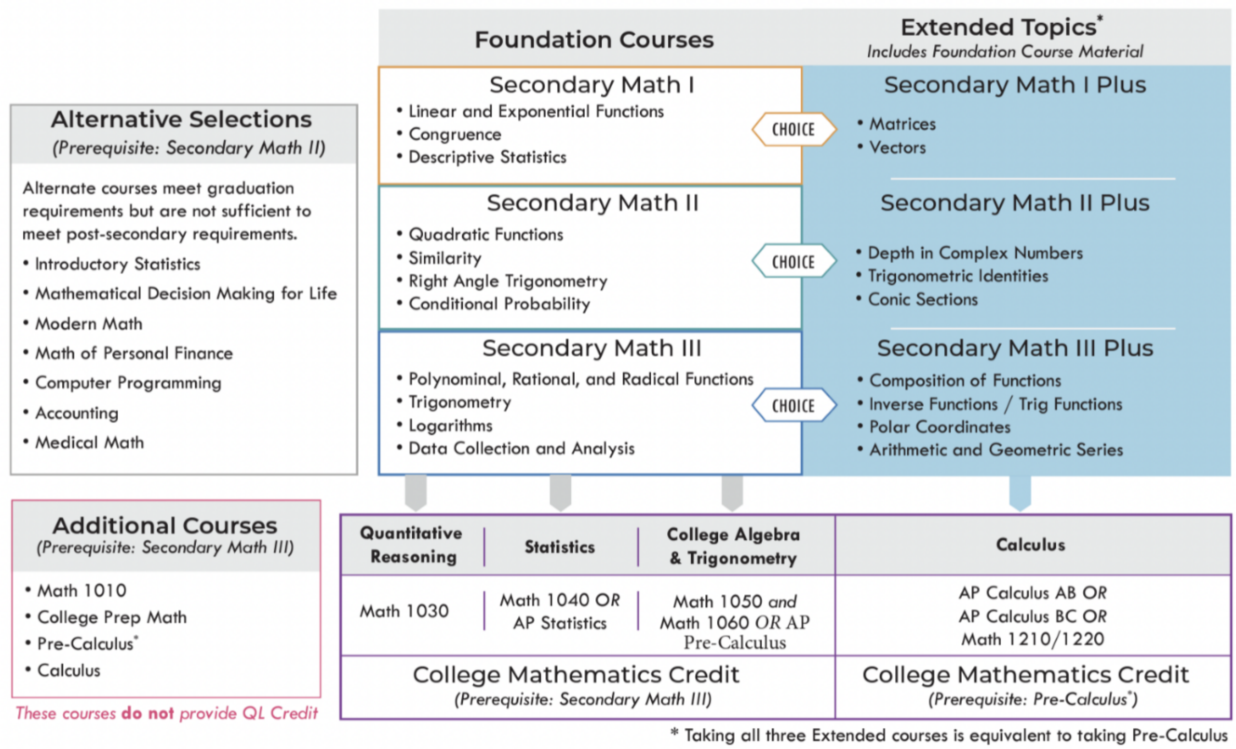
- In these pathway models, courses are "integrated" to create a more holistic, connected experience of mathematics, as in the elementary grades. This approach to integration has been in use for over a hundred years (de Araujo et al., 2013), and experts consider it consistent with the way that people use mathematics to understand and solve real-world problems (Beal et al., 1990; Schmidt et al., 2002; Shaughnessy, 2011).



Source: Author.

- The highest-achieving countries on the international Programme for International Student Assessment (PISA) test (OECD, 2016) typically use an integrated approach for high school mathematics.
- Initial evidence regarding the effectiveness of an integrated U.S. mathematics curriculum has been promising, with students in the integrated curriculum scoring significantly higher than those in the siloed curriculum on a test of common mathematics objectives in several studies (Chávez et al., 2015; Grouws et al., 2013; Tarr et al., 2013).

- High schools in Tennessee, Georgia, North Carolina, West Virginia, and Utah have transitioned to an integrated model (Will, 2014). As of 2015, half of the 30 largest districts in California had opted for an integrated high school mathematics curriculum (Harlow, 2015). In Utah, students take a series of "Foundation Courses" (Secondary Math I, Secondary Math II, Secondary Math III) in which mathematical topics deemed essential to prepare students for college-level mathematics are covered in a conceptually integrated manner.



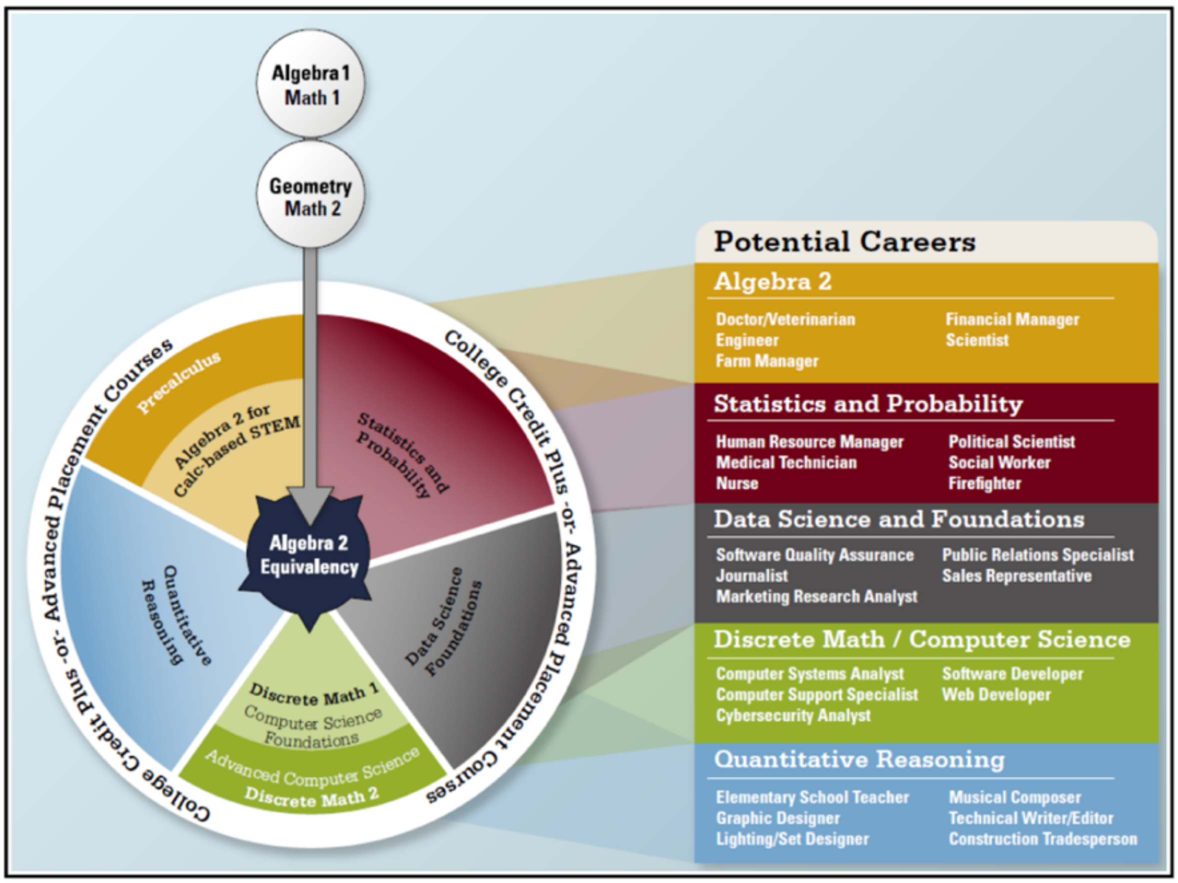
Source: Utah High School Mathematics Graduation Pathways, p. 1.

Creating Algebra II-equivalent courses and year three/four course sequences

- An **Algebra II-equivalent course** is a rigorous third-year mathematics course that is designed and taught "at a level equivalent to that of Algebra II desired to prepare students for advanced courses other than Calculus" (Dana Center, 2020, p. 39).
- The course does not rely on existing Algebra II standards, but is built from a definition of **Algebra II equivalence** and the identification of the preparation students will need to successfully transition from high school to postsecondary education. Such third-year courses should:
    - Focus on sense-making, authentic use and modeling, data analysis, computational thinking, and functions to reason through and tackle unfamiliar problems and to prepare for higher-level future work.
    - Require students to make decisions based on analysis of messy, uncertain situations.

- Include challenging, meaningful, and relevant tasks, some of which require the use of technological tools.
- Emphasize the importance, relevance, and application of mathematics for solving real-world problems. That is, the course teaches students to understand important mathematics deeply (Dana Center, 2020, p. 40).

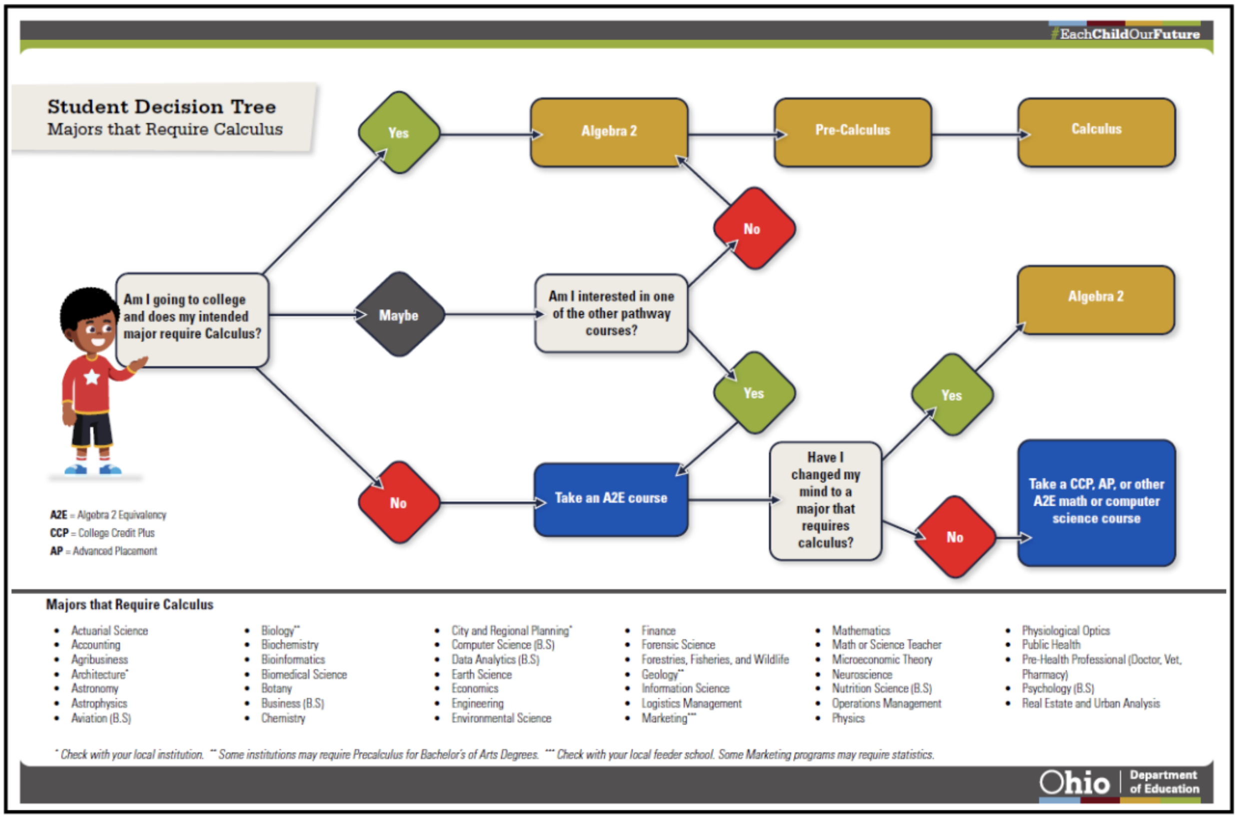
- Many rigorous third-year options could meet these criteria, either as stand-alone courses or as part of a two-year sequence that students take across years three and four.
- Third- and fourth-year course sequences should be designed in partnership with postsecondary institutions at the state, regional, or local level to align with modern higher education mathematics pathways and support students' attainment of their long-term goals (Dana Center, 2020).



Ohio high school mathematics course options (Patchnowski & Cannelongo, 2021, p. 5).

- Ohio bases Algebra II equivalence on "the level of thinking, reasoning, and rigor" (Patchnowski & Cannelongo, 2021, p. 5), rather than equivalent content. The pathways' architects elected to address the state's identified Essential Concepts over three years, recommending that all students study algebra and geometry in their first two years of high school, then select one of four "Algebra II-equivalent" courses designed to align with local postsecondary math pathways: (1) Quantitative

Reasoning, (2) Data Science Foundations, (3) Statistics and Probability, and/or (4) Discrete Math/Computer Science. Districts are able to choose which of these options they offer.



Ohio high school mathematics pathways "student decision tree" (Patchnowski & Cannelongo, 2021, p. 6).

- Alabama students cover Essential Concepts in years one (Algebra I with Probability) and two (Algebra II with Statistics), then select from three specialized mathematics classes (Precalculus, Mathematical Modeling, or Applications of Finite Mathematics) "designed to prepare them for future success in the postsecondary study of mathematics, in careers, and in their lifelong use and enjoyment of mathematics" (Mackey, 2019, p. 106). All three courses are designed to allow students to progress to further studies in mathematics, and they can be taken together or in any order. After taking at least one of the three specialized courses, students may elect to take Precalculus, AP Statistics, AP Computer Science, or AP Calculus if they are offered. Oregon's system is similar.
- Schools should allow students to move between different courses or course sequences (for example, from Algebra II to an Algebra II–equivalent course such as Quantitative Reasoning, Statistics, or Data Science, or vice versa) if they decide to (Dana Center, 2020). Supports for doing so may come in the form of bridge models or just-in-time additional instruction within the course. Yearlong transition courses for fourth-year high school students designed in collaboration with postsecondary partners are another strategy for ensuring that these students are prepared for entry-level credit-

bearing college mathematics courses such as College Algebra, Statistics, or Quantitative Reasoning (Dana Center, 2020).

Courses/sequences for targeted "Beyond Essential Concepts"

There are many innovative, meaningful, and rigorous mathematics courses being developed across the U.S. at the state and local levels (NCTM, 2018). A few include the following:

- Precalculus and Calculus. A precalculus/calculus pathway should be available for students who wish to pursue majors and careers in engineering or other STEM fields that require calculus, but should not preclude or limit students pursuing other mathematical avenues or majors/careers in other fields. Students and parents should be well-educated regarding productive and unproductive beliefs about calculus and informed about the benefits and challenges of taking calculus in high school (Bressoud, 2017; Bressoud et al., 2015; NCTM, 2018).
- Statistics. In recent years, new technology has made it much more practical and efficient to teach statistical topics and reasoning skills at the K–12 level, on its own or as part of a two-year data science sequence. This and the inherent real-world applicability of statistics makes it a natural choice for a rigorous third- or fourth-year high school mathematics course (Franklin, 2013).
- Financial Literacy/Financial Algebra. This is a rigorous mathematics course that addresses high school mathematics standards related to functions, number systems, and mathematical modeling through topics such as saving and investing, banking, credit and debt, income taxes, insurance and risk management, money management, and planning for retirement. Los Angeles Unified School District (LAUSD) (along with other California districts) offers a Financial Algebra sequence rigorous enough to satisfy the University of California system's Algebra I/II admission requirement. Similar courses are also offered in West Virginia (Cann, 2018), Washington (Mortensen, 2022), Massachusetts (Lowell High School, 2022), and New Mexico (Arizona State University Course Approval, n.d.).
- Mathematical Modeling. Mathematical modeling is essential to providing high school students with the knowledge, skills, and dispositions needed to make greater sense of the world (NCTM, 2018). The Common Core State Standards for Mathematics (Council of Chief State School Officers, 2010) devotes an entire high school conceptual category to mathematical modeling, with standards drawn from across the other high school categories, underscoring its uniquely pervasive nature in mathematics.
- Data Science. Interest has increased in recent years around data science as a field as well as an increasingly consequential area of mathematics in which global citizens should be literate (LaMar & Boaler, 2021). Preliminary research suggests that these courses may be more inclusive than traditional courses (Lue, 2019); lead to a greater sense of confidence, empowerment, and belonging for many students, particularly those who have not experienced success in mathematics in the past (Heinzman, 2022); and support students developing skills and habits of mind that enable them to use open-source data to understand their community (Gould et al., 2016). As of 2021, California, Georgia, New York, Oregon, Virginia, and Washington had all released

updates to high school mathematics pathway options that mention data science as a possible third- or fourth-year mathematics course (California Department of Education, 2021), with similar efforts underway in Illinois, Maryland, Michigan, Ohio, Texas, and Utah. Center X at the University of California, Los Angeles, has developed an Introduction to Data Science course and is currently supporting its implementation in the Los Angeles Unified School District (UCLA Center X, 2019).

- Advanced Mathematical Decision Making (AMDM) Using Quantitative Reasoning (Dana Center, 2017a). AMDM is a rigorous fourth-year high school mathematics course designed by the Dana Center at the University of Texas at Austin to prepare students for a range of future options in non-algebraically intensive college majors or for entering workforce training programs. The curriculum emphasizes statistics, quantitative reasoning, modeling, and financial applications, and prepares students to use a variety of mathematical tools and approaches to model a range of situations and solve a variety of quantifiable problems.
- Statway/Quantway Dual Enrollment (Carnegie Math Pathways, 2022; Klipple, 2021). The Carnegie Math Pathways (CMP) program began as a set of rigorous yet accessible credit-bearing postsecondary math courses designed to accelerate college math students to and through college-level mathematics. Statway is a rigorous statistics-focused course taught using contextualized curricula and a pedagogical approach that combines collaborative learning and social emotional student supports; Quantway is a rigorous quantitative reasoning course taught in a similar fashion (Carnegie Math Pathways, 2022; Klipple, 2021). CMP students succeed at three to four times the rate of their peers in traditional developmental math sequences in half the time or less, results that have held year after year, across sex, race, and ethnicity (Silva & White, 2013; Edwards & Beattie, 2016; Strother & Klipple, 2019; CMP, 2022). Both courses are now available at the high school level as part of a dual enrollment model in which high schools can accept Statway or Quantway as an alternative to Algebra II, with the assurance that the credit will transfer to nearby colleges.

Transition courses

- High school transition curricula are typically co-created by colleges and high schools and designed for high school students who may not otherwise be on track for college-level mathematics by graduation.
- Transition courses were offered as part of secondary school programming in 39 states in 2017 versus 20 states in 2012–2013 (Barnett et al., 2018).
- Some transition curricula are aligned with mathematics pathways offered at colleges and can be paired with other college readiness interventions such as dual enrollment programs (Barnett et al., 2018).
- Positive research findings related to transition courses include:
    - Students participating in Tennessee's Seamless Alignment and Integrated Learning Support [SAILS] program were more likely than non-participants to place into a college-level math course and earn more college credits, though participation did not improve math achievement or boost the likelihood of passing college-level math (Kane et al., 2018).

  - In Florida, participation in a transition course led to a higher likelihood of enrolling in college-level courses, though it is not clear that students' pass rates in these courses improved (Mokher et al., 2017).
  - Research on New York City's At Home in College math course found that participation yielded a small positive and significant effect on passing a math gatekeeper course within one year of college entry and a small positive and significant impact (one credit) on the number of college course credits earned in the first year (Trimble et al., 2017).
  - A companion study on West Virginia's former transition course found negative impacts from participation, possibly because the course was less rigorous than the alternative math course (Pheatt et al., 2016).
  - ACT scores in Arkansas and Mississippi were found to have improved after completing the Southern Regional Education Board's Math Ready transition course, with 62 percent improving their math or science ACT scores, a statistically significant gain (Southern Regional Education Board, 2019).
- While transition courses have been increasing in popularity, their design and implementation varies from state to state (Barnett et al., 2018).

Course substitutions

- Some states and districts allow students to substitute courses in fields such as computer science, engineering, or robotics for mathematics. While there has been a growing call to teach computer science and coding across grades K–12, NCTM (2015, 2018) warns that schools and education agencies need to exercise caution in determining when and if computer science courses can be used to fulfill mathematics requirements; they should address mathematics standards and enable students to meet core mathematics requirements.
- A course should only be allowed to fulfill a mathematics requirement if the criteria below are met—that is, the course:
  - Requires clarity and precision in reasoning;
  - Has focused and significant mathematics learning standards;
  - Maintains the integrity of the mathematical standards;
  - Is part of a coherent mathematical learning progression (i.e., they are courses that prepare students to continue their study of mathematics; they are not dead-end courses); and
  - Approaches the mathematics in an instructionally balanced way that includes attention to conceptual understanding, procedural fluency, problem solving, and mathematical reasoning/critical thinking practices (NCTM, 2018, p. 84).
- In many cases, engineering computer science courses meet the first criteria but not the other four.
- To ensure equity, pathways and courses offered beyond established "Essential Concepts" must be consistent with one another with respect to mathematical rigor, demand for reasoning, relevance, and the postsecondary opportunities that they afford students.

Career and Technical Education (CTE)-Focused Models

- As of 2019, at least 10 states allowed students to replace one or more required math courses with approved integrated, applied, interdisciplinary, occupational, or technical courses (Macdonald et al., 2019), with many states stipulating that these courses must include content that aligns with the state's core math standards.
- In determining whether a CTE course can be used to fulfill a mathematics requirement, NCTM (2018) takes much the same position as above regarding computer science or other STEM courses. Equity again requires that any and all pathways and courses offered beyond established "Essential Concepts" be consistent with one another with respect to mathematical rigor, demand for reasoning, relevance, and the postsecondary opportunities that they afford students.
- Students should never be placed in the position of using a less rigorous, less mathematically meaningful course to fulfill a mathematics requirement because adults have deemed the student incapable of doing so through standard means. If a student is struggling to succeed in mathematically rigorous courses, this should be addressed using strategies described below in Theme 5, and never by removing students from grade-level-appropriate mathematics or placing them in a course that may limit or curtail their mathematical future.

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Appendix A: Methods

This scan was conducted for technical assistance purposes and, as such, was not intended to meet What Works Clearinghouse standards of review, but rather to identify recent peer-reviewed research and policy/guidance documents that support understanding of issues related to high school mathematics course pathways that support the needs, interests, and goals of a diverse student body. Below we list information on how resources were identified. The annotated bibliography includes the identification method for each resource included.

Keywords and search strings used in the search

Theme 1: "high school mathematics pathways"; "history of high school mathematics pathways"; "history of high school mathematics graduation requirements"; "impact of U.S. high school mathematics curriculum"; "impact of U.S. high school mathematics graduation requirements"; "high school mathematics tracking"; "high school mathematics ability grouping"; "high school postsecondary mathematics gatekeeper"

Theme 2: "changes in mathematics pathways"; "changes in high school mathematics curricula"; "modern mathematics pathways"; "high school mathematics calls for change"; "high school postsecondary mathematics articulation"

Theme 3: "effective high school mathematics pathways"; "high school mathematics pathways recommendations"; "supporting success in high school mathematics pathways"

Theme 4: "integrated math pathways"; "traditional math pathways"; "alternative math pathways"; "geometry first high school mathematics"; "high school mathematics essential concepts"; "high school data science"; "high school discrete finite math"; "high school financial algebra"; "dual enrollment mathematics"; "transition courses high school postsecondary mathematics"

Theme 5: "mathematics pathways"; "mathematics curriculum"; "mathematics courses"; and "graduation requirements" in conjunction with names of states

Search of databases

ERIC, Education Source (i.e., EBSCO Host), Google, Google Scholar

Similar prior literature review

A similar review on diverse high school mathematics pathways conducted in late 2022 provided some resources.

Similar technical assistance

Similar recent efforts to support districts in reviewing and diversifying the secondary mathematics pathways available to students helped to identify some resources.

Criteria for inclusion

When reviewing resources, we considered four main factors:

- **Date of the publication:** Empirical research studies, briefs, literature reviews, and policy/guidance documents published in 2013 or later were prioritized. Additional, older resources were included when the information remained relevant or supported more recent work.
- **Student grade range:** The most current information was included for grades 6–12 and postsecondary, with an emphasis on grades 9–12.
- **Source and funder of the report/study/brief/article:** Priority was given to IES, nationally funded, and certain other vetted sources known for strict attention to research protocols. However, in an effort to identify relevant studies, the search was not limited to peer-reviewed journals, but instead also included gray literature including conference presentations, technical reports, news articles and press releases, state policy and guidance documents, and professional development and practical resources that draw on empirical research (e.g., NCTM, 2014, 2018).
- **Methodology:** Sources included randomized controlled trial studies, surveys, literature reviews, professional development resources, conference presentations, technical reports, news articles and press releases, and policy/guidance documents. Priority for inclusion generally was given to randomized controlled trial study findings but the reader should note the following factors when basing decisions on these resources: numbers of participants, sample selection, and sample representation. The annotated bibliography includes our summaries and abstracts from the articles.

Appendix B: Annotated Bibliography

3—Academic Senate of the California State University Quantitative Reasoning Task Force. (2016). *Final report*. https://www.calstate.edu/csu-system/faculty-staff/academic-senate/Documents/reports/QRTF.FinalReport.KSSF.pdf

In its 2015–16 term, the Academic Senate of the California State University (CSU) convened a Quantitative Reasoning Task Force to review the CSU's expectations for student proficiency in quantitative reasoning upon high school and college graduation, and to recommend changes to existing policies and practices. (See Appendix A, Academic Senate CSU Resolution 3230-15.) The CSU's existing standards for statewide curricula in quantitative reasoning have been in place for many years, and this suggests they may lag behind current thinking and best practices in the field. But there is also evidence indicating that these dated policies may be acting as barriers to some students, particularly those from traditionally underserved populations and in the California Community Colleges. The work of the Task Force was guided by the principle that any educational policy enacted by the CSU must balance access and opportunity to achieve equity. That is, genuine equity lies in providing students from all backgrounds with equitable prospects not only for admission and graduation (access), but also for meaningful degrees that prepare them for high-value careers after graduation (opportunity). The Task Force included faculty and administration representing the CSU, the University of California, the California Community Colleges, the California Department of Education, employers, and the Office of the Lieutenant Governor. Its final recommendations were prepared by a subset of the Task Force holding offices in the Academic Senate CSU, and designated "drafting members." (See the Task Force membership given in Appendix B.) Members of the Task Force conducted an extensive literature review, met with invited advisors, and participated in a national forum programmed by the U.S. Department of Education and hosted at the CSU Office of the Chancellor. This report details the final recommendations of the Quantitative Reasoning Task Force.

3—Achieve. (2020). *The Algebra II variable: State policies for graduation requirements, assessments, and alignment to postsecondary expectations*.

In this research brief, the authors explore inconsistencies that exist both within and between states, and urge reflection on the need for stakeholders to clarify the purpose and role of Algebra II. Specifically, they provide an overview of state-level mathematics policies across the United States with respect to Algebra II course requirements, state assessments, and alignment to postsecondary expectations as they examine the following: (a) graduation requirements: Which mathematics courses/experiences do states expect of students before they graduate from high school?; (b) assessment: How do states assess students' mathematics proficiency and how does Algebra II fit in with those assessment requirements?; and (c) alignment to postsecondary expectations: How do states' high school graduation requirements and assessments align to in-state postsecondary education institutions?

1—ACT. (2016). *The condition of college & career readiness 2016.* https://www.act.org/content/dam/act/unsecured/documents/CCCR_National_2016.pdf

This report examines the progress of the 2016 ACT tested graduating class relative to college and career readiness. The report shows that 64 percent of students in the 2016 US graduating class took the ACT test, and that of those students, only 41 percent met the ACT College Readiness Benchmark in mathematics.

1—Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices*. NCTM.

This book invites K–8 teachers to reflect on their own and their students' multiple identities and consider the rich possibilities for learning that may result when teachers draw on these identities. The authors hold that reflecting on identity and re-envisioning learning and teaching through this lens especially benefits students who have been marginalized by race, class, ethnicity, or gender. The authors encourage teachers to reframe instruction by using five equity-based mathematics teaching practices.

1— Aguirre, J., Herbel-Eisenmann, B., Celedon-Pattichis, S., Civil, M., Wilkerson, T., Stephan, M., Pape, S., & Clements, D.H. (2017). Equity within mathematics education research as a political act: Moving from choice to intentional collective professional responsibility. *Journal for Research in Mathematics Education*, *48*(2), 124–147.

In 2005, the NCTM Research Committee devoted its commentary to exploring how mathematics education research might contribute to a better understanding of equity in school mathematics education (Gutstein et al., 2005). In that commentary, the concept of equity included both conditions and outcomes of learning. Although multiple definitions of equity exist, the authors of that commentary expressed it this way: "The main issue for us is how mathematics education research can contribute to understanding the causes and effects of inequity, as well as the strategies that effectively reduce undesirable inequities of experience and achievement in mathematics education" (p. 94). That research commentary brought to the foreground important questions one might ask about equity in school mathematics and some of the complexities associated with doing that work. It also addressed how mathematics education researchers (MERs) could bring a "critical equity lens" (p. 95, hereafter referred to as an "equity lens") to the research they do. 10 years from now, where is the MER community in terms of including an equity lens in mathematics education research? Gutiérrez (2010/2013) argued that a sociopolitical turn in mathematics education enables us to ask and answer harder, more complex questions that include issues of identity, agency, power, and sociocultural and political contexts of mathematics, learning, and teaching. A sociopolitical approach allows us to see the historical legacy of mathematics as a tool of oppression as well as a product of our humanity.

1—Akos, P., Shoffner, M., & Ellis, M. (2007). Mathematics placement and the transition to middle school. *Professional School Counseling*, *10*(3), 2156759X0701000304. https://journals.sagepub.com/doi/abs/10.1177/2156759X0701100108?journalCode=pcxa

Early adolescents make early career decisions in the form of curriculum choices for high school, and these choices can influence future postsecondary education and career paths. This exploratory study examined relationships between school and demographic variables and 522 eighth graders' choices of high school academic tracks. Analysis of variance and chi-square analyses identified statistically significant relationships between curricular choices and various school and demographic variables. The potential for aspiration gaps and implications for school counselor educational and career planning are discussed.

4—Alberta Education. (2006). *Common curriculum framework for K-9 mathematics: Western and northern Canadian protocol.* https://open.alberta.ca/dataset/465efca1-3b52-4355-8fbd-88ce553b4384/resource/6d788800-5b0b-4e7b-a501-b249ee03e306/download/ccfkto9.pdf

This report describes the mathematics curricular framework for western and northern Canada for grades K–9.

4—Alberta Education. (2008). *Common curriculum framework for 10–12 mathematics: Western and northern Canadian protocol.* https://education.alberta.ca/media/564028/math10to12.pdf

This report describes the mathematics curricular framework for western and northern Canada for grades 10–12.

3—American Educational Research Association. (2006). Do the math: Cognitive demand makes a difference. *Research Points: Essential Information for Education Policy 4*(2), 1–4.

At the time this report was released, extending high expectations to all students in mathematics was a relatively new idea. The authors describe two challenges in mathematics education today: raising the floor by expanding achievement for all, and lifting the ceiling of achievement to better prepare future leaders in mathematics, as well as in science, engineering, and technology. Although these goals are not mutually exclusive, this research brief tackles the challenge of ensuring that whole groups of students are not excluded from higher mathematics learning. The brief provides a history and data on mathematics course-taking and describes the relevance of mathematical tasks to supporting cognitive demand. Three recommendations for policymakers are included.

2—American Mathematical Association of Two-Year Colleges. (2014). *Position statement of the American Mathematical Association of Two-Year Colleges: The appropriate use of Intermediate Algebra as a prerequisite course.* https://amatyc.org/page/PositionInterAlg.

This report by the American Mathematical Association of Two-Year Colleges discusses the appropriate use of intermediate algebra as a prerequisite course and makes several arguments resulting in a position that prerequisite courses other than intermediate algebra can adequately prepare students for courses of study that do not lead to calculus.

1—Antonovics, K., Black, S. E., Cullen, J. B., & Meiselman, A. Y. (2022). *Patterns, determinants, and consequences of ability tracking: Evidence from Texas Public Schools* (Working Paper No. 30370). National Bureau of Economic Research. https://doi.org/10.3386/w30370

Schools often track students to classes based on ability. Proponents of tracking argue it is a low-cost tool to improve learning since instruction is more effective when students are more homogeneous, while opponents argue it exacerbates initial differences in opportunities without strong evidence of efficacy. In fact, little is known about the pervasiveness or determinants of ability tracking in the U.S. To fill this gap, the authors used detailed administrative data from Texas to estimate the extent of tracking within schools for grades 4 through 8 over the years 2011–2019. They found substantial tracking; tracking within schools overwhelms any sorting by ability that takes place across schools. The most important determinant of tracking is heterogeneity in student ability, and schools operationalize tracking through the classification of students into categories such as gifted and disabled and curricular differentiation. When the authors examined how tracking changes in response to educational policies, they saw that schools decrease tracking in response to accountability pressures. Finally, when they explored how exposure to tracking correlates with student mobility in the achievement distribution, they found positive effects on high-achieving students with no negative effects on low-achieving students, suggesting that tracking may increase inequality by raising the ceiling.

4—Arizona State University Course Approval (n.d.). *Financial Literacy*. https://courseapproval.asu.edu/content/financial-literacy-12

This website describes Financial Literacy as a mathematics course that satisfies the fourth-year mathematics requirement and is designed to apply algebra, geometry, and consumer topics to real life.

4—Arkansas Department of Education (n.d.). *Quantitative literacy*. https://sites.google.com/pdarkansas.net/quantitative-literacy/home

This website provides information on the state's focus on quantitative literacy. On the website, quantitative literacy, or numeracy, is defined as the ability to use numbers and solve problems in real life and is a critical skill to function in 21st-century society. A strong foundation in mathematics is the center of STEM education and provides a way to describe and reason about models of real-world phenomena.

4—Arrington, K. L. (2018). *Texas's House Bill 5 as modern tracking structure: Social stratification reified?* [Unpublished doctoral dissertation]. University of Texas at Austin.

In 2013, Texas policymakers passed House Bill 5 (HB 5), which changed high school graduation requirements to a multi-tiered set of plans called the Foundation High School Program (FHSP). This hierarchical set of graduation plans groups students based on a chosen career endorsement and offers different content instruction based on their choices, mirroring tracking structures that categorize students into groups and then provide those groups with dissimilar instructional experiences. This project investigated whether HB 5 is achieving the hope of the bill's authors—to increase student engagement through allowing students to choose programs tailored to their career aspirations—or if the policy functionally operates as tracking. This study used a quantitative analysis of the data available through the Texas Education Agency (TEA) to look for descriptive patterns in the offerings and outcomes for students using the predictor variables of the type of or urbanicity of the district and the racial and socioeconomic composition of each district. This study found significant differences in the endorsements offered by districts based on urbanicity of the district, specifically differences between rural districts and the rest of the state. The study found differences in who was enrolled in FHSP while enrollment was considered optional, with significant differences by year and for those students enrolled in rural districts as well as for students in districts with higher proportion of African American/Black and Hispanic/Latino students. There are significant differences in graduates under FHSP who earned the distinguished level of achievement based on these predictors and specific differences in the odds of students in suburban districts with higher proportions of African American/Black students graduating under FHSP and earning the distinguished level of achievement. Implications indicate that FHSP operates as a means to uphold the system of student tracking.

1—Asim, M., Kurlaender, M., & Reed, S. (2019). *12th grade course-taking and the distribution of opportunity for college readiness in mathematics.* Policy Analysis for California Education, PACE. https://files.eric.ed.gov/fulltext/ED600439.pdf

In this report, the authors explore the patterns in mathematics course-taking among California public high school seniors. They describe what courses students are enrolled in and how course participation varies by key student characteristics, such as race/ethnicity, socioeconomic status, and performance level on the state's 11th grade assessments. The authors also explore course-taking patterns for students eligible for California's public four-year colleges—the California State University (CSU) and the University of California (UC)—and for applicants and admitted students at the CSU and UC. The findings demonstrate that although a large majority of college-bound students enrolled in math in their final year of high school, advanced math pathways were not equally accessed among high school seniors. These disparities in enrollment patterns by race/ethnicity and school characteristics likely contribute to disparities in postsecondary access and success.

3—Attewell, P., & Domina, T. (2008). Raising the bar: Curricular intensity and academic performance. *Educational Evaluation and Policy Analysis*, *30*(1), 51–71.

Using national transcript data, the authors examine inequality in access to an advanced curriculum in high school and assess the consequences of curricular intensity on test scores and college entry. Inequalities in curricular intensity are primarily explained by student socioeconomic status effects that operate within schools rather than between schools. They find significant positive effects of taking a more intense curriculum on 12th-grade test scores and in probabilities of entry to and completion of college. However, the effect sizes of curricular intensity are generally modest, smaller than advocates of curricular upgrading policies have implied.

1—Aughinbaugh, A. (2012). *A comparison of college attendance and high school coursework from two cohorts of youth*. U.S. Bureau of Labor Statistics. https://www.bls.gov/opub/btn/volume-1/a-comparison-of-college-attendance-and-high-school-coursework-from-two-cohorts-of-youth.htm

This analysis examines how the relationship between high school courses and college attendance may have changed between the late 1970s and the late 1990s. The article uses two data sources: the National Longitudinal Survey of Youth 1979 (NLSY79), a nationally representative sample of 12,686 men and women born from 1957 to 1964 and living in the United States at the time of the initial survey, and the National Longitudinal Survey of Youth 1997 (NLSY97), a nationally representative sample of 8,984 men and women born from 1980 to 1984 and living in the United States at the time of the initial survey.

1—Bailey, T., Jeong, D.W., & Cho, S. W. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review, 29*(2), 255–270. https://eric.ed.gov/?id=EJ876583

After being assessed, many students entering community colleges are referred to one or more levels of developmental education. While the need to assist students with weak academic skills is well known, little research has examined student progression through multiple levels of developmental education and into entry-level college courses. The purpose of this paper is to analyze the patterns and determinants of student progression through sequences of developmental education starting from initial referral. The results indicate that fewer than one half of the students who are referred to remediation actually complete the entire sequence to which they are referred. About 30 percent of students referred to developmental education do not enroll in any remedial course, and only about 60 percent of referred students actually enroll in the remedial course to which they were referred. The results also show that more students exit their developmental sequences because they "did not enroll in" the first or a subsequent course than because they "failed or withdrew from" a course in which they were enrolled. Men, older students, African American students, part-time students, and students in vocational programs are less likely to progress through their full remedial sequences.

4—Baker, D., Mehlberg, S., & MacNeille, B. (2018). *College readiness math initiative: Bridge to College impact evaluation*. BERC Group. https://www.sbctc.edu/resources/documents/about/agency/initiatives-projects/bridge-to-college-impact-eval-mar2018.pdf

College Spark Washington (CSW) is a grant-making organization dedicated to improving educational outcomes for low-income students in Washington State. In 2014, CSW launched a multifaceted Math Initiative designed to support college readiness. The goal of the initiative was to prepare students to transition into college-level math and English language arts (ELA) without the need for remediation or other placement courses. CSW is in the implementation stage of this multi-pronged strategy focused on college readiness expectations. The seven-year initiative includes strategies for students who perform at all levels on the Smarter Balanced Assessment (SBA). Despite sustained efforts to eliminate barriers to postsecondary success, several barriers remain for students in Washington State. Mandatory testing in high school, placement testing in college, and placement into pre-college courses once enrolled increase the likelihood that students will not persist in college. Bridge to College (BtC) high school courses were designed to reduce the number of barriers students experience in their pursuit of postsecondary opportunities. These courses are intended to be one intervention among many to help students prepare for and succeed in college-level math and ELA coursework. The report details the impact evaluation conducted from fall 2017 to spring 2018. Overall, students who had taken high school BtC courses responded positively when asked about the support and instruction offered in these courses. Many felt the courses helped them develop a growth mindset and become more efficacious students. Quantitative data support these perspectives. Despite some promising preliminary outcomes and perceptions of the BtC courses, students and college faculty reported inconsistent experiences using these courses as alternative placement options in college.

3—Balfanz, R., Mac Iver, D. J., & Byrnes, V. (2006). The implementation and impact of evidence-based mathematics reforms in high-poverty middle schools: A multi-site, multi-year study. *Journal for Research in Mathematics Education, 37*(1), 33–64.

This article reports on the first four years of an effort to develop comprehensive and sustainable mathematics education reforms in high poverty middle schools. In four related analyses, we examine the levels of implementation achieved and impact of the reforms on various measures of achievement in the first three schools to implement the Talent Development (TD) Middle School Model's mathematics program, which combines coherent research-based instructional materials from the University of Chicago School Mathematics Project with a multi-tiered teacher support system of sustained professional development and in-class coaching. A moderate level of implementation was achieved. TD students outperformed students from control schools on multiple measures of achievement. The average effect size by the end of middle school was 0.24.

1, 2, 3—Barnett, E.A., Fay, M.P., Liston, C., & Reyna, R. (2022). *The role of higher education in high school math reform*. Community College Research Center. https://ccrc.tc.columbia.edu/media/k2/attachments/higher-education-high-school-math-reform.pdf

Change in high school math depends a great deal on policies, practices, and norms at the higher education level. To better understand this topic, researchers from the Community College Research Center (CCRC) and the Education Strategy Group (ESG) conducted interviews with representatives from national secondary math education organizations as well as individuals from each of three states—Georgia, Texas, and Washington—who are engaged in efforts to reform high school math. The interviewees are affiliated with Launch Years, an initiative aimed at helping states and school systems change the structure and content of secondary math coursework. In this report, the authors describe several domains of higher education practice that tend to present challenges to high school math reform. They discuss potential solutions to these challenges and consider ways that higher education can facilitate high school reform efforts. The authors then present short case studies of secondary math reform efforts in the three states (all of which have also undertaken math pathways reforms—at least to some extent—at the college level) with a focus on the influence of higher education in effecting change. The report concludes with a brief discussion of an overarching theme central to much of this work: the importance of sustained conversation and collaboration between math educators and administrators from both sectors.

4—Barnett, E. A., Chavarín, O., & Griffin, S. (2018). *Math transition courses in context: Preparing students for college success.* Community College Research Center. https://academiccommons.columbia.edu/doi/10.7916/d8-72b2-sj54/download

Drawing on data from several sources, including interviews from persons involved in the development of transition curricula in 11 states, this brief describes the design, implementation, and effectiveness of math transition curricula and how they fit into the current educational reform landscape. It identifies key trends and factors involved in the development of the curricula. Some transition curricula are aligned with math pathways options and may incorporate contextualized learning. In some cases, transition curricula may also be used along with other college readiness interventions, such as dual enrollment programs.

1, 2, 3—Barnett, E.A., Fay, M.P., Liston, C., & Reyna, R. (2022). *The role of higher education in high school math reform*. Community College Research Center. https://ccrc.tc.columbia.edu/media/k2/attachments/higher-education-high-school-math-reform.pdf

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1—Barta, J., Eglash, R., & Barkley, C. (2014). *Math is a verb: Activities and lessons from cultures around the world.* NCTM.

For many people in different cultures, mathematics is not simply something they learn in school but something they do as an intrinsic part of their everyday lives. This book provides guidance for teachers who would like to enhance their mathematics instruction by integrating it with examples and activities from cultures throughout the world. It provides culturally situated examples, each linked to Common Core objectives that show how mathematics can be more than a story problem or an exercise in a worksheet with little or no context. The eleven chapters provide a range of activities from around the world that teach students key math concepts while introducing them to a diversity of cultures. Each chapter has activities for specific grade bands (K–3, 4–8, and 9–12), and all activities are designed to encourage students to discover connections among math concepts, world cultures, and their own daily lives and communities.

4—Beal, J., Dolan, D., Lott, J., & Smith, J. P. (1990). *Integrated mathematics: Definitions, issues, and implications.* Exxon Education Foundation (Eric Document Reproduction Service No. ED 347 071). http://files.eric.ed.gov/fulltext/ED347071.pdf

The “Integrated Mathematics Project” was conducted under the sponsorship of the Montana Council of Teachers of Mathematics and designed to examine the issues of teaching secondary school mathematics in an integrated manner to all students. The purpose was to improve mathematics literacy in the general population in accordance with suggestions by both the 1987 Mathematical Sciences Education Board (MSEB) draft report, “A Framework for the Revision of the K–12 Mathematics Curriculum,” and the 1989 National Council of

Teachers of Mathematics (NCTM) publication, "Curriculum and Evaluation Standards for School Mathematics." A questionnaire was developed to determine the extent of the interest in, as well as the curriculum structure and content of, the pedagogical strategies critical to an integrated secondary mathematics program and to its expected outcomes and implementation. Responses from 54 percent of state supervisors ($n = 27$), 31 percent of district supervisors ($n = 140$), 33 percent of mathematics teacher educators ($n = 164$), and 28 percent of mathematics teachers ($n = 140$) were compiled to develop a first report. The data analysis focused on the following areas of concern: (a) a definition of integrated mathematics; (b) extent of interest in integrated mathematics at the state, district, and teacher preparation levels, including expected objectives for students and teachers; and (c) implications of adopting such a program in secondary schools for professional organizations, colleges and universities, state departments of education, school systems, testing organizations, curriculum developers and publishers, and funding agencies. The rest of the report discusses each of three areas in turn. Appendices, forming about two-thirds of the document, include questionnaires used, national demographic data, survey results, and a packet of materials sent to participants who reviewed the second draft of the policy report. A separately bound executive summary accompanies the report.

1—Beasley, M. A., & Fischer, M. J. (2012). Why they leave: The impact of stereotype threat on the attrition of women and minorities from science, math, and engineering majors. *Social Psychology of Education, 15*, 427–448. https://link.springer.com/content/pdf/10.1007/s11218-012-9185-3.pdf?pdf=button

This paper examines the effects of group performance anxiety on the attrition of women and minorities from science, math, and engineering majors. While past research has relied primarily on the academic deficits and lower socioeconomic status of women and minorities to explain their absence from these fields, the authors focus on the impact of stereotype threat—the anxiety caused by the expectation of being judged based on a negative group stereotype. Using data from the National Longitudinal Survey of Freshmen, the findings indicate that minorities experience stereotype threat more strongly than whites, although women do not suffer from stereotype threat more than men. The findings also reveal that stereotype threat has a significant positive effect on the likelihood of women, minorities, and surprisingly, white men leaving science, technology, engineering, and math majors.

1—Beilock, S.L., & Willingham, D.T. (2014). Math anxiety: Can teachers help students reduce it? Ask the cognitive scientist. *American Educator*, *38*(2), 28. http://files.eric.ed.gov/fulltext/EJ1043398.pdf

Cognitive science is an interdisciplinary field of researchers from psychology, neuroscience, linguistics, philosophy, computer science, and anthropology who seek to understand the mind. In this regular "American Educator" column, the authors consider findings from this field that are strong and clear enough to merit classroom application. This issue discusses math anxiety. Math anxiety is not limited to a minority of individuals nor to one country. International comparisons of high school students show that some students in every country are anxious about math. It is perhaps not surprising that there is an inverse relationship between anxiety and efficacy: countries where students are less proficient in math (as measured by the Program for International Student Assessment) tend to have higher levels of math anxiety. Topics discussed in this month's column include when and how math anxiety emerges, which students tend to be most susceptible to math anxiety, social influences that may impact math anxiety, and what teachers can do to remediate math anxiety.

4—Berk, D., Taber, S. B., Gorowara, C. C., & Poetzl, C. (2009). Developing prospective elementary teachers' flexibility in the domain of proportional reasoning. *Mathematical Thinking and Learning, 11*(3), 113-135. https://www.researchgate.net/profile/Dawn-Berk/publication/232852594_Developing_Prospective_Elementary_Teachers%27_Flexibility_in_the_Domain_of_Proportional_Reasoning/links/563230fc08ae3de9381fb987/Developing-Prospective-Elementary-Teachers-Flexibility-in-the-Domain-of-Proportional-Reasoning.pdf

Flexibility in the use of mathematics procedures consists of the ability to employ multiple solution methods across a set of problems, solve the same problem using multiple methods, and choose strategically from among methods so as to reduce computational demands. The purpose of this study was to characterize prospective elementary teachers' (n = 148) flexibility in the domain of proportional reasoning before formal instruction and to test the effects of two versions of an intervention that engaged prospective teachers in comparing different solutions to proportion problems. Results indicate that (a) participants exhibited limited flexibility before formal instruction, (b) the intervention led to significant gains in participants' flexibility that were retained six months after instruction, and (c) varying the source of the problem solutions that participants compared had no discernible effects on their flexibility. Implications for mathematics teacher preparation and for research on flexibility development are provided.

1—Berry, R. Q., III, Ellis, M., & Hughes, S. (2014). Examining a history of failed reforms and recent stories of success: Mathematics education and Black learners of mathematics in the United States. *Race Ethnicity and Education*, *17*(4), 540–568. https://www.academia.edu/download/38889739/Berry-Ellis-Hughes-ArticleonHistoryofMathReforms.pdf

There is a long history indicating that during times of reform, the interests and needs of Black children are in many ways dismissed. This history culminated in the 1990s in what are described today as the "Math Wars." The underlying narrative focuses on America's national security, technological interests, social efficiency, and the perpetuation of white privilege. There are intense debates focusing on curriculum, teaching, learning, and assessment, but little debate on understanding the realities of children's lives. Through a hybrid historical–critical race theory (CRT) lens informed largely by the work of Derrick Bell, this article makes the case that Black children have not benefited from reform agendas in mathematics, which are often situated in the larger political and social space. Moreover, the CRT analysis coupled with the historical critical methods of this article attempt to unveil the underpinning of the "mathematics for all" message, often touted in policy documents, as having done little to understand the influence of mathematics teaching and learning for Black children. The article also draws from Bell's work to offer counter-narratives through examples of the brilliance of the Black children who are thriving with mathematics, even within a context that tends to ignore them. The article concludes with implications for considering and addressing what we name the "opportunity to learn mathematics gap."

1, 3, 4—Berry, R.Q., III, & Larson, M. R. (2019). The need to catalyze change in high school mathematics. *Phi Delta Kappan 100*(6), 39–44. https://journals.sagepub.com/doi/abs/10.1177/0031721719834027?journalCode=pdka

Results on the National Assessment for Education Progress and the Program for International Student Assessment show that high school mathematics instruction is past due for a redesign. Despite calls for reform going back at least four decades, the structure of math instruction has remained largely the same. In April 2018, the National Council of Teachers of Mathematics released *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* to promote discussion of the changes needed. Robert Berry and Matthew Larson, current and past presidents of NCTM, describe the arguments within this report, asserting that the math curriculum needs to help students understand the mathematics that's part of daily life, that tracking of students and teachers should be abandoned, that instruction should involve all students as doers of math, and that all students should experience a common curriculum.

1—Best Colleges. (2020). *Bachelor's in math education program guide.* https://www.bestcolleges.com/features/math-education-degree-programs/

In this tool, authors provide information about schools that offer the option to earn a bachelor's in math education from an online school. Authors ranked the top online bachelor's in math education programs and compared schools by cost, convenience, and academic quality.

1—Bettinger, E. P., & Long, B. T. (2005). Remediation at the community college: Student participation and outcomes. *New Directions for Community Colleges*, *2005*(129), 17–26. https://eric.ed.gov/?id=EJ761022

This chapter explores the characteristics and features of remedial education at community colleges, examines participation in these courses, and reviews findings on the effects of remediation on student decisions and outcomes.

2—Blair, R., Kirkman, E.E., & Maxwell, J.W. (2018). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: Fall 2015 CBMS survey*. American Mathematical Society. http://www.ams.org/profession/data/cbms-survey/cbms2015

Every five years since 1965, the Conference Board of the Mathematical Sciences (CBMS) has sponsored a national survey of undergraduate mathematical and statistical sciences in the nation's two- and four-year colleges and universities. The 2015 CBMS survey, conducted with National Science Foundation support, is the 11th report in this series of now 50 years of data. The CBMS surveys study two-year college mathematics programs and the undergraduate programs of mathematics departments and statistics departments at four-year colleges and universities. Three different instruments are sent to a stratified random sample of these three populations, and this report presents the estimates computed using the responses to these questionnaires.

1—Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning.* Lawrence Erlbaum.

https://www.routledge.com/Experiencing-School-Mathematics-Traditional-and-Reform-Approaches-To-Teaching/Boaler/p/book/9780805840056

This is a revised edition of *Experiencing School Mathematics*, first published in 1997. *Experiencing School Mathematics* reports case studies of two schools that taught mathematics in totally different ways. Three hundred students were followed over three years, providing a wide range of data, including observations, interviews, questionnaires, and assessments, to show the ways students' beliefs and understandings were shaped by the different approaches to mathematics teaching. The interviews that are reproduced in the book give insight into what it meant to be a student in the classrooms of the two schools. Questions are raised about and new evidence is provided for

- the ways in which "traditional" and "reform oriented" mathematics teaching approaches can impact student attitude, beliefs, and achievement;
- the effectiveness of different teaching methods in preparing students for the demands of the "real world" and the 21st century;
- the impact of tracking and heterogeneous ability grouping; and
- gender and teaching styles--the potential of different teaching approaches for the attainment of equity.

1—Boaler, J. (2008). *What's math got to do with it? How parents and teachers can help children learn to love their least favorite subject*. Penguin.

In this book, the author outlines concrete, research-based solutions with the potential to improve mathematics teaching and learning, including classroom approaches, essential strategies for students, and advice for parents.

1—Boaler, J. (2011). Changing students' lives through the de-tracking of urban mathematics classrooms. *Journal of Urban Mathematics Education 4*(1), 7–14. https://www.sfusdmath.org/uploads/2/4/0/9/24098802/boaler_de-tracking.pdf

The author discusses how teaching environments that encourage high achievement from all students provide a range of possibilities for student learning that go beyond content knowledge. The author claims that heterogeneous classrooms based upon cooperation among students change student perceptions of who they are and who they can be, change perceptions of the nature of mathematics, teach students about the different qualities and contributions of students who are different from themselves, and challenge the racial segregation that continues in schools.

4—Boaler, J., Cordero, M., & Dieckmann, J. (2019) Pursuing gender equity in mathematics competitions: A case of mathematical freedom. *MAA Focus, 39* (1), 18-20. http://digitaleditions.walsworthprintgroup.com/publication/?i=566588&article_id=3302571&view=articleBrowser

The authors describe the results of a study of the Consortium for Mathematics and its Applications, a competition designed to explore mathematical modeling. To understand issues related to the exclusion of girls and women in the competition (and, therefore, in mathematics more broadly), the authors investigated features of the competition that created gender-equitable experiences. Using a multi-method design (including observations of the competition, interviews with and surveys of mathematics faculty, and interviews with and surveys of participating students), the authors explore questions such as "Why did you enter the competition?" Findings suggested three themes contributed to the equitable environment: the collaborative nature of the competition, multidimensional mathematics and modeling, and freedom to create. The authors explore each of these themes and present statistical findings from student surveys. The authors conclude that mathematical competitions can provide girls and women with opportunities for different kinds of engagement with mathematics.

2—Boaler, J., & Levitt, S.D. (2019, October 23). Opinion: Modern high school math should be about data science—not Algebra 2. *The Los Angeles Times*. https://www.youcubed.org/wp-content/uploads/2019/10/LA-times-op-ed.pdf

This *Los Angeles Times* opinion piece presents an argument for creating more modern and relevant mathematical experiences for students by putting data and its analysis at the center of high school mathematics. The authors share information about the focus on data in other countries, describe how international assessments also include issues of data literacy, and describe the Los Angeles Unified School District's effort to update the way math is taught through the inclusion of data science courses for students.

1—Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experiences of ability grouping—disaffection, polarisation and the construction of failure. *British Educational Research Journal*, *26*(5), 631–648. https://discovery.ucl.ac.uk/id/eprint/10001139/1/Boaler2000Students631.pdf

This paper reports findings from the first two years of a four-year longitudinal study into the ways that students' attitudes toward and achievement in mathematics are influenced by ability-grouping practices in six schools. Through the use of questionnaires administered to the whole cohort of 943 students, interviews with 72 students, and approximately 120 hours of classroom observation, the relative achievement in and the changes in attitudes toward mathematics are traced as the students move from year 8 to year 9, with students in four of the six schools moving from mixed-ability grouping to homogenous ability groups or "sets." Ability grouping was associated with curriculum polarization. This was enacted through restriction of opportunity to learn for students in lower sets, and students in top sets being required to learn at a pace that was, for many students, incompatible with understanding. The same teachers employed a more restricted range of teaching approaches with "homogeneous" groups than with mixed-ability groups, which impacted upon the students' experiences in profound and largely negative ways. Almost all of the students interviewed from "setted groups" were unhappy with their placement.

2—Boatman, A. (2021). Accelerating college remediation: Examining the effects of math course redesign on student academic success. *The Journal of Higher Education, 92*(6), 927–960. https://www.tandfonline.com/doi/abs/10.1080/00221546.2021.1888675?role=button&needAccess=true&journalCode=uhej20

Exploiting a statewide cutoff on a remediation placement exam along with data on student outcomes prior to and after a course redesign effort in Tennessee, the author estimates the effects of three different institutional redesign efforts (acceleration, modularization, and corequisite math remediation) on students' short-, moderate-, and long-term academic success. Findings indicate that students exposed to accelerated and corequisite developmental math courses had more positive outcomes than their peers exposed to traditional developmental math courses, but those in modularized courses did not. The magnitude of the estimated effect differs by the type of math redesign and the level of academic need of the students. These results provide insight into the extent to which the particular instruction and delivery methods of developmental courses affect performance in college-level math, credit accumulation, and persistence to degree.

1, 4—Box, J. F. (1985). *R. A. Fisher: The life of a scientist*. Wiley Publishing.

This biography describes a man who made important achievements in mathematical statistics, especially regarding the problems of estimation, analysis and design of experiments, and inductive inference.

1—Brelias, A. (2015). Mathematics for what? High school students reflect on mathematics as a tool for social inquiry. *Democracy & Education, 23*(1), 1–11. https://democracyeducationjournal.org/cgi/viewcontent.cgi?article=1163&context=home

This study examines high school students' views of mathematics as a tool for social inquiry in light of their classroom experiences using mathematics to explore social issues. A critical theoretical perspective on mathematics literacy is used to ascertain the ways in which their views challenge or affirm the dominant image of mathematics in society. The study concludes that mathematics applications addressing social justice issues are promising vehicles for developing students' appreciation of mathematics as a social problem-solving tool, an awareness of its limitations, and a healthy skepticism toward its uses.

1—Bressoud, D. (Ed.). (2017). *The role of calculus in the transition from high school to college mathematics*. Mathematical Association of America & National Council of Teachers of Mathematics. https://www.maa.org/sites/default/files/RoleOfCalc_rev.pdf

In March of 2016, a group of high school teachers, mathematicians, mathematics and science education researchers, state and district supervisors of mathematics, and representatives of organizations with a stake in the issues surrounding calculus in high school, which included the College Board and the National Academy of Sciences, met for three days in Washington, DC, to clarify what we know and what we need to know about the role of calculus in the transition from high school to college mathematics. This is a summary of the issues they identified.

1—Bressoud, D., Mesa, V., & Rasmussen, C. (2015). *Insights and recommendations from the MAA national study of college calculus.* Mathematics Association of America. https://eric.ed.gov/?id=EJ1078558

Over the past five years, the Mathematical Association of America, with support from the National Science Foundation, has explored the teaching of mainstream Calculus 1 at the postsecondary level, where "mainstream" means those courses that can be used as part of the prerequisite stream to more advanced postsecondary mathematics. The authors surveyed 213

colleges and universities, 502 instructors, and more than 14,000 students to learn who takes Calculus 1 in college, why they take it, their preparation for this class, and their experience in this class. The authors also began to identify the characteristics of those classes that are most successful in encouraging students to continue their pursuit of mathematics. Following up on these surveys, teams of researchers visited 20 of these institutions, including community and technical colleges, liberal arts colleges, and public and private universities, to see firsthand what some of the best programs were doing. The paper presents findings from this study, potentially of interest to those who are preparing students to succeed in college-level mathematics.

1—Bromberg, M., & Theokas, C. (2016). *Meandering toward graduation: Transcript outcomes of high school graduates.* The Education Trust. https://files.eric.ed.gov/fulltext/ED566663.pdf

Most students will need to earn a postsecondary credential in order to achieve a family-sustaining wage in today's economy. However, given unequal preparation at the outset of high school and the differing demands of different postsecondary settings, achieving this aspiration is not without its challenges. While graduation rates have reached an all-time high and postsecondary enrollment rates are steadily rising, thousands of those new college students are testing into remedial reading, writing, or math courses because they don't have the foundation to perform at the levels demanded in college classes. Employers, too, report that high school graduates don't have the basic foundational skills to start in entry-level positions. To better understand this paradox, authors explored the most recent national database of high school transcripts to better understand the experiences and preparation of the nation's graduates. They conclude that students are "meandering toward graduation" and that rather than ensuring students have access to a cohesive curriculum that aligns high school coursework and students' future goals, high schools are prioritizing credit accrual, which treats graduation as the end goal.

1—Brown, M., Brown, P., & Bibby, T. (2008). I would rather die: Reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education, 10*(1), 3–18. https://www.tandfonline.com/doi/pdf/10.1080/14794800801915814?needAccess=true&role=button

Improving participation rates in specialist mathematics after the subject ceases to be compulsory at age 16 is part of government policy in England. This article provides independent and recent support for earlier findings concerning reasons for nonparticipation, based on free response and closed items in a questionnaire with a sample of over 1,500 students in 17 schools, close to the moment of choice. The analysis supports findings that perceived difficulty and lack of confidence are important reasons for students not continuing with mathematics, and that perceived dislike and boredom, and lack of relevance, are also factors. There is a close relationship between reasons for nonparticipation and predicted grade, and a weaker relation to gender. An analysis of the effects of schools demonstrates that enjoyment is the main factor differentiating schools with high and low participation indices. Building on discussion of these findings, ways of improving participation are briefly suggested.

3—Bryk, A. S., Gomez, L. M., Grunow, A., & LeMahieu, P. G. (2015). *Learning to improve: How America's schools can get better at getting better*. Harvard Education Press. https://www.carnegiefoundation.org/resources/publications/learning-to-improve/

Using ideas borrowed from improvement science, the authors show how a process of disciplined inquiry can be combined with the use of networks to identify, adapt, and successfully scale up promising interventions in education. Organized around six core principles, the book shows how "networked improvement communities" can bring together researchers and practitioners to accelerate learning in key areas of education. Examples include efforts to address the high rates of failure among students in community college remedial math courses and strategies for improving feedback to novice teachers.

2—Burdman, P. (2015, April). *Degrees of freedom: Diversifying math requirements for college readiness and graduation* (Report 1 of a 3-part series). PACE: Policy Analysis for California Education and LearningWorks. https://www.edpolicyinca.org/publications/degrees-freedom-diversifying-math-requirements-college-readiness-and-graduation-report-1-3-part-series

Since the mid-20th century, the standard U.S. high school and college math curriculum has been based on two years of Algebra and a year of Geometry, preparing students to take classes in Precalculus, followed by Calculus. That pathway became solidified after the 1957 launch of the Soviet satellite Sputnik motivated reforms in U.S. science and engineering education to boost the nation's technological prowess. Students' math pursuits have been differentiated primarily by how far or how rapidly they proceed along a clearly defined trajectory that has changed little since then. But evolutions in various disciplines and in learning sciences are calling into question the relevance and utility of this trajectory as a requirement for all students. The emerging movement is toward differentiated "math pathways" with distinct trajectories tied to students' goals. Alternatives emphasizing statistics, modeling, computer science, and quantitative reasoning that are cropping up in high schools and colleges are beginning to challenge the dominance of the familiar math sequence. The drive toward acknowledging the importance of multiple domains within math is prompted largely by two developments: (a) **Technological tectonics**: The advent of new technologies is creating novel applications of math in various academic disciplines, elevating the importance of statistics, data analysis, modeling, and computer science in the undergraduate curriculum. It is also leading faculty members outside of math departments to pay more attention to their students' quantitative preparation. (b) **Demand for deeper learning**: Learning scientists and math educators are emphasizing the importance of students' developing the capacity to use math skills and knowledge to solve problems in various contexts rather than simply learning isolated procedures and facts. American students' poor performance in traditional math sequences as well as the high proportion of college students taking remedial math have some reformers asking whether more applied courses would better lend themselves to the effective instruction needed to support college success. The dialogues converge in broader conversations about how colleges' expectations shape what students need to learn in K–12 schools—as well as in community colleges, which send hundreds of thousands of transfer students annually to four-year institutions. Decisions about math requirements and expectations will have a major impact on the academic opportunities of millions of students nationally.

1—Burdman, P. (2018). *The mathematics of opportunity: Rethinking the role of math in educational equity.* Just Equations. https://justequations.org/resource/the-mathematics-of-opportunity-rethinking-the-role-of-math-in-educational-equity

In Just Equations' report *The Mathematics of Opportunity,* the authors argue that traditional approaches to mathematics education can contribute to inequity and highlight emerging approaches to change that equation. The goal is to ensure that math instruction is more equity-oriented and that math requirements are more valid and aligned across educational systems. Quantitative reasoning skills are an important underpinning for achievement in school, at work, and in life. But, perhaps because it plays such a primary role in fostering such skills, mathematics can be wielded in ways that arbitrarily close doors to educational advancement.

Even as math expectations can serve as a foundation for academic success, they can also operate as arbitrary filters that needlessly stop many students in their educational tracks, especially students of color and others who are traditionally marginalized in the education system. In fact, surveys have shown that a majority of the U.S. population dislikes and fears mathematics. The architecture of math opportunity for too long has been shaped by misconceptions about math ability that magnify existing inequities. These, in turn, are reinforced by math's use as a marker or pedigree that confers access to opportunities. Designing a new mathematics of opportunity entails rethinking the way content requirements, instructional approaches, and assessment practices combine to form policies that can determine students' future educational opportunities.

4—Burdman, P. (2019). *Re-thinking the role of Algebra 2 in college readiness*. Just Equations. https://justequations.org/news/rethinking-the-role-of-algebra-two/

The author discusses the problems with relying on Algebra II course-taking in high school as a key predictor of future college enrollment, and how new course pathways such as statistics and data science may be a better fit for most students. She describes current efforts in many states to diversify available pathways and meet the needs of more students.

3—Burkhardt, H. (1990). On specifying a national curriculum. In I. Wirszup & R. Streit (Eds.), *Developments in school mathematics worldwide*, vol. 2 (pp. 98–11). National Council of Teachers of Mathematics. https://www.mathshell.com/papers/pdf/hb_1990_spec_nat_curr.pdf

The author discusses the many challenges involved in establishing a national curriculum, including faithful implementation by classroom teachers. He notes that in cases where the changes sought are substantial, this problem has yet to be solved anywhere worldwide, and that in terms of the patterns of classroom learning activity and student performance, a qualitative mismatch between stated intentions and outcomes is the norm.

2—Burning Glass, IBM, & BHEF: Business–Higher Education Forum. (2017). *The quant crunch: How the demand for data science skills is disrupting the job market.* Burning Glass Technologies. http://www.bhef.com/publications/quant-crunch-how-demand-data-science-skills-disrupting-job-market

This report is the result of a research partnership between Burning Glass Technologies, BHEF, and IBM, motivated by the need to close the data science and analytics skills gap through data-driven insights and increased collaboration between higher education and industry. It defines the data science and analytics (DSA) landscape, presents research findings about the skills gap, adds context to the DSA jobs and skills that are disrupters, and offers recommendations to alleviate the DSA talent shortage.

3—Burris, C. C., Heubert, J. P., & Levin, H. M. (2006). Accelerating mathematics achievement using heterogeneous grouping. *American Educational Research Journal*, *43*(1), 137–154. https://journals.sagepub.com/doi/abs/10.3102/00028312043001105?journalCode=aera

This longitudinal study examined the effects of providing an accelerated mathematics curriculum in heterogeneously grouped middle school classes in a diverse suburban school district. A quasi-experimental cohort design was used to evaluate subsequent completion of advanced high school math courses as well as academic achievement. Results showed that probability of completion of advanced math courses increased significantly and markedly in all groups, including minority students, students of low socioeconomic status, and students at all initial achievement levels. Also, the performance of initial high achievers did not differ statistically in heterogeneous classes relative to previous homogeneous grouping, and rates of participation in advanced placement calculus and test scores improved.

1—Burris, C. C., & Weiner, K. G. (2005). Closing the achievement gap by detracking. *Phi Delta Kappan 86*(8), 594–598. https://nepc.colorado.edu/sites/default/files/publications/EPSL-0505-111-EPRU.pdf

Achievement follows from opportunities, Ms. Burris and Mr. Welner assert, and the persistent practice of tracking denies a range of opportunities to large numbers of students. The authors argue that a disproportionate number of minority students are tracked, which is one of the underlying reasons that the achievement gap has remained so persistent. The authors describe how a diverse suburban district in New York narrowed the gap by offering its high-track curriculum to all students.

2—Business-Higher Education Forum. (2019). *Reskilling America's workforce: Exploring the nation's future STEM workforce needs; recommendations for federal agency engagement.* http://www.bhef.com/publications/reskilling-america's-workforce-exploring-nations-future-stem-workforce-needs

This report from a workshop funded by the National Science Foundation recommends how federal agencies can more effectively support STEM workforce needs. Business executives and CEOs, college and university presidents, professional associations, state leaders, and heads of federal agencies recognize that developing a STEM-capable workforce is critical for the nation. Innovation is occurring on a daily basis to create and understand the best methods to train, reskill, and educate our nation's workforce, with stakeholders moving rapidly in the same direction. At this critical juncture, there is an even more important role for federal agencies to serve as amplifiers, investors, and leaders in propelling the development of our nation's workforce now and in the future. As such, authors call on leaders in the federal government along with business and academia to build the pathways, partnerships, and programs that will help the country reach scale and meet the evolving needs of the future workforce in the digital economy.

2—Business–Higher Education Forum & Burning Glass Technologies. (2018). *The new foundational skills of the digital economy: Developing the professionals of the future*. https://www.burning-glass.com/wp-content/uploads/New_Foundational_Skills.pdf

This paper reports on a search for the New Foundational Skills of the digital economy. How and when do evolving skills change the job market? Which skills are in demand in both digitally intensive jobs, and more broadly? Which skills retain their value over time? If such a set of emergent, critical skills exists, how do the skills interact, and what do they mean for job seekers and incumbent employees, educators, and employers? To find out, the Business-Higher Education Forum (BHEF) commissioned Burning Glass Technologies to examine skills in the job market by drawing from a set of more than 150 million unique U.S. job postings, dating back to 2007. The research identified 14 skills that have become foundational in the new economy, which converge in three interrelated groups: human skills, business skills, and digital skills. Human skills have a long history of close study, so BHEF and Burning Glass plan to subject these two other major segments of the skills continuum—business enabler and digital building block skills—to similar scrutiny.

1—Byun, S. Y., Irvin, M. J., & Bell, B. A. (2015). Advanced math course taking: Effects on math achievement and college enrollment. *The Journal of Experimental Education*, *83*(4), 439–468. https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4620065/

Using data from the Educational Longitudinal Study of 2002–2006, the authors investigated the effects of advanced math course-taking on math achievement and college enrollment and how such effects varied by socioeconomic status and race/ethnicity. Results from propensity score matching and sensitivity analyses showed that advanced math course-taking had positive effects on math achievement and college enrollment. Results also demonstrated that the effect of advanced math course-taking on math achievement was greater for low socioeconomic status students than for high socioeconomic status students, but smaller for Black students than for white students. No interaction effects were found for college enrollment. Limitations, policy implications, and future research directions are discussed.

4—California Department of Education (2021). *Teaching & learning—high school graduation requirements.* https://www.cde.ca.gov/ci/gs/hs/cefhsgradreq.asp

The website shows state and district requirements that students must fulfill to receive a high school diploma. State-mandated graduation course requirements (the state minimums) are as follows:

- Three years of English
- Two years of mathematics (including Algebra I)
- Three years of social science (including U.S. history and geography; world history, culture, and geography; one semester of American government; and one semester of economics)
- Two years of science (including biology and physical science)

- Two years of physical education
- One year of foreign language or visual and performing arts or, commencing with the 2012–13 school year, career technical education. For purposes of satisfying the minimum course requirement, a course in American Sign Language shall be deemed a course in foreign language

Students who successfully complete Algebra I in middle school must still complete a minimum of two years of mathematics in high school. Recognizing that these 13 courses of preparation are state minimum requirements, local school boards often set local graduation requirements that exceed the state-mandated requirements.

1—Callahan, R. M. (2005). Tracking and high school English learners: Limiting opportunity to learn. *American Educational Research Journal*, *42*(2), 305–328. https://www.academia.edu/download/66947266/Tracking_and_High_School_English_Learner20210504-7584-1sf1fdn.pdf

Programs and policies related to the education of English learners are often based on the belief that fluency in English is the primary, if not sole, requirement for academic success. While English is in fact necessary for academic success in U.S. schools, so is a strong base in content-area academics. This study investigated the effects of track placement and English proficiency on secondary English learners' academic achievement while taking students' previous schooling and length of time enrolled in U.S. schools into account. In the case of a variety of outcomes, track placement was a better predictor of achievement than proficiency in English. Results indicate that track placement is a better predictor of English learners' academic performance than proficiency in English, highlighting the importance of quality instruction for English learners.

4—Cann, V. L. (2018). Harrison County high schools to offer financial algebra as core credit for upcoming school year. *The Exponent Telegram.* https://www.wvnews.com/theet/news/harrison-county-high-schools-to-offer-financial-algebra-as-core-credit-for-upcoming-school-year/article_d34e6b54-2917-5d5d-8ae6-f7a023f450c2.html

This article describes how, beginning in July 2018, all Harrison County high school students were able to take Financial Algebra for a math credit rather than elective credit. "While remaining true to the connection with algebra concepts, Financial Algebra provides a more concrete connection with concepts," Harrison Assistant Superintendent Dr. Donna Hage said. "It is important that all of our students take at least one math course per year, so utilizing this course for a math credit also helps our school system fulfill that philosophy with an additional course for a math credit."

2—Carnegie Math Pathways. (2023). *Our approach.* https://www.carnegiemathpathways.org/approach/

The Carnegie Math Pathways program "combines research and experience from educators to create an approach that supports students who have previously struggled with math to deeply understand and even love the subject." On average, Pathways students achieve success in

college mathematics courses at three times the rate of their peers by engaging with relevant content in a productively challenging and supportive environment. The Carnegie Math Pathways website describes the program's approach and results.

3, 4—Carnegie Math Pathways. (2022). *Moving mathematical mountains: A decade of educator-led change to make math a gateway to success for all students.* https://carnegiemathpathways.org/wp-content/uploads/2022/08/Moving-Mathematical-Mountains-A-Decade-of-Educator-Led-Change-to-Make-Math-a-Gateway-to-Success-for-All-Students.pdf

In 2010, the Carnegie Math Pathways Networked Improvement Community was established to bring together educators and researchers, using an improvement science framework to remake the entry-level college math system into a more equitable and empowering gateway to college success and completion for every student. Over the last decade, Carnegie Math Pathways (CMP) has helped propel tens of thousands of students to and through gateway college mathematics, increasing outcomes by an average of three- and four-fold compared to those of traditional math sequences. These increases hold across sex, race, and ethnicity, with students experiencing higher confidence in themselves as math learners and doers and higher graduation rates compared to students in standard remedial math courses. CMP programs have helped colleges and systems transform their math offerings and advance their college completion goals. In the process, we have amassed a community of over 1,000 educators, who, through their commitment to promoting a more equitable approach to math learning and teaching, have driven the creation and continuous improvement of innovative course solutions and instructional resources. The collective efforts of the Pathways Network have helped shape a national math pathways movement, prompting institutions to critically rethink their entry-level and foundational math offerings and expanding the body of evidence proving the effectiveness of corequisites and how to support their successful implementation. This report highlights the impact and key learnings from over 10 years of Carnegie Math Pathways implementation. Among the findings are that alternative math pathways create more equitable and empowering math opportunities for students.

1—Carnevale, A. P., & Desrochers, D. M. (2003). Preparing students for the knowledge economy: What school counselors need to know. *Professional School Counseling*, *6*(4), 228–236. https://www.jstor.org/stable/42732435

This paper presents tips for school counselors in guiding students to good career choices and future workplace environments. It discusses the need for counselors to be constantly aware of trends in employment opportunities, the role of counselors in explaining the significance of educational attainment of graduates, and advice on developing the skills and abilities that employers of the future might require from graduates and job applicants.

1—Carnevale, A. P., & Strohl, J. (2010). How increasing college access is increasing inequality, and what to do about it. In Richard D. Kahlenberg (Ed.), *Rewarding strivers: Helping low-income students succeed in college*. Century Foundation Press. https://cew.georgetown.edu/how-increasing-college-access-is-increasing-inequality-and-what-to-do-about-it-2

The authors share their research on a coherent and concrete way for colleges and universities to provide a leg up to economically disadvantaged students in selective college admissions. This research was referenced in the affirmative action Supreme Court case *Fisher vs. University of Texas* in two amicus briefs: Fisher I: Brief of the American Association for Affirmative Action as Amicus Curiae in Support of Respondent and Fisher II: Brief of Richard D. Kahlenberg as Amicus Curiae in Support of Neither Party.

3—Céledon-Pattichis, S., White, D. Y., & Civil, M. (Eds.). (2017). *Access and equity: Promoting high quality mathematics in PreK–grade 2*. NCTM. https://tdsb.insigniails.com/Library/ItemDetail?l=&i=2623323&proID=1946&ti=11

Employing mathematical modeling to engage students at all levels: Mathematical modeling plays an increasingly important role both in real-life applications—in engineering, business, the social sciences, climate study, advanced design, and more—and within mathematics education itself. This 2016 volume of Annual Perspectives in Mathematics Education (APME) focuses on this key topic from a wide variety of perspectives and distinguishes it from modeling mathematics. The book's 25 chapters are grouped into seven sections: Understanding Models and Modeling; Using Models to Represent Mathematics; Teaching and Learning about Mathematical Modeling; Mathematical Modeling as a Vehicle for STEM Learning; Designing Modeling-Oriented Tasks and Curricula; Assessing Mathematical Modeling; Supporting Teachers' Learning about Mathematical Modeling. Written by leading mathematics educators, mathematicians, and researchers, the chapters in this volume include reflections on the meaning and practice of modeling, case studies, research observations, and proven strategies for implementation in the classroom and in teacher education.

1—Cha, S. H. (2015). Exploring disparities in taking high level math courses in public high schools. *KEDI Journal of Educational Policy*, *12*(1). https://www.kedi.re.kr/eng/kedi/cmmn/file/fileDown.do?menuNo=200067&atchFileId=FILE_000000000003468&fileSn=1&bbsId=

The study examines the disparity in which mathematics courses students take at public high schools under the more rigorous graduation requirements policy in the U.S. Using nationally representative data from the 2005 National Assessment of Educational Progress–High School Transcript Study (2005 NAEP-HSTS), the study used a two-level hierarchical generalized linear model (HGLM) to explore the effect of both student and school characteristics on the likelihood that students take high-level mathematics courses. The study found that Black and Hispanic students, students from low-income families, and students with low educational expectations are less likely to take high-level mathematics courses than their counterparts. At the school level, school socioeconomic level, measured by free and reduced-price lunch program eligibility, was negatively related to the likelihood of taking high-level mathematics courses. Students at suburban schools had higher odds than those at rural schools of taking Advanced Mathematics, Calculus, and Advanced Placement Calculus.

3,4—Charles A. Dana Center at the University of Texas at Austin. (2017a). *Advanced quantitative reasoning/advanced mathematical decision making (AQR/AMDM).*

http://www.utdanacenter.org/pre-kindergarten-12-education/tools-for-teaching-and-learning/advanced-quantitative-reasoning-advanced-mathematical-decision-making/

This website describes a 12th-grade capstone course that engages students in relevant problems and prepares them for higher education and the workplace by supporting their quantitative reasoning, statistical reasoning, and use of modeling tools to solve problems in applied situations. The course materials are designed for a yearlong course to follow Algebra II or Integrated Mathematics 3 that emphasizes statistics, quantitative reasoning, modeling, and financial applications. The materials prepare students to use a variety of mathematical tools and approaches to model a range of situations and solve problems.

2—Charles A. Dana Center at the University of Texas at Austin. (2017b). *Dana Center Mathematics Pathways (DCMP) Model.* www.utdanacenter.org/higher-education/dcmp/dcmp-model/

This website describes how the Dana Center Mathematics Pathways (DCMP) model supports students' mathematics access, ability to obtain degree completion, and equitable outcomes that empower individuals to engage productively in a society and economy that increasingly rely on data and quantitative reasoning.

The DCMP has worked in more than 30 states with numerous higher education systems and hundreds of colleges and universities to develop and implement mathematics pathways that

- are aligned to students' goals;
- accelerate student progress toward completion;
- integrate student learning supports; and
- use evidence-based curriculum and pedagogy.

2—Charles A. Dana Center at the University of Texas at Austin. (2018). *Creating structural change for student success: State mathematics task force accomplishments and progress*. https://dcmathpathways.org/resources/creating-structural-change-student-success-state-mathematics-task-force-accomplishments

This 2018 report provides an update of the current state efforts and progress toward increased student persistence and success as a result of implementing multiple mathematics pathways. This report specifically explores examples of innovative approaches to drafting and implementing task force recommendations from thirteen states working in collaboration with the Dana Center. Furthermore, this report provides a synopsis of key focus areas of state-level task force recommendations and concrete examples of customized state-level supports for the sustainability and scale math pathways

1, 2, 3, 4—Charles A. Dana Center at the University of Texas at Austin. (2019). *What is rigor in mathematics really?* [White paper]. https://dcmathpathways.org/resources/what-is-rigor-in-mathematics-really

When the focus for entry-level mathematics shifted, over a decade ago, from access to success, it catalyzed demand for accelerated multiple mathematics pathways alongside the algebraic-intensive pathway. Subsequent successes have shown more than three times the success rates for students in one third of the time for some programs, highlighting the concern about maintaining rigor within the discipline. In response to this concern, the Charles A. Dana Center engaged in a study of the meaning and intention of rigor in mathematics education. This paper first explores the meaning of rigor in mathematics education through a synthesis of interviews with leading mathematicians and educators, and presents a review of the literature in higher education and K–12. It concludes by offering recommendations for a shared definition of rigor and its implications for curriculum and instruction.

1, 2, 3, 4, 5—Charles A. Dana Center at the University of Texas at Austin. (2020). *Launch Years: A new vision for the transition from high school to postsecondary mathematics.* https://www.utdanacenter.org/launchyears

This website describes the Launch Year Initiative, designed to support students to overcome the barrier of degree completion. The initiative supports the scaling of mathematics pathways from high school through postsecondary education and into the workplace, aligned to students' goals and aspirations. Twenty states are joining this work, along with national organizations and leaders in mathematics education and educational equity. The states joining the Launch Years Initiative are Arizona, Arkansas, California, Colorado, Georgia, Indiana, Kansas, Louisiana, Maine, Maryland, Massachusetts, Michigan, New York, North Carolina, Oklahoma, Oregon, Rhode Island, Utah, Washington, and Wisconsin. Through these twenty states, the Launch Years Initiative has the potential to directly impact hundreds of thousands of students across the country. States are focusing on different areas, including designing and implementing postsecondary and high school mathematics pathways, modern math courses and content, equitable impact, and advising practices.

Along with states, leaders from major national mathematics organizations are collaborating to strengthen and advocate for work that improves the experiences and outcomes for students transitioning from high school into postsecondary education. The Launch Years Math Organizations Leadership Network (LY-MathLN) includes members from the Alliance of Indigenous Math Circles, American Mathematical Association of Two-Year Colleges, American Statistical Association, Association of Mathematics Teacher Educators, Association of State Supervisors of Mathematics, Benjamin Banneker Association, Conference Board of Mathematical Sciences, Mathematical Association of America, National Council of Teachers of Mathematics, NCSM: Leadership in Mathematics Education, and TODOS: Mathematics for ALL.

1,4,5—Charles A. Dana Center at the University of Texas at Austin. (2022). *Re-envisioning mathematics pathways to expand opportunities: The landscape of high school to postsecondary course sequences*. https://edstrategy.org/wp-content/uploads/2022/07/Re-Envisioning-Mathematics-Pathways-to-Expand-Opportunities_FINAL.pdf

This report includes analyses of states' available middle and high school student course-taking data to examine whether recent high school mathematics pathways reforms have influenced students' mathematics course enrollment. It also examines how students' mathematics course-taking patterns vary within and across states and how state policy levers such as graduation course requirements might be influencing students' mathematics course-taking decisions. The report includes a discussion of recent changes to states' standards and

policies for adopting instructional materials as well as updates on the student assessment landscape in mathematics. It also provides lessons learned and guidance from the field as well as recommendations and considerations for strategies to center equity and incorporate data into states' mathematics pathways efforts. Finally, the report includes noteworthy state-specific highlights, mathematics focus group insights, and key questions for additional research.

4—Chávez, Ó., Tarr, J. E., Grouws, D. A., & Soria, V. M. (2015). Third-year high school mathematics curriculum: Effects of content organization and curriculum implementation. *International Journal of Science and Mathematics Education* S97-S120. https://www.researchgate.net/profile/Douglas-Grouws/publication/271921576_THIRD-YEAR_HIGH_SCHOOL_MATHEMATICS_CURRICULUM_EFFECTS_OF_CONTENT_ORGANIZATION_AND_CURRICULUM_IMPLEMENTATION/links/57fbc08408ae51472e7e7e05/THIRD-YEAR-HIGH-SCHOOL-MATHEMATICS-CURRICULUM-EFFECTS-OF-CONTENT-ORGANIZATION-AND-CURRICULUM-IMPLEMENTATION.pdf

In this empirical study, the authors examined the effect of curriculum organization in US high schools where students could freely choose to study mathematics from textbooks that employed one of two types of content organization: an "integrated" approach or a (traditional) "subject-specific" approach. The study involved 2,242 high school students, enrolled in either Year 3 Mathematics or Algebra 2, in 10 schools in five geographically dispersed states. Taking account of curriculum implementation and students' prior mathematics learning, the authors analyzed two end-of-year outcome measures: a test of common objectives and a standardized achievement test. Their hierarchical linear models with three levels showed that students in the integrated curriculum scored significantly higher than those in the subject-specific curriculum on the common objectives test. In both outcome measures, gender and prior achievement were significant student-level predictors. In the standardized achievement test, ethnicity was a moderating factor. At the teacher level, in addition to curriculum type, teachers' orientation and free and reduced-price lunch eligibility were significant moderating factors. Opportunity to learn, implementation fidelity, teacher experience, and professional development were not significant predictors.

1—Chen, X. (2016). *Remedial coursetaking at U.S. public 2- and 4-year institutions: Scope, experiences, and outcomes* (NCES 2016-405). National Center for Education Statistics. https://files.eric.ed.gov/fulltext/ED568682.pdf

This report attempts to contribute to the literature with a descriptive analysis of beginning postsecondary students' course-taking spanning the six-year period between 2003 and 2009, documenting the scope, intensity, timing, and completion of remedial course-taking and its association with various postsecondary outcomes among students who began at public two- and four-year institutions. Remedial education programs may include support services in addition to pre-college-level coursework, both of which are designed to get underprepared students ready for college-level work. However, this report focuses only on remedial coursework (not support services), using the terms "remedial course taking," "college remediation," or "simply remediation" interchangeably to describe students' participation in college preparatory coursework at the postsecondary level. The study addresses the following questions: (a) What percentage of 2003–04 beginning postsecondary students at public two-

and four-year institutions took remedial courses from 2003 to 2009? What types of remedial courses did they take? What was the average number of remedial courses taken?; (b) Who took remedial courses? When did students take these courses? What were their completion rates?; and (c) Did remedial course completers and noncompleters experience different postsecondary outcomes than students who had similar demographic backgrounds, academic preparation, and enrollment characteristics but did not take any remedial courses? The data for this report were drawn from the 2004/09 Beginning Postsecondary Students Longitudinal Study (BPS:04/09) and its associated 2009 Postsecondary Education Transcript Study (PETS:09). It was found that not all students who enrolled in remedial courses passed them. The relationship between students' participation in and completion of remedial coursework and their subsequent college outcomes varied by their level of academic preparedness.

1—Chen, X., & Soldner, M. (2013). *STEM attrition: College students' paths into and out of STEM fields* (NCES 2014-001). https://files.eric.ed.gov/fulltext/ED544470.pdf

This report presents an examination of students' attrition from STEM fields over the course of six years in college using data from the 2004/09 Beginning Postsecondary Students Longitudinal Study (BPS:04/09) and the associated 2009 Postsecondary Education Transcript Study (PETS:09). In this report, the term "STEM attrition" refers to enrollment choices that result in potential STEM graduates (i.e., undergraduates who declare a STEM major) moving away from STEM fields by switching majors to non-STEM fields or leaving postsecondary education before earning a degree or certificate. The purpose of this study is to gain a better understanding of this attrition by (a) determining rates of attrition from STEM and non-STEM fields; (b) identifying characteristics of students who leave STEM fields; (c) comparing the STEM course-taking and performance of STEM leavers and persisters; and (d) examining the strength of various factors' associations with STEM attrition. Appended are (a) Glossary; (b) Technical Notes and Methodology; (c) Classification of Major Field of Study in BPS:04/09; and (d) Classification of Postsecondary STEM Courses in BPS:04/09.

1—Cheryan, S., Master, A., & Meltzoff, A. N. (2015). Cultural stereotypes as gatekeepers: Increasing girls' interest in computer science and engineering by diversifying stereotypes. *Frontiers in psychology*, 49. https://www.frontiersin.org/articles/10.3389/fpsyg.2015.00049/full

The authors propose that students' stereotypes about the culture of these fields—including the kind of people, the work involved, and the values of the field—steer girls away from choosing to enter them. Computer science and engineering are stereotyped in modern American culture as male-oriented fields that involve social isolation, an intense focus on machinery, and inborn brilliance. These stereotypes are compatible with qualities that are typically more valued in men than women in American culture. As a result, when computer science and engineering stereotypes are salient, girls report less interest in these fields than their male peers. However, altering these stereotypes—by broadening the representation of the people who do this work, the work itself, and the environments in which it occurs—significantly increases girls' sense of belonging and interest in the field. Academic stereotypes thus serve as gatekeepers, driving girls away from certain fields and constraining their learning opportunities and career aspirations.

4—Code.org. (2017). *Code.org Advocacy Coalition*. https://code.org/advocacy.

This website provides a clickable map where interested individuals can find a national overview and state-specific data on the computer science landscape.

3—Cohen, J. (2022). *Change agents: Transforming schools from the ground up.* Corwin Press.

Partners in School Innovation has worked with educators in implementing meaningful and lasting change across twenty-two school districts and eight states, to focus on helping underperforming schools through specialized adult learning tools; a results-oriented cycle of inquiry; professional development systems focused on coaching and collaboration; implementing improvement science; understanding the roles of race, class, culture, and power in schools; and more. This book presents those research-based practices through narratives chronicling the efforts of real-life educators.

1—Colorado Department of Education. (2023). *Graduation guidelines*. https://www.cde.state.co.us/postsecondary/graduationguidelines

The website shows state and district requirements that students must fulfill to receive a high school diploma to help students and their families plan for success after high school. Students choose from a Menu of Options—embedded in each school district's graduation requirements—to demonstrate their readiness for career, college, and the military, based on at least one measure in reading, writing, and communicating, and one measure in mathematics. Graduation Guidelines give students the chance to experience all kinds of learning—in and out of the classroom—and to achieve education and workplace credentials. Graduation Guidelines begin with the implementation of Individual Career and Academic Plans (ICAP), Essential Skills, and Colorado Academic Standards for all content areas.

4—Common Core Standards Writing Team. (2022). *Progressions for the Common Core State Standards for Mathematics* (February 28, 2023). Institute for Mathematics and Education, University of Arizona. https://mathematicalmusings.org/wp-content/uploads/2023/02/Progressions.pdf

The 2010 Common Core State Standards in mathematics began with narrative documents describing the progression of a topic across a number of grade levels, informed both by educational research and the structure of mathematics. Those documents were then organized into grade-level standards, and subsequent work focused on refining and revising the grade-level standards rather than refining the progressions documents. The 2023 Progressions for the Common Core State Standards are updated versions of the earlier progressions drafts, revised and edited to correspond with the standards by members of the original Progressions work team, together with other mathematicians, statisticians, and education researchers not involved in the initial writing. They note key connections among standards, point out cognitive difficulties and pedagogical solutions, and provide additional detail. The Progressions also provide additional resources for curriculum that illustrates the range and types of mathematical work described by the standards, discussions of individual standards, classroom tasks, teacher professional development, and understanding the importance of modeling and language in students' mathematical development.

1—Complete College America. *Corequisite remediation: Spanning the completion divide.* http://ccaspanning.wpengine.com/wp-content/uploads/2016/01/CCA-SpanningTheDivide-ExecutiveSummary.pdf.

This report describes an overview of the challenges of traditional student remediation and ways in which states are providing students a "bridge to success" through corequisite remediation. By "corequisite remediation," the authors refer to the process where students are doubling and tripling gateway college courses in half the time or better. States highlighted include Georgia, West Virginia, Tennessee, Indiana, and Colorado. Data on remediation is provided for all 50 states.

2—Conference Board of the Mathematical Sciences. (2016). *Active learning in post-secondary mathematics education.* https://www.cbmsweb.org/2016/07/active-learning-in-post-secondary-mathematics-education/

In this position statement, CBMS calls on institutions of higher education, mathematics departments and the mathematics faculty, public policymakers, and funding agencies to invest time and resources to ensure that effective active learning is incorporated into postsecondary mathematics classrooms. They further call on professional societies and funding agencies to continue their support of training and resources for the use of active learning. The authors believe that using active learning methods in a way that builds on the extensive previous and ongoing work to modernize mathematics curriculum and pedagogy will lead to richer and more meaningful mathematical experiences for both students and teachers.

4—Confrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. *Learning Mathematics 26,* 31–60.

Conventional treatments of functions start by building a rule of correspondence between x-values and y-values, typically by creating an equation of the form $y = f(x)$. The authors call this a *correspondence* approach to functions. However, in their work with students they have found that a *covariational* approach is often more powerful, where students working in a problem situation first fill down a table column with x-values, typically by adding 1, then fill down a y-column through an operation they construct within the problem context. Such an approach has the benefit of emphasizing rate of change. It also raises the question of what it is that should be called "rate" across different functional situations. The authors make two initial conjectures, first that a rate can be initially understood as a *unit per unit* comparison and second that a unit is the *invariant relationship between a successor and its predecessor*. Based on these conjectures they describe a variety of multiplicative units, then propose three ways of understanding rate of change in relation to exponential functions. Finally, they argue that rate is different from ratio and that an integrated understanding of rate is built from multiple concepts.

3—Cortes, K., Goodman, J., & Nomi, T. (2013). A double dose of algebra: Intensive math instruction has long-term benefits. *Education Next*, *13*(1), 70–77. https://go.gale.com/ps/i.do?id=GALE%7CA313012651&sid=googleScholar&v=2.1&it=r&linkaccess=fulltext&issn=15399664&p=AONE&sw=w&casa_token=1JXFJ1rUiEEAAAAA:3

L54nUnrFoLtfd_WRLOCBq7H9uD0UdTPYo-ICn_imj89vV1pUgE9yk32lp9iJKYWUmKtgrGInL6NxxGjf6k

This study examines the impact of Chicago Public School's double-dose algebra policy on such longer-run outcomes as advanced math coursework and performance, ACT scores, high-school graduation rates, and college enrollment rates. Using data that track students from 8th grade through college enrollment, authors analyze the effect of this innovative policy by comparing the outcomes for students just above and just below the double-dose threshold. These two groups of students are nearly identical in terms of academic skills and other characteristics, but differ in the extent to which they were exposed to this new approach to algebra. Comparing the two groups thus provides unusually rigorous evidence on the policy's impact. The study provides evidence of positive and substantial long-run impacts of intensive math instruction on college entrance exam scores, high school graduation rates, and college enrollment rates. Authors also show that the intervention was most successful for students with relatively high math skills but relatively low reading skills. Although the intervention was not particularly effective for the average affected student, the fact that it improved high school graduation and college enrollment rates for even a subset of low-performing and at-risk students is extraordinarily promising when targeted at the appropriate students. In this case, those were students with only moderately low math skills but below-average reading skills.

1—Cotton, T. (2004, June). Inclusion through mathematics education. *Mathematics Teaching, 187*, 35–40. https://www.atm.org.uk/mathematics-teaching-journal-archive/3886

Cotton discusses the role that access to rigorous, rich mathematics education plays in societal inclusion and participation, including the impacts of inclusion or lack thereof on the beliefs and attitudes of students toward mathematics and learning mathematics.

3, 4—Council of Chief State School Officers. (2010). *Common Core State Standards for mathematics.* https://learning.ccsso.org/wp-content/uploads/2022/11/ADA-Compliant-Math-Standards.pdf

This document lays out the Common Core State Standards for mathematics across all grade levels, including both content and practices standards.

4—Cox, D. C. (2013). Similarity in middle school mathematics: At the crossroads of geometry and number. *Mathematical Thinking and Learning, 15*(1), 3–23. https://www.tandfonline.com/doi/abs/10.1080/10986065.2013.738377

In this study, students' understanding of presimilarity is examined based on a set of clinical interviews of 21 students ages 12–13 years. Students were asked to scale a series of geometric figures and were found to use a variety of strategies including some that incorporated both geometric and numeric reasoning. Tasks were developed that manipulated the characteristics of figures that students were required to attend to in order to explore the boundaries of numeric reasoning and to maximize the degree to which visual reasoning could be brought to bear on the task. Contrary to the literature, student use of visual reasoning did not indicate less developed conceptions of similarity. In fact, visually based strategies supported students as they reflected on and sought to improve wholly numeric strategies.

Analysis of the interview data indicated that providing students with tasks that required them to scale more complex geometric figures improved their capability to attend to the quantifiable features of shape and to the numeric relationships between them.

1—Covay Minor, E. (2015). Classroom composition and racial differences in opportunities to learn. *Journal of Education for Students Placed at Risk (JESPAR)*, *20*(3), 238–262. https://www.tandfonline.com/doi/abs/10.1080/10824669.2015.1043009

Black and white advanced math students leave high school with disparate math skills. One possible explanation is that minority students are exposed to different learning opportunities, even when they are taking classes with the same title. Using a convenience sample of the Mathematics Survey of the Enacted Curriculum (SEC), this study found that math teachers in classrooms with a minority racial composition spend their instructional time emphasizing different topics and instructional tasks than teachers in classrooms that have a predominantly white racial composition. Racial differences continue to exist when school socioeconomic level and teacher-reported classroom achievement level are included. Students in classrooms with minority racial compositions have different learning opportunities compared to those of their peers, which may explain racial differences in returns to advanced math course-taking.

1—Cundiff, J. L., Vescio, T. K., Loken, E., & Lo, L. (2013). Do gender-science stereotypes predict science identification and science career aspirations among undergraduate science majors? *Social Psychology of Education, 16*, 541–554. https://www.researchgate.net/profile/Jessica-Cundiff/publication/257665340_Do_gender-science_stereotypes_predict_science_identification_and_science_career_aspirations_among_undergraduate_science_majors/links/02e7e528e2b22b70dc000000/Do-gender-science-stereotypes-predict-science-identification-and-science-career-aspirations-among-undergraduate-science-majors.pdf

The research examined whether gender-science stereotypes were associated with science identification and, in turn, science career aspirations among women and men undergraduate science majors. More than 1,700 students enrolled in introductory science courses completed measures of gender-science stereotypes (implicit associations and endorsement of male superiority in science), science identification, and science career aspirations. Results were consistent with theoretically based predictions. Among women, stronger gender-science stereotypes were associated with *weaker* science identification and, in turn, *weaker* science career aspirations. By contrast, among men, stronger gender-science stereotypes were associated with *stronger* science identification and, in turn, *stronger* science career aspirations, particularly among men who were highly gender identified. These two sets of modest but significant findings can accumulate over large populations and across critical time points within a leaky pipeline to meaningfully contribute to gender disparities in STEM domains.

1—D'Ambrosio, U. (2012). A broad concept of social justice. In A. A. Wager & D. W. Stinson (Eds.), *Teaching Mathematics for Social Justice: Conversations with Educators* (pp. 201–213). NCTM.

In this chapter, the author encourages mathematics educators to think about a broader conception of social justice, one that aims toward human well-being, as stated by the

International Council of Scientific Unions (human well-being: a context- and situation-dependent state, comprising basic material for a good life, freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience), leading to a broader vision of teaching mathematics for social justice.

1—Darling-Hammond, L. (2007). Third annual Brown lecture in education research—The flat earth and education: How America's commitment to equity will determine our future. *Educational researcher*, *36*(6), 318–334. https://www.researchgate.net/publication/250183418_Third_Annual_Brown_Lecture_in_Education_Research-The_Flat_Earth_and_Education_How_America's_Commitment_to_Equity_Will_Determine_Our_Future#fullTextFileContent

In the knowledge-based economy that characterizes the 21st century, most previously industrialized countries are making massive investments in education. The United States ranks poorly on many leading indicators, however, primarily because of the great inequality in educational inputs and outcomes between white students and non-Asian "minority" students, who comprise a growing share of the U.S. public school population. Standards-based reforms have been launched throughout the United States with promises of greater equity, but while students are held to common standards—and increasingly experience serious sanctions if they fail to meet them—most states have not equalized funding and access to the key educational resources needed for learning. The result of this collision of new standards with old inequities is less access to education for many students of color, rather than more. This article outlines current disparities in educational access; illustrates the relationships between race, educational resources, and student achievement; and proposes reforms needed to equalize opportunities to learn.

1—Darling-Hammond, L. (2015). *The flat world and education: How America's commitment to equity will determine our future*. Teachers College Press.

Darling-Hammond examines issues like equality of spending, testing in K–12 education, and teacher preparation. She focuses on the successful educational strategies around the world, including a high degree of personalization that allows stronger, closer relationships among students, faculty, staff, and parents.

1, 2, 3, 4, 5—Daro, P., & Asturias, H. (2019). *BRANCHing out: Designing high school math pathways for equity*. Just Equations. https://justequations.org/resource/branching-out-designing-high-school-math-pathways-for-equity

This report outlines a new vision for high school mathematics: advanced math pathways that align with students' areas of interest. The authors argue that traditional high school math sequences filter too many students out of STEM (science, technology, engineering, and mathematics) opportunities while simultaneously erecting irrelevant math hurdles for students with other interests. The burdens of the mismatch, including placement into dead-end high school and college remedial courses, fall heaviest on students of color and low-income students. Equity requires new, rigorous offerings as alternatives to existing STEM-oriented courses. The authors call these "BRANCH" courses. In outlining the need for these pathways,

the authors note that they must be designed to articulate with postsecondary policies —and that, in some cases, postsecondary admissions and readiness policies need to be revised to support the move to multiple math pathways in college. The authors also note the relevance of pathway content to students' interests as well as flexibility for students to switch pathways if those interests change. Pathway design must also support educators to address the role of bias and privilege in traditional school structures and dislodge harmful preconceptions about who can and cannot do math. Furthermore, the authors also underscore the need for intentional strategies to recruit students to pathways in equitable ways and to provide support that addresses uneven prior opportunities and vulnerable math identities.

2—Daugherty, L., Mendoza-Graf, A., Gehlhaus, D., Miller, T., & Gerber, R. (2021). *How does corequisite remediation change student experiences? Results from a randomized study in five Texas community colleges*. https://www.rand.org/pubs/research_briefs/RBA810-1.html

RAND Corporation researchers compared the experiences of Texas community college students in English corequisite remediation courses with the experiences of students in stand-alone developmental reading and writing courses. This research brief summarizes research findings and highlights the implications that higher education policymakers, college administrators, and teachers should consider as they continue to design approaches to corequisite remediation.

4—de Araujo, Z., Jacobson, E., Singletary Lee, L., Wilson, P., Lowe, L., & Marshall, A. M. (2013). Teachers' conceptions of integrated mathematics curricula. *School Science and Mathematics 113*(6), 285–296. https://www.academia.edu/download/33707647/de_Araujo_et_al_2013_SSM_teachers_conceptions.pdf

In this qualitative research study, authors sought to understand teachers' conceptions of integrated mathematics. Participants were teachers in the first year of implementation of a state-mandated high school integrated mathematics curriculum. Primary data sources included focus group and individual interviews. Authors found that the teachers had varied conceptions of what the term "integrated" meant in reference to mathematics curricula. These varied conceptions led to the development of the Conceptions of Integrated Mathematics Curricula Framework, describing the different conceptions of integrated mathematics held by the teachers. The four conceptions—integration by strands, integration by topics, interdisciplinary integration, and contextual integration—refer to the different ideas teachers connect as well as the time frame over which these connections are emphasized. Results indicate that even when teachers use the same integrated mathematics curriculum, they may have varying conceptions of which ideas they are supposed to connect and how these connections can be emphasized. These varied conceptions of integration among teachers may lead students to experience the same adopted curriculum in very different ways.

2—Deloitte & Manufacturing Institute. (2015). *The skills gap in U.S. manufacturing 2015 and beyond*. Deloitte Development LLC. https://www.themadeinamericamovement.com/wp-content/uploads/2017/04/Deloitte-MFG-Institute.-The-Skills-Gap-in-the-US-MFG-21015-and-Beyond.pdf

This report addresses the question "Is U.S. manufacturing prepared to meet the talent needs of 2015 and beyond?" The report begins with the assertion that a strong manufacturing industry is fundamental to our nation's economic prosperity, yet a widening skills gap results in low productivity and has significant negative impact on business and the economy. The authors describe the changing nature of work, challenges in recruiting high-skilled workers, and ways to close the gap between workforce demand and worker supply including the earlier use of STEM initiatives to build the workforce pipeline.

4—DePiper, J. N., Driscoll, M. (2018). Teacher knowledge and visual access to mathematics. In S. Uzzo, S. Graves, E. Shay, M. Harford, & R. Thompson (Eds.), *Pedagogical content knowledge in STEM: Advances in STEM education.* Springer. https://link.springer.com/chapter/10.1007/978-3-319-97475-0_5

The authors propose that there exists mathematical knowledge for teaching (MKT) specific to visual representations (VRs), abbreviated MKT-VR. They define a VR as a graphic creation, such as a diagram or drawing, that illustrates quantities and shows quantitative relationships or that illustrates geometric properties of figures and shows geometric relationships. A teacher with strong MKT-VR will, for example, be able to use and understand VRs in his or her own problem solving and will have mathematical knowledge specific to teaching students to use, analyze, and solve problems with VRs. The Visual Access to Mathematics (VAM) project seeks to help teachers understand the value of VRs, specifically when teaching and learning ratio and proportional reasoning content. This chapter lays out a theoretical framework that authors anticipate using to guide and benchmark future research.

1—Desai, S., Kurtz, B., & Safi, F. (2021). Mathematics Heritage Project: An exploration empowering students' mathematical identities. *Journal of Humanistic Mathematics 11*(2), 106–122. https://scholarship.claremont.edu/cgi/viewcontent.cgi?article=1824&context=jhm

The International Study Group on Ethnomathematics (ISGEm) supports incorporating cultural diversity of mathematical practices to promote the teaching and learning of school mathematics. Through *The Mathematics Heritage Project*, students at a middle school in the southeastern United States developed unique creations to connect with the mathematics connected to their identities and self-identified cultural group. Upon reflection, students reported an increased awareness of the relevance of mathematics in their lives and a sense of ownership that is both meaningful and modern.

4—Diekman, A. B., Brown, E. R., Johnston, A. M., & Clark, E. K. (2010). Seeking congruity between goals and roles: A new look at why women opt out of science, technology, engineering, and mathematics careers. *Psychological Science, 21*(8), 1051–1057.

Although women have nearly attained equality with men in several formerly male-dominated fields, they remain underrepresented in the fields of science, technology, engineering, and mathematics (STEM). The authors argue that one important reason for this discrepancy is that STEM careers are perceived as less likely than careers in other fields to fulfill communal goals (e.g., working with or helping other people). Such perceptions might disproportionately affect women's career decisions, because women tend to endorse communal goals more than men. The authors found that STEM careers, relative to other careers, were perceived to impede communal goals. Moreover, communal-goal endorsement negatively predicted

interest in STEM careers, even when controlling for past experience and self-efficacy in science and mathematics. Understanding how communal goals influence people's interest in STEM fields thus provides a new perspective on the issue of women's representation in STEM careers.

4—Diekman, A. B., Clark, E. K., Johnston, A. M., Brown, E. R., & Steinberg, M. (2011). Malleability in communal goals and beliefs influences attraction to STEM careers: Evidence for a goal congruity perspective. *Journal of Personality and Social Psychology, 101*, 902–918.

The goal congruity perspective posits that two distinct social cognitions predict attraction to STEM fields. First, individuals may particularly value communal goals (e.g., working with or helping others), due to either chronic individual differences or the salience of these goals in particular contexts. Second, individuals hold beliefs about the activities that facilitate or impede these goals, or *goal affordance stereotypes*. Women's tendency to endorse communal goals more highly than do men, along with consensual stereotypes that STEM careers impede communal goals, intersect to produce disinterest in STEM careers. The authors provide evidence for the foundational predictions that gender differences emerge primarily on communal rather than agentic goals (Studies 1a and 3) and that goal affordance stereotypes reflect beliefs that STEM careers are relatively dissociated from communal goals (Studies 1b and 1c). Most critically, they provide causal evidence that activated communal goals decrease interest in STEM fields (Study 2) and that the potential for a STEM career to afford communal goals elicits greater positivity (Study 3). These studies thus provide a novel demonstration that understanding communal goals and goal affordance stereotypes can lend insight into attitudes toward STEM pursuits.

1—Dossey, J. A., McCrone, S. S., & Halvorsen, K T. (2016). *Mathematics education in the United States, 2016: A capsule summary fact book.* NCTM.

Determining what is happening in mathematics education in a country as large and complex as the United States is difficult—even for those who are familiar with U.S. education. To promote and extend understanding, this report

- gives general information about education in the United States;
- describes the intended, implemented, and attained curricula in school mathematics;
- surveys the emergence of the Common Core State Standards for Mathematics;
- examines U.S. participation in international studies of mathematics achievement; and
- previews the Every Student Succeeds Act, the federal education law passed by the U.S. Congress at the end of 2015.

The fact book also includes sections detailing programs for high-achieving students, programs for mathematics teacher education, and resources for additional information about mathematics education in the United States.

1—Douglas, D., & Attewell, P. (2017). School mathematics as gatekeeper. *The Sociological Quarterly*, *58*(4), 648–669.

https://www.tandfonline.com/doi/pdf/10.1080/00380253.2017.1354733?needAccess=true&role=button

The authors find that the gatekeeper function of mathematics is not a function of general academic prowess and operates separately from advantages attributable to family background. Moreover, mathematics is especially important for educational attainment among youth from higher socioeconomic status families. Human capital theory implies that schooling's emphasis on math reflects its functional importance in the workplace. However, authors' analyses of skills used in workplaces indicate that very small proportions of educated workers ever use "school math." Authors draw out the implications of this disjuncture both for theories of class reproduction and for human capital theory.

2—Douglas, D., Logue, A. W., & Watanabe-Rose, M. (2022). The long-term impacts of corequisite mathematics remediation with statistics: Degree completion and wage outcomes. *Educational Researcher 52*(1), 7–15. https://journals.sagepub.com/doi/abs/10.3102/0013189X221138848

This study examined the seven-year results of a randomized controlled trial of corequisite remediation with college-level statistics. Students assigned to the corequisite group were 50 percent more likely to complete associate's degrees within three years, and 100 percent more likely to complete bachelor's degrees within five years. Corequisite students also earned, on average, $3,000–$4,500 more in years 5–7. Mediation analysis further suggests that the wage treatment effect was due to shortened time to degree.

1—Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology*, *7*, 508. https://www.frontiersin.org/articles/10.3389/fpsyg.2016.00508/full

This paper focuses on what research has revealed about mathematics anxiety in the last 60 years and what still remains to be learned. The authors discuss what mathematics anxiety is, how distinct it is from other forms of anxiety, and how it relates to attitudes to mathematics. They discuss the relationships between mathematics anxiety and mathematics performance. They describe ways in which mathematics anxiety is measured, both by questionnaires and by physiological measures. They discuss some possible factors in mathematics anxiety, including genetics, gender, age, and culture. Finally, they describe some research on treatment. They conclude with a brief discussion of what still needs to be learned.

4—Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5–10.* Heinemann. https://www.heinemann.com/products/e09313.aspx

This book discusses essential, practical ideas for helping students cultivate geometric habits of mind that lead to success in this subject. The book focuses on rigorous, problem-based teaching that encourages students to deepen their thinking in three key geometric strands:

- geometric properties,
- geometric transformations, and
- measurement of geometric objects.

2—Dudley, U. (2010). What is mathematics for? *Notices of the AMS*, *57*(5), 608–613. https://community.ams.org/journals/notices/201005/rtx100500608p.pdf

The author discusses various rationales for studying school mathematics subjects beyond arithmetic, such as algebra, geometry, and trigonometry, including reasoning and appreciating mathematics as a human cultural pursuit. The author argues against the common response that studying algebra, geometry, trigonometry, and other common high school mathematics courses is critically important for most people's job performance.

2—EdReports. (n.d.). *High school mathematics report.* https://www.edreports.org/reports/math/hs

EdReports rates mathematics textbooks and curricula according to criteria aligned with the Common Core State Standards for Mathematics Publishers' Criteria document and is a resource that schools and districts can use to determine the extent to which various published instructional materials are high-quality.

1—EdSource. (2012). *Passing when it counts: Math courses present barriers to student success in California community colleges.* https://edsource.org/wp-content/publications/pub12-Math2012Final.pdf

What makes math completion especially challenging is that students often come to math with a high degree of anxiety, frequently rooted in earlier failures. Succeeding in math at the outset could provide students with "early momentum" that would contribute to their overall success in college. Conversely, lack of success could discourage them from completing their studies. Increasing the success rates in more advanced math courses remains a continuing challenge for California and the nation. This issue brief underscores the need to make math success a priority at not only the developmental or remedial level, but at all levels of math instruction.

2—Education Commission of the States (ECS). (2018). *50-State comparison: Developmental education policies* [Web resources and reports]. https://www.ecs.org/50-state-comparison-developmental-education-policies

To provide a national perspective on developmental education assessment and placement policies, Education Commission of the States researched state-level and postsecondary system policies to create this comprehensive resource. Users can click on a series of questions for a 50-State Comparison showing how states and postsecondary systems approach these policies, or view a specific state's approach.

4—Edwards, A. R., & Beattie, R. L. (2016). Promoting student learning and productive persistence in developmental mathematics: Research frameworks informing the Carnegie Pathways. *NADE Digest, 9*(1), 30–39. https://files.eric.ed.gov/fulltext/EJ1097458.pdf

This paper focuses on two research-based frameworks that inform the design of instruction and promote student success in accelerated, developmental mathematics pathways. These are Learning Opportunities, or productive struggle on challenging and relevant tasks, deliberate practice, and explicit connections; and Productive Persistence, or promoting students' academic and social mindsets, and good strategies. These frameworks are the foundations of the highly successful Carnegie Pathways (Statway and Quantway), two distinct pathways that take students who place into developmental mathematics through college-level mathematics in one year. In this paper, we describe these research-based frameworks and discuss examples of high-impact practices derived from them.

4—Engel, J. (2017). Statistical literacy for active citizenship: A call for data science education. *Statistics Education Research Journal, 16*(1), 44–49. https://iase-web.org/ojs/SERJ/article/download/213/117

Data—abundant quantitative information about the state of society and the wider world—are around us more than ever. The authors argue that recent trends in the public discourse point toward a post-factual world that seems content to ignore or misrepresent empirical evidence. The authors argue that as statistics educators, we are challenged to promote understanding of statistics about society. In order to re-root public debate to be based on facts instead of emotions and to promote evidence-based policy decisions, the authors argue that statistics education should embrace two areas widely neglected in secondary and tertiary education: understanding of multivariate phenomena, and thinking with and learning from complex data.

5—Erickson, T., Wilkerson, M., Finzer, W., & Reichsman, F. (2019). Data moves. *Technology Innovations in Statistics Education, 12*(1).

When experienced analysts explore data in a rich environment, they often transform the dataset. For example, they may choose to group or filter data, calculate new variables and summary measures, or reorganize a dataset by changing its structure or merging it with other information. Such actions background, highlight, or even fundamentally change particular features of the data, allowing different types of questions to be explored. We call these actions *data moves*. In this paper, we argue that paying explicit attention to data moves, as well as their purposes and consequences, is necessary for educators to support student learning about data. This is especially needed in an era when students are expected to develop critical literacy around data and engage in purposeful, self-directed exploration of large and often complex datasets.

1—Ernest, P. (1991). Mathematics teacher education and quality. *Assessment and Evaluation in Higher Education*, *16*(1), 56–65. https://www.tandfonline.com/doi/abs/10.1080/0260293910160107

A model of a teacher's mathematics-related belief-system is presented, and the issue of the contrast between espoused and enacted beliefs is discussed. "Quality" in mathematics teacher education raises all of these issues, as well as the aims, goals, and means of the teacher preparation process itself. The paper concludes by arguing against the "apprenticeship" model of mathematics teacher education, for depriving student teachers of theory and of practical research experience. "Quality" in mathematics teaching and teacher education depends on both theory and practice in systematic cooperation.

4—Evans, C. D., & Diekman, A. B. (2009). On motivated role selection: Gender beliefs, distant goals, and career interest. *Psychology of Women Quarterly, 33*(2), 235–249.

Despite widespread changes in occupational opportunities, men and women continue to show divergent preferences for careers. This research invoked a motivational framework to explain sex-differentiated career interest. From a role congruity perspective (Diekman & Eagly, 2008), the internalization of gender roles leads people to endorse gender-stereotypic goals, which then lead to interest in occupations that afford the pursuit of those goals. Three studies provided evidence for the hypotheses. Study 1 found that male- and female-stereotypic careers were perceived to afford different goals. Studies 2 and 3 found that men and women endorsed different goals and that this gender-normative goal endorsement predicted gender-stereotypic career interest. In addition, structural equation modeling (Study 3) indicated that internalization of gender roles fully accounted for sex-differentiated goal endorsement. These findings thus extend the social role theory framework to consider processes related to self-selection into specific social roles.

1—Finkelstein, N., Fong, A., Tiffany-Morales, J., Shields, P., & Huang, M. (2012). College bound in middle school and high school? How math course sequences matter. *Center for the Future of Teaching and Learning at WestEd*. https://files.eric.ed.gov/fulltext/ED538053.pdf

Prior research confirms that success in high-level mathematics in high school is predictive of postsecondary success and careers in STEM fields. This study, funded by the S.D. Bechtel, Jr. Foundation and the Noyce Foundation, digs deeper into this middle and high school connection as it applies to STEM, in order to better understand the degree to which California students stay on the trajectory for STEM-related attendance eligibility at California's public universities and, if students veer off the trajectory, to better understand when and why. Thus, researchers examined math and science course-taking patterns for a representative cohort of some 24,000 California students who were enrolled in grade 7 in 2004/05 and stayed in their district through grade 12 in 2009/10. Although the study looked at students' science course-taking, this report focuses more tightly on the mathematics-related findings, partly because it turns out that course-taking patterns and performance in science are quite similar to, though less complex than, those in mathematics and partly because mathematical understanding, while not sufficient, is essential to student success in some key high school science courses, such as chemistry and physics. The math findings include: (a) Math performance in grade 7 is predictive of high-school math course-taking. (b) While the majority of students who achieved at least Proficient on their math California Standards Test (CST) are those who took Algebra 1 in grade 8, Geometry in grade 9, and Algebra 2 in grade 10, in general this accelerated pathway does not support students who are not proficient in math in grade 7. (c) Many students repeat algebra, but few repeaters achieve proficiency on their second attempt. (d) Districts are keenly aware of poor student performance in mathematics but less aware of course-taking patterns. (e) Districts feel great urgency to improve algebra outcomes.

1—Flores, A. (2007). Examining disparities in mathematics education: Achievement gap or opportunity gap? *High School Journal 91*(1), 29–42. https://eaop.ucsd.edu/198/achievement-gap/Examining%20Disparities%20in%20Mathematics%20Education%20-%20Achievement%20Gap%20or%20Opportunity%20Gap.pdf

The so-called achievement gap in mathematics is reframed as a problem of unequal opportunities to learn experienced by many low-income students and many Latino and African American students. First, data are presented showing striking and persistent differences on standardized tests among students of different ethnic groups and socioeconomic levels. Then evidence is presented demonstrating that opportunities to learn mathematics are not equally distributed among all students. Specifically, data show that African American, Latino, and low-income students are less likely to have access to experienced and qualified teachers, more likely to face low expectations, and less likely to receive equitable per student funding. The final section discusses how teachers and schools can provide more equitable opportunities to learn mathematics for all students.

1,4—Franklin, C. (2013). Guest editorial: Common Core State Standards and the future of teacher preparation in Statistics. *The Mathematics Educator 22*(2), 3–10. https://ojs01.galib.uga.edu/tme/article/download/1980/1885

This editorial describes the evolution of the argument to include statistics as a part of the K–12 curriculum and the Common Core State Standards in Mathematics. The author provides multiple recommendations related to teacher preparation that provides opportunities to cultivate their statistical thinking skills at the elementary, middle, and high school levels.

3—Fullan, M. (2020). *Leading in a culture of change.* John Wiley & Sons.

This guide helps readers learn about the five components of change leadership—moral purpose, understanding change, building relationships, creating and sharing knowledge, and creating coherence—and mobilize others to accomplish shared goals in often difficult conditions. This book aims to help leaders from across sectors understand the dynamics of change and navigate the end-to-end change process. The second edition includes more precise definitions of the core competencies of change, contemporary case studies of their development and practical application, and increased guidance on their effective use through new concrete examples.

2—GAISE College Report ASA Revision Committee. (2016). *Guidelines for assessment and instruction in statistics education (GAISE) college report 2016.* https://www.amstat.org/asa/files/pdfs/GAISE/GaiseCollege_Full.pdf

The revised report describes changes in the world of statistics education and statistical practice since 2005 and suggests a direction for the future of introductory statistics courses. The work was informed by outreach to the statistics education community and by reference to the statistics education literature. The authors continue to endorse the six recommendations outlined in the original GAISE College Report with simplified language and some reordering of the recommendations to focus first on what to teach in introductory courses and then on how to teach those courses. Two new emphases were also added to the first recommendation. The revised recommendations are as follows: (a) Teach statistical thinking. Teach statistics as an investigative process of problem solving and decision making. Give students experience with multivariable thinking. (b) Focus on conceptual understanding. (c) Integrate real data with a context and purpose. (d) Foster active learning. (e) Use technology to explore concepts and analyze data. (f) Use assessments to improve and evaluate student learning. This report includes an updated list of learning objectives for students in introductory courses, along with suggested topics that might be omitted from or deemphasized in an introductory course. Some appendices were substantially expanded and updated, and new appendices provide details about the evolution of introductory statistics courses, examples involving multivariable thinking, and ideas for implementing the GAISE recommendations in a variety of different learning environments.

1—Gamoran, A. (2009). Tracking and inequality: New directions for research and practice. In M. W. Apple, S. J. Ball, & L. A. Gandin (Eds.), *The Routledge international handbook of the sociology of education* (pp. 213–228). Routledge.

Building on past research, the author discusses recent advances in the study of tracking that indicate promising new directions for research and practice. First, new international scholarship has extended knowledge about the consequences of tracking for student achievement to contexts beyond the United States and United Kingdom, where most prior research had been conducted. Second, recent studies of attempts to reduce or eliminate tracking and ability grouping have yielded important insights about why tracking is resistant to change and how some of the obstacles to detracking may be surmounted. Third, a new wave of research on classroom assignment and instruction has pointed toward approaches that, while not resolving the tension between commonality and differentiation, may capture the benefits of differentiation for meeting students' varied needs without giving rise to the consequences for inequality that commonly accompany tracking and ability grouping. These findings in turn call for new research and experimentation in practice.

1—Ganga, E., Mazzariello, A., & Edgecombe, N. (2018). *Developmental education: An introduction for policymakers.* Education Commission of the States (ECS) and CAPR: Center for the Analysis of Postsecondary Readiness. https://academiccommons.columbia.edu/doi/10.7916/D8GM9KP0/download

This ECS/CAPR brief discusses the importance of and challenges surrounding developmental education and suggests ways in which policymakers can address these challenges. Three key findings include: (a) More than two thirds of community college students and 40 percent of four-year college students take at least one developmental course. (b) Students who take developmental courses are less likely to complete a program and earn a degree or certificate. (c) Remediation as traditionally taught has had, at best, modest effects on improving outcomes for students who enter college with weak academic skills.

1, 3—Gao, N. (2021). *Does raising high school graduation requirements improve student outcomes?* Public Policy Institute of California. https://www.ppic.org/wp-content/uploads/does-raising-high-school-graduation-requirements-improve-student-outcomes-february-2021.pdf

In this report, authors review district graduation policies for the 2018–19 school year and examine the relationship between math and science requirements and student outcomes. Overall, they find:

- Most districts have graduation requirements that exceed state minimums, which include two years of math and two years of science. During the 2018–19 school year, 59 percent of districts—enrolling 66 percent of the state's K–12 population—required three or four years of math; 22 percent required three or four years of science.
- Descriptively, districts with larger shares of Latinos, African Americans, low-income students, English learners, and students with non-college-educated parents are about as likely to have higher math requirements (three or four years) as are other districts.
- Higher math graduation requirements are associated with better outcomes, particularly for students in high-need, high-poverty, and high-minority schools. These requirements

do not appear to lead to lower high school graduation rates or higher dropout rates. Outcomes for students in rural schools and the lowest-performing schools do not appear to be affected.

The overall impact of science requirements is less certain: higher requirements are associated with a lower dropout rate, but enrollment in advanced courses and proficiency on standardized tests do not appear to be affected. Notably, students in high-poverty schools with higher requirements are 31 percent more likely to take advanced science courses and 2 percent more likely to graduate. Outcomes for students in rural schools appear unchanged.

3—Gao, N., & Johnson, H. (2017). *Improving college pathways in California.* Public Policy Institute of California. https://eric.ed.gov/?redir=http%3a%2f%2fwww.ppic.org%2fwp-content%2fuploads%2fr_1117ngr.pdf

In this study, the authors examine college pathways in high school, at California's community colleges, and at California State University (CSU). They find that most students exit the pathway in the last two years of high school or the first two years of college. In addition, they find: (a) Most of California's high school graduates are not prepared for college. Even with significant increases over the past 10 years, only 45 percent of the graduating class of 2016 complete the college preparatory courses—known as the a-g courses—required to be considered for admission to CSU or the University of California. (b) Even academically prepared students are falling off the pathway. Among the authors' sample of high schools, students who successfully pass the first college preparatory math course, 34 percent do not take the next one—even though 13 percent earned an A and 22 percent a B in the class. (c) Eliminating progression problems in the sample of high schools would increase the a-g completion rate by 18 percentage points. (d) Students historically underrepresented in higher education are more likely to drop off the pathway at every stage. (e) CSU does not have adequate capacity to enroll qualified students.

1—Gayles, J. G., & Ampaw, F. (2014). The impact of college experiences on degree completion in STEM fields at four-year institutions: Does gender matter? *The Journal of Higher Education, 85*, 439–468. https://www.tandfonline.com/doi/abs/10.1080/00221546.2014.11777336

Degree attainment at the undergraduate level for women in science, technology, engineering, and mathematics (STEM) continues to be an issue of national concern, particularly when trying to explain disparaging gender differences in persistence. Thus, the purpose of this study was to examine factors that influence degree attainment for students in STEM majors at four-year colleges and universities. The authors were interested in differential effects across gender and educational experiences, such as interacting with faculty and social involvement with peers, on degree attainment. Results support that the effects of the college experience on degree attainment in STEM are conditional on gender. Recommendations for policy and practice are offered.

1—Geist, E. (2010). The anti-anxiety curriculum: Combating math anxiety in the classroom. *Journal of Instructional Psychology*, *37*(1). https://www.andrews.edu/sed/gpc/faculty-research/montagano-research/the-anti-anxiety-cur.pdf

Negative attitudes toward mathematics and what has come to be know as "math anxiety" are serious obstacles for children in all levels of schooling today. In this paper, the literature is reviewed and critically assessed in regard to the roots of math anxiety and its especially detrimental effect on children in "at-risk" populations such as low socioeconomic status and females. The effects of teachers' and parents' assumptions, family support, and parents' level of educational attainment are addressed. The paper also addresses the curricular issues that may lead to math anxiety such as high-stress instructional methods and "timed testing."

1—Getz, A., Ortiz, H. R., Hartzler, R., & Leahy, F. (2016). *The case for mathematics pathways*. https://dcmathpathways.org/sites/default/files/resources/2016-11/The%20Case%20for%20Mathematics%20Pathways.pdf

This brief presents the case that a mathematics pathways solution can significantly increase student success by addressing two structural drivers of the problem: (a) the mismatch of content, and (b) long, multi-semester course sequences. Mathematics pathways refer to developmental and college-level course sequences that align to a student's academic and career goals and that accelerate student completion of a gateway college-level math course.

3—Gojak, L. M. (2013). *What's all this talk about rigor?* https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/What_s-All-This-Talk-about-Rigor_/

This article summarizes conversation between the former president of the National Council of Teachers of Mathematics and a group of math coaches who are working with elementary teachers on implementation of the Common Core Standards for Mathematics. The article aims to clear up the confusion around the term "rigor" by providing an overview of the term as used in the Common Core Standards, and examples of what rigor might look like in instruction compared to nonrigorous instruction. The author also describes the importance of everyone being able to engage in rigorous learning experiences and how to support classroom teachers to work toward greater rigor in their mathematics instruction.

4—Goldenberg, E. P., Cuoco, A. A. & Mark, J. (1998). A role for geometry in general education. In *Designing learning environments for developing understanding of geometry and space* (pp. 3–44). Routledge.

This volume reflects an appreciation of the interactive roles of subject matter, teacher, student, and technologies in designing classrooms that promote understanding of geometry and space. Although these elements of geometry education are mutually constituted, the book is organized to highlight, first, the editors' vision of a general geometry education; second, the development of student thinking in everyday and classroom contexts; and third, the role of technologies. Rather than looking to high school geometry as the locus—and all too often, the apex—of geometric reasoning, the contributors to this volume suggest that reasoning about space can and should be successfully integrated with other forms of mathematics, starting at the elementary level and continuing through high school. Reintegrating spatial reasoning into the mathematical mainstream—indeed, placing it at the core of K–12 mathematics environments that promote learning with understanding—will mean increased attention to problems in modeling, structure, and design and reinvigoration of traditional topics such as measure, dimension, and form. Further, the editors' position is that the teaching of geometry and spatial visualization in school should not be compressed into a characterization of Greek geometry, but should include attention to contributions to the mathematics of space that developed subsequent to those of the Greeks.

1—Goldhaber, D., Lavery, L., & Theobald, R. (2015). Uneven playing field? Assessing the teacher quality gap between advantaged and disadvantaged students. *Educational researcher*, *44*(5), 293–307. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=7f790a9d5b7c0e1afcf088052c28814229382b7c

Policymakers aiming to close the well-documented achievement gap between advantaged and disadvantaged students have increasingly turned their attention to issues of teacher quality. A number of studies have demonstrated that teachers are inequitably distributed across student subgroups by input measures, like experience and qualifications, as well as output measures, like value-added estimates of teacher performance, but these tend to focus on either individual measures of teacher quality or particular school districts. In this study, the authors present a comprehensive, descriptive analysis of the inequitable distribution of both input and output measures of teacher quality across various indicators of student disadvantage across all school districts in Washington State. They demonstrate that in elementary school, middle school, and high school classrooms, virtually every measure of teacher quality we examine—experience, licensure exam scores, and value added—is inequitably distributed across every indicator of student disadvantage—free or reduced-price lunch status, underrepresented minority, and low prior academic performance. Finally, the authors decompose these inequities to the district, school, and classroom levels and find that patterns in teacher sorting at all three levels contribute to the overall teacher quality gaps.

1—Gonzalez, H. B., & Kuenzi, J. J. (2012). *Science, technology, engineering and mathematics (STEM) education: A primer.* Congressional Research Service. https://fayllar.org/pars_docs/refs/480/479491/479491.pdf

This report is intended to serve as a primer for outlining existing STEM education policy issues and programs. It includes assessments of the federal STEM education effort and the condition of STEM education in the United States, as well as an analysis of several of the policy issues central to the contemporary federal conversation about STEM education. Appendix A contains frequently cited data and sources and Appendix B includes a selection of major STEM-related acts.

1, 3—Goodman, J. (2019). The labor of division: Returns to compulsory high school math coursework. *Journal of Labor Economics*, *37*(4), 1141–1182. https://www.nber.org/system/files/working_papers/w23063/w23063.pdf

Authors illustrate that state changes in minimum high school math requirements substantially increase Black students' completed math coursework and their later earnings. The marginal student's return to an additional math course is 10 percent, roughly half the return to a year of high school, and is partly explained by a shift toward more cognitively skilled occupations. White students' coursework and earnings are unaffected. Rigorous standards for quantitative coursework can close meaningful portions of racial gaps in economic outcomes.

1—Goodman, M. J., Sands, A. M., & Coley, R. J. (2015). America's skills challenge: Millennials and the future. *Educational Testing Service*. https://files.eric.ed.gov/fulltext/ED589564.pdf

Recent research reveals an apparent paradox for U.S. millennials (born after 1980, ages 16–34): while they may be on track to be the most educated U.S. generation ever, they consistently score below many of their international peers in literacy, numeracy, and problem solving in technology-rich environments. These findings also represent a decrease in literacy and numeracy skills when compared to results from previous years of U.S. adult surveys. The authors propose that there needs to be a greater focus on skills—not just educational attainment—or we are likely to experience adverse consequences that could undermine the fabric of our democracy and community.

1—Gordon, S. P. (2008). What's wrong with college algebra? *Primus*, *18*(6), 516–541. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=952cd9478d141ba46991e286479b910d1160af74

Most college algebra courses are offered in the spirit of preparing the students to move on toward calculus. In reality, only a vanishingly small fraction of the million students a year who take these courses ever get to calculus. This article argues for the need to change the focus in college algebra to one that better meets the actual needs of the students and of the other disciplines that require college algebra of their students—by focusing on conceptual understanding, data, and modeling and problem solving.

4—Gould, R., Machado, S., Ong, C., Johnson, T., Molyneux, J., Nolen, S., Tangmunarunkit, H., Trusela, L., & Zanontian, L. (2016, July). Teaching data science to secondary students: The Mobilize Introduction to Data Science curriculum. In J. Engel (Ed.), *Promoting understanding of statistics about society: Proceedings of the Roundtable Conference of the International Association of Statistics Education*, Berlin. http://iase-web.org/documents/papers/rt2016/Gould.pdf

Making sense of data is complex, and the knowledge and skills required to understand "Big Data"—and many open data sources—go beyond those taught in traditional introductory statistics courses. The Mobilize project has created and implemented a course for secondary students, Introduction to Data Science (IDS), that aims to develop computational and statistical thinking skills so that students can access and analyze data from a variety of traditional and nontraditional sources. Although the course does not directly address open source data, such data are used in the curriculum, and an outcome of the project is to develop skills and habits of mind that allow students to use open source data to understand their community. This paper introduces the course and describes some of the challenges in its implementation.

3—Gresham, R., & Little, M. E. (2012). *RTI and mathematics: Practical tools for teachers in K–8 classrooms*. Pearson Higher Ed. https://www.pearson.com/en-us/subject-catalog/p/rti-and-mathematics-practical-tools-for-teachers-in-k-8-classrooms/P200000001140

This resource provides mathematics educators with sound, research-based knowledge and expertise for successfully implementing Response to Intervention (RTI) in mathematics and addressing the challenges involved. *RTI and Mathematics* clarifies and describes the issues of

RTI, the connections among teachers' knowledge and skills and their use with RTI, and the role of the teacher within the classroom and school, and provides evidence-based content, scenarios, examples, resources, and activities; modeling description; and reflection upon the key learning outcomes of RTI. Included is an illustrative continuous case study of a mathematics teacher as she implements RTI in her classroom, and a myriad of information and resources educators can use immediately within K–8 classrooms in order to implement RTI in mathematics successfully.

5—Grissom, J. A., & Redding, C. (2016). Discretion and disproportionality: Explaining the underrepresentation of high-achieving students of color in gifted programs. *Aera Open, 2*(1), 2332858415622175. https://eric.ed.gov/?id=EJ1194583

Students of color are underrepresented in gifted programs relative to white students, but the reasons for this underrepresentation are poorly understood. The authors investigate the predictors of gifted assignment using nationally representative, longitudinal data on elementary students. They document that even among students with high standardized test scores, Black students are less likely to be assigned to gifted services in both math and reading, a pattern that persists when controlling for other background factors, such as health and socioeconomic status and characteristics of classrooms and schools. The authors then investigate the role of teacher discretion, leveraging research from political science suggesting that clients of government services from traditionally underrepresented groups benefit from diversity in the providers of those services, including teachers. Even after conditioning on test scores and other factors, Black students indeed are referred to gifted programs, particularly in reading, at significantly lower rates when taught by non-Black teachers, a concerning result given the relatively low incidence of assignment to own-race teachers among Black students.

4—Grouws, D. A., Tarr, J. E., Chavez, Ó., Sears, R., Soria, V. M., & Taylan, R. D. (2013). Curriculum and implementation effects on high school student'' mathematics learning from curricula representing subject-specific and integrated content organizations. *Journal for Research in Mathematics Education 44*(2), 416–463. https://www.academia.edu/download/46230156/Curriculum_and_Implementation_Effects_on20160604-783-548pis.pdf

This study examined the effect of two types of mathematics content organization on high school students' mathematics learning while taking account of curriculum implementation and student prior achievement. The study involved 2,161 students in 10 schools in five states. Within each school, approximately one half of the students studied from an integrated curriculum (Course 1) and one half studied from a subject-specific curriculum (Algebra 1). Hierarchical linear modeling with three levels showed that students who studied from the integrated curriculum were significantly advantaged over students who studied from a subject-specific curriculum on three end-of-year outcome measures: Test of Common Objectives, Problem Solving and Reasoning Test, and a standardized achievement test. Opportunity to learn and teaching experience were significant moderating factors.

1—Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning, 4*(2&3), 145–187.

In this article, the author addresses the need for a more clearly articulated research agenda around equity issues by proposing a working definition of equity and a focal point for research. More specifically, she asserts that rather than pitting them against each other, we must coordinate (a) efforts to get marginalized students to master what currently counts as "dominant" mathematics with (b) efforts to develop a critical perspective among all students about knowledge and society in ways that ultimately facilitate (c) a positive relationship between mathematics, people, and equity on the planet. She makes this argument partly by reviewing the literature on (school) contexts that engage marginalized students in mathematics. She then argues that the place that holds the most promise for addressing equity is a research agenda that emphasizes enabling the practice of teachers and that draws more heavily on design-based and action research, thereby redefining what the practice of mathematics means along the way. Specific research questions are offered.

1—Hallinan, M. T., Bottoms, E., & Pallas, A. M. (2003). Ability grouping and student learning. *Brookings Papers on Education Policy 6*, 95–140. https://www.jstor.org/stable/20067255

Criticisms by M. T. Hallinan concerning Slavin's work are considered. Unambiguous indicators of instructional quality in different ability groups are difficult to obtain, and student achievement, measured by standardized tests, is an adequate indicator. Slavin's research provides no evidence of student achievement advantage from ability grouping.

2—Handel, M. (2016). What do people do at work? A profile of U.S. jobs from the survey of workplace Skills, Technology, and Management Practices (STAMP). *Job Tasks, Work Skills and the Labour Market 49*,177–197. https://labourmarketresearch.springeropen.com/articles/10.1007/s12651-016-0213-1

This paper describes the survey of Skills, Technology, and Management Practices (STAMP), which emphasizes the use of behaviorally specific questions in order to improve the quality of job measures. Such measures yield better understanding of the absolute levels of job demands compared to items or scales with arbitrary units that lack definite meaning outside the framework of a particular survey. STAMP measures reveal most workers use relatively simple levels of math on their jobs, but there is a bifurcation of jobs in terms of the complexity of reading and especially writing that is required. Aside from managerial and professional occupations, the absolute level of academic skills required on most jobs does not appear to be very high. Likewise, computer use is widespread, but most people use computers for fairly mundane office duties rather than more complex tasks; few workers use any kind of automated production equipment on their jobs. Well-developed employee involvement practices, such as self-directed teams, cover about one-fifth to one-quarter of the workforce. Very few workers report being affected by outsourcing, and the numbers affected by technological displacement are almost imperceptible.

3— Hao, Z., & Cowan, B. W. (2017, September). *The effects of graduation requirements on risky health behaviors of high school students* (Working Paper No. 23803). National Bureau of Economic Research. https://www.nber.org/system/files/working_papers/w23803/w23803.pdf

Using national survey data from the Youth Risk Behavior Surveillance System, authors studied the effects of mathematics and science high school graduation requirements (HSGR) on high school students' risky health behaviors—specifically on drinking, smoking, and marijuana use. Authors found that an increase in mathematics and science HSGR has significant negative impacts on alcohol consumption among high school students, especially males and nonwhite students. The effects of math and science HSGR on smoking and marijuana use are also negative but generally less precisely estimated. Results suggest that curriculum design may have potential as a policy tool to curb youth drinking.

4—Harlow, J. (2015). *California districts moving to new "integrated" high school math pathway. EdSource.* https://edsource.org/2015/california-districts-moving-to-new-integrated-high-school-math-pathway/89288

This article describes California's move to include a sequence of courses that represents a significant departure from how high school math has traditionally been taught in California and the nation. The new "integrated pathway" combines and reorders content from traditional Algebra 1, Geometry, and Algebra 2 courses in a three-year sequence. While the state does not keep track of which approach districts are adopting, an EdSource review of the state's 30 districts with the largest high school enrollments showed that the traditional approach—Algebra 1, geometry, Algebra 2—is still used in 15 of them. The remaining 15 have embraced the integrated Math I, II, and III pathway. The article goes on to describe implications of this pathway decision for students and teachers.

4—Heinzman, E. (2022). "I love math only if it's coding": A case study of student experiences in an introduction to data science course. *Statistics Education Research Journal*, *21*(2), 5. https://www.iase-web.org/ojs/SERJ/article/download/43/463

Many important voices—including the National Council for Teachers of Mathematics (NCTM), the Dana Center's Launch Years Initiative, and others—advocate for expanding the traditional course offerings in high school mathematics and statistics to include courses such as Introduction to Data Science (IDS). To date, research on the IDS course has primarily focused on pedagogy, professional learning for teachers, and the curriculum. This mixed-methods case study expands understanding by analyzing the perspective of IDS students at a California public high school. Self-determination theory provides a useful frame for interpreting how these students experience the IDS course. The theory focuses on conditions for students to engage in meaningful learning: competence (self-efficacy), autonomy (agency), and relatedness (a sense of belonging). The findings from this case study suggest the IDS students feel confident, empowered, and part of a vibrant community, unlike in previous mathematics and statistics courses they may have completed, and use specific language to describe their joy in problem solving and the accessibility of the course. These findings have implications for the development and refinement of any high school data science course, including IDS.

4—Hemmi, K., Bråting, K., & Lepik, M. (2021). Curricular approaches to algebra in Estonia, Finland and Sweden: A comparative study. *Mathematical Thinking and Learning, 23*(1), 49–71. https://www.researchgate.net/publication/340282724_Curricular_approaches_to_algebra_in_Estonia_Finland_and_Sweden_-_a_comparative_study#fullTextFileContent

The aim of the study is to investigate and compare approaches to algebra in the Estonian, Finnish, and Swedish national core curricula (grades 1–9). Despite the similarities in the

school systems of these neighboring countries, the analysis reveals three quite different curricular approaches. The Estonian approach shows influences of the Russian Davydov School. The Finnish approach to some extent resembles the traditional one whereby algebra is addressed first at the lower secondary level and then in a formal manner. However, there are also characteristics typical of the functional view, which dominates the Swedish curriculum. Here, as opposed to the documents from the other two countries, a transition to more formal sophisticated methods at the secondary level is not visible at all. Authors discuss the results in relation to earlier research and students' different learning outcomes in light of international evaluations.

2—Henson, L., Huntsman, H., Hern, H., & Snell, M. (2017). *Leading the way: Cuyamaca College transforms math remediation.* CAP: The California Acceleration Project. https://accelerationproject.org/Portals/0/Documents/Cap_Leading_the_Way_Web_Final.pdf

This report highlights Cuyamaca College, the first community college in California to completely transform math remediation—from how it assesses and places students into math courses, to the courses it offers, to what happens in the classroom. Most students at Cuyamaca can now complete their math requirements in one semester. Students in math-intensive majors take no more than one semester of math that doesn't count toward a bachelor's degree. And math faculty are teaching with "brains-on" activities and collaborative pedagogy. Completion of transferable, college-level math has increased nearly sevenfold among students who would have previously taken remedial courses, with dramatic gains for all racial and ethnic groups. Cuyamaca's experience points the way for the rest of the state, revealing what's possible when colleges step up to transform their systems on behalf of students.

1—Hodgen, J., Pepper, D., Sturman, L., & Ruddock, G. (2010). *Is the UK an outlier? An international comparison of upper secondary mathematics education*. Nuffield Foundation. https://www.nuffieldfoundation.org/sites/default/files/files/Country_profiles_outlier_NuffieldFoundation18_04_11.pdf

The Nuffield Foundation for Educational Research (NFER) commissioned a study to gather evidence regarding the extent to which the United Kingdom is unique in requiring students to study mathematics after age 16. The research was carried out by Dr Jeremy Hodgen and David Pepper from King's College London and Linda Sturman and Graham Ruddock from NFER. The research addresses a number of questions about policy and participation in upper secondary mathematics education in 24 countries (mainly from the Organisation for Economic Co-operation and Development), including the four countries of the United Kingdom. The results are summarized in this report and the associated country profiles that underpin it. The findings are stark. In England, Wales, and Northern Ireland fewer than one in five students study any mathematics after the age of 16 (Scotland does slightly better). In 18 of the 24 countries, more than half of students in the age group study mathematics; in 14 of these, the participation rate is over 80 percent; and in 8 of these every student studies mathematics.

2—Hofstra University Center for STEM Research. (2018, January 12–14). *Needed Math Conference proceedings: A National Science Foundation Advanced Technological Education funded* *conference.* http://www.neededmath.org or https://www.hofstra.edu/academics/colleges/seas/ctl/needed-math/index.html

This project website provides information about Needed Math (NM), a three-year full-scale research and development project submitted to the Advanced Technological Education (ATE) program as Targeted Research on Technician Education. The aims of the project are (a) to identify contextual examples of important mathematical concepts, skills, and processes (mathematical competencies) that technicians in selected manufacturing sectors are expected to be able to apply; and (b) to develop and evaluate a replicable mechanism for industry to communicate with community college faculty about the mathematics needed.

1—Hopkins, G. (2009). Is ability grouping the way to go—or should it go away? *Education World, 24.* http://www.educationworld.com/a_admin/admin/admin009.shtml

This article discusses some of the frequent and ongoing points of contention between proponents and opponents of tracking/ability grouping, including whether or not it is a fair and effective practice and whether or not heterogenous classrooms do all students a disservice because teachers are not as able to tailor instruction to students' current ability or achievement level.

4—Hord, C., Gregson, S. A., Walsh, J. B., & Marita, S. (2018). Using visual modeling tools to reach students with learning disabilities. *Ohio Journal of School Mathematics, 79*(1).

Teachers can use electronic visual modeling tools to help students with learning disabilities visualize and understand mathematical concepts such as proportions, dilations, and scale factors. In this article, the authors describe strategies for using static and dynamic visuals for supporting the memory and processing of students with learning disabilities as they engage in challenging mathematics.

5—Howell, J. S., Kurlaender, M., & Grodsky, E. (2010). Postsecondary preparation and remediation: Examining the effect of the early assessment program at California State University. *Journal of Policy Analysis and Management, 29*(4), 726–748.

In this paper, the authors investigate how participation in the Early Assessment Program (EAP), which provides California high school juniors with information about their academic readiness for college-level work at CSU campuses, affects their college-going behavior and need for remediation in college. Using administrative records from CSU Sacramento and the California Department of Education, they authors find that participation in the Early Assessment Program reduces the average student's probability of needing remediation at CSU by 6.1 percentage points in English and 4.1 percentage points in mathematics. Rather than discouraging poorly prepared students from applying to Sacramento State, EAP appears to lead students to increase their academic preparation while still in high school.

3—Howson, G., Keitel, C., & Kilpatrick, J. (1981). *Curriculum development in mathematics.* Cambridge University Press. https://www.academia.edu/7444115/_Geoffrey_Howson_Christine_Keitel_Jeremy_Kilpatr_Book_Fi_org_

In the mid-1970s, the curriculum development boom in mathematics was to end almost as rapidly as it had begun. In this book the authors, who come from countries with differing educational traditions and patterns, consider these developments in their historical, social, and educational context. They give not only a descriptive account of developmental work in a variety of countries, its aims, and the patterns of management utilized, but also attempt to identify trends and characteristics and thus provide a theoretical base for criticism and analysis. The book includes several case studies, including extracts from such renowned authors as Bruner, Dieudonne, and Piaget.

1—Huckstep, P. (2000). The utility of mathematics education: Some responses to scepticism. *For the Learning of Mathematics*, *20*(2), 8–13. https://www.jstor.org/stable/40248321

In this paper, the author highlights some general difficulties that emerge in justifying the learning of mathematics on the strength of its supposed utility. The author discusses the view that mathematics is of crucial importance in every student's education, focusing on use and cognition, use and compulsion, and use and self-consciousness.

2—Hughes, K. L., & Zoellner, J. (2019a). *Emerging solutions in mathematics education for nursing* [White paper]. Charles A. Dana Center at the University of Texas at Austin. https://dcmathpathways.org/resources/emerging-solutions-1-mathematics-education-nursing

While a shortage of nursing faculty continues to be a hindrance on the supply side of nursing education, a consideration on the demand side is whether any program prerequisites or curricular requirements serve as unnecessary obstacles to nursing degree program entry, progression, or completion. This brief examines the mathematics content and requirements for nursing degrees, as compared with the mathematics used by nurses in the field; considers whether they pose a barrier to access; and offers some emerging solutions suggested or implemented by a variety of agencies, institutions, and states.

2—Hughes, K. L., & Zoellner, J. (2019b). *Mathematics for social work: Recommendations from professional organizations and sample requirements from U.S. institutions of higher education*. Program-of-Study Brief Number 4. Charles A. Dana Center at the University of Texas at Austin.
https://dcmathpathways.org/resources/program-study-issue-brief-mathematics-social-work

Updated in 2019 with national data, new research, and refreshed analyses, this program-of-study brief discusses what constitutes relevant mathematics for social work majors by examining institutional requirements and recommendations from professional organizations.

3—Hull, T., Balka, D., & Miles, R. (2013). Mathematical rigor in the Common Core. *Principal Leadership, 14*(2), 50–55. https://eric.ed.gov/?id=EJ1018772

The authors discuss how the main driving forces around teaching and learning mathematics are a combination of changes in assessment and advances in technology being spurred on by introduction of content in the Common Core State Standards for Mathematical Practice. They discuss how technological advances are expanding the ways mathematics teachers can assess students' learning and how the end result is that classroom assessments will become more rigorous. The shift in classroom instruction and assessments will demand that students learn

and perform differently. To meet this shift in assessments and demand for rigor, authors argue that classroom instruction must change, specifically in the direction of embracing the mathematical shifts in the common core and the Standards for Mathematical Practice. They argue that the practices associated with the standards, when used intentionally and deeply, promote mathematical rigor, but that to be attained, mathematical rigor must be clearly understood as it relates to the practices and forthcoming assessments. This article defines rigor, provides a rigor comparison chart showing current practices versus future demands, and provides two sample problems that accentuate the differences between a current classroom and a future, rigorous classroom.

3—Hull, T. H., Miles, R. H., & Balka, D. S. (2014). *Realizing rigor in the mathematics classroom*. Corwin Press. https://sk.sagepub.com/books/realizing-rigor-in-the-mathematics-classroom

The framework the authors of this book provide for analyzing how rigor is valued in the classroom captures the major problems and successes for learning mathematics. Rigor is a habit of mind and as such requires daily attention in the process of mathematics learning. Thinking, reasoning, and understanding of mathematics are all a basis of rigor. The authors believe that there are three widely agreed-upon principles that have demonstrated the nature of rigorous learning experiences:

1. Good teaching is central to improving achievement.
2. Teachers must identify rigorous, well-defined curriculum standards, benchmarks, and corresponding assessments.
3. All stakeholders must hold high expectations for student performance.

The authors make the case for powerful mathematics instruction that relates conceptual development and mathematical connections. This type of instruction supports themes identified by the National Research Council as ones that support rigor. The authors also develop a Rigor Comparison Chart that lists current descriptions and thoughts about rigor versus future demands of rigorous mathematical teaching and learning. It is noted many times that mathematical rigor can be achieved by effectively implementing the Common Core State Standards for Mathematics and the Standards for Mathematical Practice.

1—Hyde, J. S., & Mertz, J. E. (2009). Gender, culture, and mathematics performance. *Proceedings of the National Academy of Sciences*, *106*(22), 8801–8807. https://www.pnas.org/doi/abs/10.1073/pnas.0901265106

Using contemporary data from the United States and other nations, the authors address three questions: Do gender differences in mathematics performance exist in the general population? Do gender differences exist among the mathematically talented? Do females exist who possess profound mathematical talent? In regard to the first question, contemporary data indicate that girls in the United States have reached parity with boys in mathematics performance, a pattern that is found in some other nations as well. Focusing on the second question, studies find more males than females scoring above the 95th or 99th percentile, but this gender gap has significantly narrowed over time in the United States and is not found among some ethnic groups and in some nations. Furthermore, data from several studies indicate that greater male variability with respect to mathematics is not ubiquitous. Rather, its presence correlates with several measures of gender inequality. Thus, it is largely an artifact of changeable sociocultural factors, not immutable, innate biological differences between the sexes. Responding to the third question, the authors document the existence of females who possess profound mathematical talent. Finally, they review mounting evidence that both the magnitude of mean math gender differences and the frequency of identification of gifted and profoundly gifted females significantly correlate with sociocultural factors, including measures of gender equality across nations.

1—Jahn, J. L., & Myers, K. K. (2014). Vocational anticipatory socialization of adolescents: Messages, sources, and frameworks that influence interest in STEM careers. *Journal of Applied Communication Research*, *42*(1), 85–106. https://nca.tandfonline.com/doi/abs/10.1080/00909882.2013.874568?journalCode=rjac20#.ZC5g9ezMI0Q

By high school, many students have dropped out of the pipeline that will lead to science, technology, engineering, and math (STEM) occupations. The authors examine the role of vocational anticipatory socialization (VAS)—the types of messages adolescents receive, message sources, and adolescents' frameworks—on youth's educational and vocational interests. Adolescents (37 focus groups, $N = 229$) reported that they received two types of VAS messages: *personal fulfillment* (advising students to prioritize their well-being) and *career detail* (advising students about specific aspects of an occupation). Adolescents used three career frameworks (*enjoyment, ability*, and *goal*) that filtered and often magnified VAS messages and experiences. The authors extend VAS research by identifying two primary purposes of the career advice embedded in VAS messages and three career frameworks. Practical implications are that parents can affect adolescents' beliefs about their abilities and potential enjoyment of STEM careers by supplementing *personal fulfillment* messages with *career detail* messages. Individuals in STEM occupations are in the best position to encourage adolescents by offering career detail and discussing how their career can be rewarding and how math and science classes can influence their career attainment.

5—Jessen, S. B., & Johnson, A. F. (2020). *Math pathways reforms in Maine.* Maine Educational Policy Research Institute (MEPRI), University of Southern Maine.

For this project, MEPRI researchers were asked to describe and summarize the "mathematics pathways" movement that is currently underway in the state of Maine, as well as in other states across the country. For the past several years, attention has turned toward reviewing and reframing the curricular pathways in math for students, particularly at the postsecondary level. Focus has shifted not only to math preparedness, but also to redefining the curricular pathways that lead to academic and career success. In higher education, the central principle involves redefining and reorganizing coursework around defined pathways, based on academic and career interests of students. While "traditional" math tracks would still exist in some subject areas, other subject areas, like social sciences, would lean more heavily upon statistics, for example. For K–12 institutions, the math pathways reforms have less clear implications. This study of the math pathways movement is designed as a mini case study to document the status of the reform in progress. Data collection began in August 2019 and concluded in December 2019. Data collection primarily included interviews and document/resource collection and review. Findings review math pathways as a potential means of promoting individualized student learning and career readiness, of clarifying math expectations, and of reducing the need for remedial coursework at the postsecondary level. The successes of and challenges facing individual institutions—from the Maine Community College System, to the state universities, to K–12 institutions—are also reviewed. Other challenges, including mitigating issues with tracking and ownership of the reforms, are discussed. This report concludes by recommending that policymakers consider the differences between the political and organizational structures in evaluating whether and how to incorporate math pathways at the K–12 level, the pressing need to navigate future postsecondary expectations for students in career and technical education programs, as well as a recommendation to support developing guidance structures for students.

1—Jiang, S., Simpkins, S. D., & Eccles, J. S. (2020). Individuals' math and science motivation and their subsequent STEM choices and achievement in high school and college: A longitudinal study of gender and college generation status differences. *Developmental Psychology*, *56*(11), 2137.

Math and science motivational beliefs are essential in understanding students' science, technology, engineering, and math (STEM) achievement and choices in high school and college. Drawing on the Eccles' expectancy-value theory and Arnett's emerging adulthood framework, this study examined the relations among high school students' motivational beliefs in 9th grade and their STEM course-taking and grade point average (GPA) throughout high school as well as their STEM major choice in college. In addition, we examined subgroup differences across (a) gender and (b) college generation status by testing mean-level differences as well as whether these relations between math and science motivational beliefs and STEM outcomes varied by gender and college generation status. Using nationally representative data from the High School Longitudinal Study (N = 14,040; M_{age} = 14; 51 percent female students), this study found that adolescents' math and science motivational beliefs at the beginning of high school were positively associated with STEM achievement and course-taking throughout high school and college major choices seven years later. The results showed that female and first-generation college students had lower math and science self-concept of ability and were less likely to pursue a STEM major in college. However, in most cases, the relations among indicators did not vary by gender and college generation status. This study provided insights for policymakers and practitioners that gender and college generation gaps in STEM are evident at least by the beginning of high school and carry forward to their STEM college choices.

1—Jimenez, L., Sargrad, S., Morales, J., & Thompson, M. (2016). *Remedial education: The cost of catching up.* Center for American Progress. https://www.americanprogress.org/article/remedial-education/

Across the country, millions of students enroll in college every year only to learn that they need to take classes that will not count toward their degrees because they cover material that they should have learned in high school. According to the authors' analysis for this report, these remedial courses cost students and their families about $1.3 billion across the 50 states and the District of Columbia every year. What is more, students who take these classes are

less likely to graduate. After defining remedial education, the authors briefly review the typical methods that institutions employ to identify students in need of remediation and the resulting national demographics of remediated students. Then, the report touches on national rates of progress through remedial education for major racial or ethnic and socioeconomic student groups before focusing on how much money students spend on these courses that do not count toward a degree. While there are certainly reforms to the design of remedial education in higher education institutions that could improve student retention and completion, the recommendations that conclude this report focus on other ways for the K–12 and higher education systems to eliminate the need for remedial education for recent high school graduates.

1, 3—Joensen, J. S., & Nielsen, H. S. (2009). Is there a causal effect of high school math on labor market outcomes? *Journal of Human Resources*, *44*(1), 171–198. https://www.econstor.eu/bitstream/10419/34081/1/533992362.pdf

In this paper, the authors exploit a high school pilot scheme to identify the causal effect of advanced high school math on labor market outcomes. The pilot scheme reduced the costs of choosing advanced math because it allowed for a more flexible combination of math with other courses. Authors find clear evidence of a causal relationship between math and earnings for students who are induced to choose math after being exposed to the pilot scheme. The effect partly stems from the fact that these students end up with a higher education.

4—Jones, K. (2002), Issues in the teaching and learning of geometry. In Linda Haggarty (Ed.), *Aspects of Teaching Secondary Mathematics: Perspectives on Practice* (pp. 121–139). Routledge Falmer. https://www.researchgate.net/publication/200744703_Issues_in_the_teaching_and_learning_of_geometry#fullTextFileContent

The aim of this chapter is to introduce some of the special features of geometry and its teaching and learning. The chapter examines the nature of geometry, the reasons for it being included in the school mathematics curriculum, and how it can be best taught and learned. At the end of the chapter there are commentaries and hints on many of the tasks.

2—Just Equations (2021). [Website]. https://justequations.org/

Just Equations is an organization with the goal of reconceptualizing the role of math in ensuring educational equity. It works to advance evidence-based policies and strategies that ensure all students develop the quantitative foundation they need to succeed in college and beyond. Policy goals include (a) ensuring all students have a rigorous quantitative reasoning foundation that prepares them to attend and complete college; (b) advancing postsecondary admission and access policies that eliminate arbitrary barriers to equitable college readiness and success; (c) supporting work around redesigned math pathways that reflect 21st-century needs and align with students' academic and career aspirations; and (d) supporting well-aligned, rigorous math expectations that support equitable outcomes across education systems.

4—Kahne, J., & Bowyer, B. (2017). Educating for democracy in a partisan age: Confronting the challenges of motivated reasoning and misinformation. *American educational research journal*, *54*(1), 3–34. https://journals.sagepub.com/doi/pdf/10.3102/0002831216679817

This article investigates youth judgments of the accuracy of truth claims tied to controversial public issues. In an experiment embedded within a nationally representative survey of youth ages 15 to 27 ($N = 2{,}101$), youth were asked to judge the accuracy of one of several simulated online posts. Consistent with research on motivated reasoning, youth assessments depended on (a) the alignment of the claim with one's prior policy position and to a lesser extent on (b) whether the post included an inaccurate statement. To consider ways educators might improve judgments of accuracy, the authors also investigated the influence of political knowledge and exposure to media literacy education. They found that political knowledge did not improve judgments of accuracy but that media literacy education did.

4, 5—Kane, T., Boatman, A., Kozakowski, W., Bennett, C., Hitch, R., & Weisenfeld, D. (2018). *Remedial math goes to high school: An evaluation of the Tennessee SAILS program* (CEPR Policy Brief). Harvard University, Center for Education Policy Research. https://cepr.harvard.edu/files/cepr/files/sails_research_report_final.pdf

In this report, the authors evaluate the impact of the Seamless Alignment and Integrated Learning Support [SAILS] program on students' ability to take and pass college-level math and to accumulate college-level credits. They evaluate the program's impact under two different remediation policies: first, they measure the consequences for the high school seniors of 2013–14, when Tennessee community colleges still required students to complete remediation before their college-level coursework ("pre-requisite remediation"); they also measure impacts for the seniors of 2014–15 and 2015–16, after Tennessee community colleges began allowing students to enroll in remediation concurrently with their college-level classes ("corequisite" remediation). Currently, at least eight states have a form of corequisite remediation analogous to Tennessee's. The authors use two different research designs to discern the impact of SAILS. First, because the SAILS program was rolled out in stages with high schools implementing the program in different years, they measure impacts by comparing changes in the outcomes of remediation-eligible students in schools implementing SAILS to the changes in outcomes for schools that had not yet implemented SAILS (or had implemented SAILS in a prior year). The staggered timing of SAILS implementation allows the authors to distinguish the effect of SAILS from other policy changes affecting high school students in Tennessee. Second, the authors compare outcomes for students with junior year ACT math scores just below the remediation cutoff, who were recommended for remediation, to the outcomes for those just above the same cutoff, who had similar academic achievement but escaped remediation. In other words, they take those with ACT scores just above the cutoff as a control group for measuring the effect of the remediation treatment. They do the same analysis in high schools with and without the SAILS program. Moreover, they can compare the impacts for the class of 2013–14 (who faced pre-requisite remediation) to the impacts for the class of 2015–16 (who faced corequisite remediation). The two methods rely on different assumptions and different comparison groups. The first assumes that the changes in outcomes at participating and nonparticipating high schools would have been the same if not for the SAILS program; the second assumes that those with ACT math scores just above the remediation cutoff (and those

who were not required to take remediation) are roughly similar to those just below (at least not in a way that cannot be controlled for with differences in their observed ACT scores). They present both sets of results, which yield similar findings.

3—Kanold, T. D., Larson, M. R., Kanold-McIntyre, J., Schuhl, S., Barnes, B., & Toncheff, M. (2018). *Mathematics assessment and intervention in a PLC at work*. Solution Tree Press & NCTM.

This resource is divided into two parts, each covering a key team action for mathematics in a professional learning community (PLC): (1) developing high-quality common mathematics assessments, and (2) using these assessments for formative student learning and intervention. The book features unit samples for learning standards, sample unit exams, student performance trackers, and more.

1—Keitel, C. (2015). Mathematics, knowledge and political power. In J. Maasz & W. Schlöglmann (Eds.), *New mathematics education research and practice* (pp. 11–22). Brill.

In this chapter, the author traces the historical evolution of conceptions of the social impact of mathematics, giving special attention to the influence of modern information and communication technologies. The author explores links between this development and the corresponding evolving conceptions of mathematical literacy and mathematics education for all.

1—Kelly, S. (2009). The black-white gap in mathematics course taking. *Sociology of Education, 82*(1): 47–79. https://journals.sagepub.com/doi/10.1177/003804070908200103

Using data from the National Education Longitudinal Study, this study investigated differences in the mathematics course-taking of white and Black students. Because of lower levels of achievement, prior course-taking, and lower socioeconomic status, Black students are much more likely than white students to be enrolled in low-track mathematics courses by the 10th grade. Using multilevel models for categorical outcomes, the study found that the Black-white gap in mathematics course-taking is the greatest in integrated schools where Black students are in the minority and cannot be entirely accounted for by individual-level differences in the course-taking qualifications or family backgrounds of white and Black students. This finding was obscured in prior research by the failure to model course-taking adequately between and within schools. Course placement policies and enrollment patterns should be monitored to ensure effective schooling for all students.

4—Kesar, S. (2017). *Closing the STEM gap: Why STEM classes and careers still lack girls and what we can do about it*. Microsoft Philanthropies. https://query.prod.cms.rt.microsoft.com/cms/api/am/binary/RE1UMWz

Microsoft commissioned this research to understand better what causes girls and women to lose interest in STEM subjects and careers, as well as what strategies and interventions have the greatest potential to reverse this trend. The goal is to inform the author's own work in this area and to share learnings with schools, government leaders, nonprofits, employers, and others. What was learned is that conditions and context can make a significant difference to girls, young women, and their interest in STEM. And the solution doesn't necessarily require

a curricula overhaul. We may be able to make significant strides just by showing girls and young women how STEM knowledge is applicable outside of the classroom and how it can power their aspirations to make the world a better place. Main research findings show:

- Girls and young women have a hard time picturing themselves in STEM roles. They need more exposure to STEM jobs, female role models, and career awareness and planning.
- Girls don't initially see the potential for careers in STEM to be creative or have a positive impact on the world. But even a little exposure to real-world applications of STEM knowledge dramatically changes their outlook.
- Girls who participate in STEM clubs and activities outside of school are more likely to say they will pursue STEM subjects later in their education. The kinds of experiments and experiences girls are exposed to in these activities can provide insights for how to enhance STEM instruction in the classroom.
- Encouragement from teachers and parents makes a big difference in girls' interest in STEM—especially when it comes from both teachers and parents.

Educators can foster a "growth mindset" among their female students by tapping into their willingness to work hard for results.

1—Kim, J., Kim, J., DesJardins, S. L., & McCall, B. P. (2015) Completing Algebra II in high school: Does it increase college access and success? *The Journal of Higher Education, 86*(4), 628–662. https://www.tandfonline.com/doi/abs/10.1080/00221546.2015.11777377

Noting the benefits of mathematics in students' future educational attainment and labor market success, there is considerable interest in high school requirements in terms of course-taking in mathematics at the national, state, and school district levels. Previous research indicates that taking advanced math courses in high school leads to positive college outcomes. However, these studies often fail to account for the self-selection of students into curricular pathways that may result in biased estimates of the effect of course-taking on subsequent educational outcomes. Applying an instrumental variable (IV) approach, the authors investigate how the level of math courses a student completes in high school differently affects their chances of attending and completing postsecondary education. Using longitudinal student unit record data from Florida, the results indicate that a statistical model that does not account for students' self-selection produces results different from a technique that corrects for this potential source of bias. Specifically, completing Algebra II significantly increases the probability of attending college, particularly two-year colleges, but has no significant effect on degree attainment.

1—Klein, D. W. (2002). Beyond Brown v. Board of Education: The need to remedy the achievement gap. *Journal of Law and Education 31*, 431–457. https://eric.ed.gov/?id=EJ655373

This paper addresses the need to remedy the disparity in academic achievement of Black and white students and examines why this disparity continues to exist in spite of the desegregation decrees issued under *Brown v. Board of Education*. The paper reviews how a court decides whether a school district has complied with a desegregation decree and explains why schools are being released from desegregation decrees despite achievement gaps.

3, 4—Klipple, K. (2021). *3 Things we've learned about designing effective dual enrollment math programs.* Carnegie Math Pathways. https://www.carnegiemathpathways.org/3-things-weve-learned-about-designing-effective-dual-enrollment-math-programs/

In this blog post, the author discusses features of the Carnegie Math Pathways dual enrollment program that support diverse student populations in accessing and succeeding in credit-bearing college level mathematics.

1—Knudson, J. (2019). *Pursuing equity and excellence in mathematics course sequencing and placement in San Francisco.* California Collaborative on District Reform. https://files.eric.ed.gov/fulltext/ED596435.pdf

On the heels of an Algebra for All model that failed to generate desired outcomes for students, San Francisco Unified School District adopted a policy in 2014 that dramatically changed its sequence of mathematics courses. The district completely de-tracked its middle school classes, enrolling all students in the same heterogeneously grouped courses for grades 6, 7, and 8. San Francisco's policy represents a significant departure from traditional approaches to organizing mathematics courses, and early outcomes appear to validate this approach. This brief describes the rationale behind the district's decision, the nature of the new policy, and the promising results the district has experienced so far.

4—Konold, C., Higgins, T., Russell, S. J., & Khalil, K. (2015). Data seen through different lenses. *Educational Studies in Mathematics, 88*(3), 305–325.

Statistical reasoning focuses on properties that belong not to individual data values but to the entire aggregate. The authors analyze students' statements from three different sources to explore possible building blocks of the idea of data as aggregate and speculate on how young students go about putting these ideas together. They identify four general perspectives that students use in working with data, which in addition to an aggregate perspective include regarding data as pointers, as case values, and as classifiers. Some students seem inclined to view data from only one of these three alternative perspectives, which then influences the types of questions they ask, the data representations they generate or prefer, the interpretations they give to notions such as the average, and the conclusions they draw from the data.

4—Krupa, E. E., & Confrey, J. (2017). Effects of a reform high school mathematics curriculum on student achievement: Whom does it benefit? *Journal of Curriculum Studies*, *49*(2), 191–215. https://www.tandfonline.com/doi/abs/10.1080/00220272.2015.1065911

This study compared the effects of an integrated reform-based curriculum to a subject-specific curriculum on student learning of 19,526 high school algebra students. Using hierarchical linear modelling to account for variation in student achievement, the impact of the reform-based *Core-Plus Mathematics* curricular materials on student test scores is compared to the subject-specific curriculum. Findings from this study indicate that students enrolled in integrated mathematics outperformed subject-specific students on an Algebra I exam (highly aligned with content) and performed equally on an Algebra II exam (poorly aligned). Minority students in high-need schools demonstrated higher performance when they were enrolled in integrated mathematics.

2—LaMar, T., & Boaler, J. (2021). The importance and emergence of K–12 data science. *Phi Delta Kappan, 103*(1), 49–53. https://kappanonline.org/math-importance-emergence-k12-data-science-lamar-boaler/

Tanya LaMar and Jo Boaler argue that data science education provides an opportunity to address students' lack of data literacy skills while providing much needed updates to the current mathematics curriculum. The integration of data science can provide a more equitable mathematics pipeline than the calculus-focused pathway that has excluded most students from a future in mathematics. Through data science, students can learn to answer questions that are relevant to their lives and communities, to be critical consumers of the data that surround them every day, and to wield the power of data analysis.

4—Lamon, S. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. Lester (Ed.). *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Information Age Publishing. https://www.infoagepub.com/products/Second-Handbook-Research-Mathematics-Teaching-Learning

This study investigated elementary and secondary teachers' understanding and pedagogical strategies applied to students making errors in finding a missing length in similar rectangles. It was revealed that secondary teachers had better understanding of ratio and proportion in similar rectangles than elementary teachers. While all secondary teachers solved the similar rectangles problems correctly, a large portion of elementary teachers struggled with the problem. In explaining their solution strategies, and even though similar strategies appeared both from elementary teachers and secondary teachers, a majority of secondary teachers pointed out the underlying idea of similarity, whereas less than half of the elementary teachers explained their reasoning for using ratios and proportion.

1—Lawyers' Committee for Civil Rights of the San Francisco Bay Area (2013). *Held back: Addressing misplacement of 9th grade students in Bay Area school math classes.* https://lccrsf.org/wp-content/uploads/HELD-BACK-9th-Grade-Math-Misplacement.pdf

This report describes how many 9th graders in San Mateo and Santa Clara Counties in California were being improperly placed in 9th grade Algebra I classes, despite having passed the class in 8th grade and/or having met or exceeded state standards on California Standards Tests (CSTs). Data indicate that minority students were being disparately impacted by these improper placements.

4—Leonard, A. E., & Bannister, N. A. (2018). Dancing our way to geometric transformations. *Mathematics Teaching in the Middle School, 23*(5), 258–267. https://pubs.nctm.org/view/journals/mtms/23/5/article-p258.xml

Research suggests that embodying mathematical concepts through movement supports improvements in students' spatial fluency, communication, collaboration, critical thinking, and problem-solving skills. This article describes a dance-based strategy that middle school mathematics teachers can use to leverage embodied movement in their instructional repertoires. Authors take cues from dance educators for imagining this work because they have disciplinary expertise in the practice of movement as a form of inquiry. They define dance broadly as purposeful sequences of movement and stillness that communicate meaning.

1—Levin, H., & Calcagno, J.C. (2008). Remediation in the community college: An evaluator's perspective. *Community College Review, 35*(3), 181–207. https://academiccommons.columbia.edu/doi/10.7916/D82V2Q7V/download

Remediation is the most common approach to preparing students academically and socially during their early stages of college. However, despite its profound importance and its significant costs, there is very little rigorous research analyzing its effectiveness. The goal of this article is to provide a conceptual framework for the evaluation of remedial education programs. Based on previous literature, the authors review a list of ingredients for successful interventions, present a number of approaches to remediation that make use of these ingredients, discuss alternative research designs for systematic evaluations, and enumerate basic data requirements.

1—Leyva, L. A., McNeill, R. T., Marshall, B. L., & Guzmán, O. A. (2021). "It seems like they purposefully try to make as many kids drop": An analysis of logics and mechanisms of racial-gendered inequality in introductory mathematics instruction. *The Journal of Higher Education*, *92*(5), 784–814.

Introductory mathematics courses, including precalculus and calculus, largely influence Black and Latina/o students' persistence and sense of belonging in STEM. However, prior research on instruction in these courses for advancing more equitable outcomes is limited. This paper presents findings from a study of 18 Black and Latina/o students' perceptions of introductory mathematics instruction as a racialized and gendered experience at a large, public, and historically white research university. Sociological perspectives of logics and mechanisms of inequality guided an analysis of Black and Latina/o students' group interview responses on how instruction perpetuates racial and gendered oppression. Two logics were identified: (a) instructors hold more mathematical authority than students in classrooms; and (b) calculus coursework is used to weed out students "not cut out" for STEM. These logics, coupled with the influence of broader sociohistorical forces (e.g., cultural scripts of behavior, stereotypes), gave rise to mechanisms of inequality through seemingly neutral instructional practices that reinforce racial-gendered distribution of classroom participation and STEM persistence. The findings inform implications for STEM higher education researchers and mathematics faculty to foster socially affirming STEM instruction, especially in introductory courses.

1—Leyva, L. A., Quea, R., Weber, K., Battey, D., & López, D. (2021). Detailing racialized and gendered mechanisms of undergraduate precalculus and calculus classroom instruction. *Cognition and Instruction*, *39*(1), 1–34.

Undergraduate mathematics education can be experienced in discouraging and marginalizing ways among Black students, Latino/a students, and white women. Precalculus and calculus courses, in particular, operate as gatekeepers that contribute to racialized and/or gendered attrition in persistence with mathematics coursework and pursuits in STEM. However, student perceptions of instruction in these introductory mathematics courses have yet to be systematically examined as a contributor to such attrition. This paper presents findings from a study of 20 historically marginalized students' perceptions of precalculus and calculus instruction to document features that they found discouraging and marginalizing. The authors' analysis revealed how students across different race-gender identities reported stereotyping as well as issues of representation in introductory mathematics classrooms and STEM fields shaping their perceptions of instruction. These perceptions pointed to the operation of three racialized and gendered mechanisms in instruction: (a) creating differential opportunities for participation and support; (b) limiting support from same-race, same-gender peers to manage negativity in instruction; and (c) activating exclusionary ideas about who belongs in STEM fields. The authors draw on findings to raise implications for research and practice in undergraduate mathematics education.

4—Lobato, J., Ellis, A., & Zbiek, R. M. (2010). *Developing essential understanding of ratios, proportions & proportional reasoning: Grades 6–8*. NCTM. https://www.nctm.org/Store/Products/Developing-Essential-Understanding-of-Ratios,-Proportions,-and-Proportional-Reasoning-for-Teaching-Mathematics--Grades-6-8/

This book extends discussion of ratios, proportions, and proportional reasoning—some of the most challenging topics for students, and teachers, to grasp.

1, 3—Long, M. C., Conger, D., & Iatarola, P. (2012). Effects of high school course-taking on secondary and postsecondary success. *American Educational Research Journal, 49*(2), 285–322. https://journals.sagepub.com/doi/abs/10.3102/0002831211431952?journalCode=aera

Using panel data from a census of public school students in the state of Florida, the authors examine the associations between students' high school course-taking in various subjects and their 10th-grade test scores, high school graduation, entry into postsecondary institutions, and postsecondary performance. The authors use propensity score matching (based on 8th-grade test scores, other student characteristics, and school effects) within groups of students matched on the composition of the students' course-taking in other subjects to estimate the differences in outcomes for students who take rigorous courses in a variety of subjects. The authors find substantial significant differences in outcomes for those who take rigorous courses, and these estimated effects are often larger for disadvantaged youth and students attending disadvantaged schools.

2—Long Beach City College. *LBCC Promise Pathways*. (n.d.). https://www.lbcc.edu/post/lbcc-promise-pathways

The website describes the Promise Pathways initiative of Long Beach City College (LBCC), one of LBCC's student success strategies and its signature effort in support of the Long Beach College Promise partnership with Long Beach Unified School District (LBUSD) and California State University, Long Beach. LBCC implemented several innovations including an evidence-based, multiple measures placement and prescriptive first-semester success plans with registration priority for Promise Pathways students. Students were placed in English and math courses based broadly on their high school achievement (e.g., high school GPA, last grade in the discipline) rather than traditional standardized tests.

4—Los Angeles Unified School District (n.d.). *Financial Algebra scope and sequence*. https://achieve.lausd.net/Page/11406

The website describes LAUSD's Financial Algebra course.

1—Loveless, T. (2013 September 4). Report: Algebra II and the declining significance of coursetaking. [Blog post]. The Brown Center Chalkboard Series. Brookings Institution. https://www.brookings.edu/research/algebra-ii-and-the-declining-significance-of-coursetaking/

In this blog post, the author elaborates on findings of a recent report that taking and successfully completing an Algebra II course, which once certified high school students' mastery of advanced topics in algebra and solid preparation for college-level mathematics, no longer means what it once did. They argue that credentialing integrity of Algebra II has weakened. The author presents National Assessment of Educational Progress (NAEP) data on second-year algebra completion, describes the pipeline problem in which students are underprepared for more advanced mathematics, and describes consequences of this underpreparation.

4—Lowell High School. (2022). Math sequence flow chart. https://www.lowell.k12.ma.us/Page/1222

This website illustrates three possible course-taking sequences for students at Lowell High School, in a community outside of Boston, Massachusetts. There is a college-level transitional flow, a college-level flow, and an honors flow for 9th–12th grade students.

4—Lue, R. A. (2019). Data science as a foundation for inclusive learning. *Harvard Data Science Review*, *1*(2), 1-6. https://assets.pubpub.org/6vaulw7k/b801f768-bf01-4409-bf63-be583891eb3a.pdf

This article describes how data science presents a unique opportunity to build a more inclusive framework for the teaching of STEM. The widening application of data science methods to nearly every field imaginable in the natural sciences, social sciences, and humanities opens up avenues for engagement based on what students care about and the challenges they are most interested in tackling. Data science therefore provides an opportunity to build an inclusive STEM curriculum from the ground up that connects with multiple disciplines as well as the personal passions of students.

4—Macdonald, H., Zinth, J. D., & Pompelia, S. (2019). *High school graduation requirements: What are the state's course requirements for high school graduation?* Education Commission of the States. https://www.ecs.org/high-school-graduation-requirements/

This resource provides an overview of state high school graduation requirements. Nearly all states require students to complete a certain number of units per course to earn a standard diploma in the state; but courses, units, and assessments outside of exit exams and diploma options vary. Education Commission of the States researched a range of policies—including state statutes, state regulations, department guidelines, state standards, and state board rules—to create this 50-State Comparison.

1—Mackenzie, J. C. (1894). The report of the committee of ten. *The School Review*, *2*(3), 146–155. https://ia800202.us.archive.org/12/items/cu31924032709960/cu31924032709960.pdf

Historical report of the meeting of the Committee of Ten regarding establishing educational requirements for U.S. secondary students, including the origin of the Algebra I–Geometry–Algebra II high school mathematics course sequence.

1, 4, 5—Mackey, E.G. (2019). 2019 Alabama course of study: Mathematics. Montgomery, AL: Alabama State Department of Education. https://www.alabamaachieves.org/wp-content/uploads/2021/03/2019-Alabama-Mathematics-COS-Rev.-6-2021.pdf

Society and the workplace require that all Alabama students receive a solid foundation of knowledge, skills, and understanding in mathematics. Alabama educators should focus on the teaching of mathematics in ways that enable students to expand professional opportunities; understand and critique the world; and experience the joy, wonder, and beauty of mathematics. To address this goal, the content of the 2019 Alabama Course of Study: Mathematics sets high standards for all students and reflects changes designed to better meet the needs of students and teachers in the State of Alabama. The document presents a conceptual framework, mathematical practice standards, and content standards across the grades, with a separate section on math pathways.

1—Madison, B. L., & Steen, L. A. (2009). Confronting challenges, overcoming obstacles: A conversation about quantitative literacy. *Numeracy, 2*(1), 1–25. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=fd4c4cbef5b43921d00e395af1971b70b36bba3a

An edited transcript of the opening session of a workshop on quantitative literacy held October 10–12, 2008, at Carleton College, Northfield, Minnesota. The workshop, which brought together interdisciplinary teams from two dozen colleges and universities, was sponsored by the Quantitative Inquiry, Reasoning, and Knowledge (QuIRK) Initiative at Carleton and the Washington-based Project Kaleidoscope. Two mathematicians in the forefront of quantitative literacy initiatives over the period 1997–2008, Lynn Arthur Steen and Bernard L. Madison, converse about attitudes, obstacles, changes, and accomplishments. The conversation, structured as an interview, begins with the relationship between mathematics and quantitative literacy and moves through issues central to effective education in quantitative reasoning, to the relationship of such reasoning to the U.S. financial crises of 2008.

1, 4, 5—Maine Department of Education (2019). *Teaching and learning—diploma requirements.* https://www.maine.gov/doe/learning/diplomas

The website shows state and district requirements that students must fulfill to receive a high school diploma, including a comprehensive program of study that includes instruction for all students in career and education development, English language arts, health education and physical education, mathematics, science, social studies, visual and performing arts, and world languages. All students must complete the following minimum requirements for a high school diploma:

- English—four years or the equivalent in standards achievement;
- Social studies and history, including American history, government, civics, and personal finance—two years or the equivalent in standards achievement;

- Mathematics—two years or the equivalent in standards achievement;
- Science, including at least one year of laboratory study—two years or the equivalent in standards achievement; and
- Fine arts, which may include art, music, forensics, or drama—one year or the equivalent in standards achievement.

4—Makgakga, S., & Sepeng, P. (2013). Teaching and learning the mathematical exponential and logarithmic functions: A transformation approach. *Mediterranean Journal of Social Sciences, 4*(13), 177. https://doi.org/10.5901/mjss.2013.v4n13p177

This paper discusses the benefits of using a transformation approach in the teaching and learning of exponential and logarithmic functions in grade 12 mathematics classrooms. The study followed a pre-test–intervention–post-test design with qualitative data informing quantitative data. Data collection strategies included a test (on exponential and logarithmic functions) that was administered to learners before and after the intervention. The intervention strategy was done via the teaching of exponential functions through a transformation approach in an attempt to reinforce understanding of the concept. A convenience sample of 38 experimental and 40 comparison groups of grade 12 learners participated in the study reported in this article from two different schools to avoid contaminating data collected. The findings indicated that the interventional strategy had an impact on academic performances of the learners. Furthermore, learners had begun to understand and apply the learned strategy when solving problems related to both exponential and logarithmic functions.

1—Mansfield, K. C., Welton, A. D., & Grogan, M. (2014). "Truth or consequences": A feminist critical policy analysis of the STEM crisis. *International Journal of Qualitative Studies in Education, 27*, 1155–1182.

STEM education has received significant attention in the United States and is largely fueled by rhetoric suggesting the United States is losing its global competitive edge and that there is a lack of qualified workers available to fill growing STEM jobs. However, a counter-discourse is emerging that questions the legitimacy of these claims. In response, the authors employed feminist critical policy analysis as both a theory and a method to further critique the STEM crisis discourse. They argue that the nature of the current discourse is misleading at worst and incomplete at best, and they show who is fueling the crisis discourse and who stands to win or lose as a result. The authors reveal how the crisis discourse draws attention away from the multilayered complexity of the issue and surface what is missing in the discourse to recenter public attention on protracted problems that still need dismantling.

1—Massachusetts Department of Elementary and Secondary Education (2023). *Massachusetts graduation requirements and related guidance.* https://www.doe.mass.edu/mcas/graduation.html

The website shows state and district requirements that students must fulfill to receive a high school diploma. All students must meet the Competency Determination (CD) standard, which is usually done by earning a passing score on the Massachusetts Comprehensive Assessment System (MCAS) examination.

4—McComb, I.-H. (2021). *A dual enrollment story: Expanding postsecondary options for Fremont High School students with Statway.* Carnegie Math Pathways.

This commentary describes how one school launched a dual enrollment program with a local community college in order to give more students the opportunity to continue studying math and the option to pursue a degree at a four-year institution after high school. The school partnered with the community college to offer Statway as an alternative to algebra and to be able to expand postsecondary options for students.

1—Meyer, M., Cimpian, A., & Leslie, S. J. (2015). Women are underrepresented in fields where success is believed to require brilliance. *Frontiers in Psychology*, 235.

Women's underrepresentation in STEM fields is a prominent concern in our society and many others. Closer inspection of this phenomenon reveals a more nuanced picture, however, with women achieving parity with men at the Ph.D. level in certain STEM fields, while also being underrepresented in some *non*-STEM fields. It is important to consider and provide an account of this field-by-field variability. The field-specific ability beliefs (FAB) hypothesis aims to provide such an account, proposing that women are likely to be underrepresented in fields thought to require raw intellectual talent—a sort of talent that women are stereotyped to possess less of than men. In two studies, the authors provide evidence for the FAB hypothesis, demonstrating that the academic fields believed by laypeople to require brilliance are also the fields with lower female representation. They also found that the FABs of participants with college-level exposure to a field were more predictive of its female representation than those of participants without college exposure, presumably because the former beliefs mirror more closely those of the field's practitioners (the direct "gatekeepers"). Moreover, the FABs of participants with college exposure to a field predicted the magnitude of the field's gender gap above and beyond their beliefs about the level of mathematical and verbal skills required. Finally, the authors found that beliefs about the importance of brilliance to success in a field may predict its female representation in part by fostering the impression that the field demands solitary work and competition with others. These results suggest new solutions for enhancing diversity within STEM and across the academic spectrum.

4, 5—Mokher, C. G., Leeds, D. M., & Harris, J. C. (2017). Adding it up: How the Florida college and career readiness initiative impacted developmental education. *Educational Evaluation and Policy Analyses, 40*(2), 219–242.

The Florida College and Career Readiness Initiative (FCCRI) was a statewide policy requiring college readiness testing and participation in college readiness courses for high school students. The authors used regression discontinuity to compare outcomes for students scoring just above and below test score cutoffs for assignment to FCCRI. They also examined impacts for students from a wider range of academic performance by using a before-after regression analysis to compare outcomes for targeted students before and after their schools implemented the FCCRI. The FCCRI increased the likelihood of enrolling in nondevelopmental courses for some targeted students, although results differ by academic performance. However, smaller differences in the likelihood of passing nondevelopmental courses suggest that some students were not prepared for these courses.

2—Milou, E., & Leinwand, S. (2022). The case for high school math pathways. *Mathematics Teacher: Learning and Teaching PK–12*, *115*(12), e1–e4.

The authors argue that the standard high school mathematics curriculum is not meeting the needs of the majority of high school students and that serious consideration of rigorous alternatives is a solution whose time has come.

5—Mitchell Institute. (2015). *Maine high school graduates: Trends in college-going, persistence, and completion, August 2015*. Maine Department of Education. https://mitchellinstitute.org/wp-content/uploads/2014/01/MaineCollegeGoing2015.pdf

This report summarizes data from the Maine Department of Education (DOE) and the National Student Clearinghouse (NSC) on college enrollment among recent Maine public high school graduates. It presents statistics at the state and superintendent region levels on initial college-going after high school graduation, persistence to the second year of college, and college degree completion. School-level data are available at the DOE's Data Warehouse website: http://dw.education.maine.gov/DirectoryManager/Web/Maine_report/MaineLanding.aspx; select Research & Reports, then College Going Reports.

2—Montana Department of Labor and Industry. (2015). *Labor Day report.* State of Montana. https://www.doleta.gov/performance/results/AnnualReports/docs/2017_State_Plans/Economic_Reports/Montana/MT%20PY%2015-Labor%20Day%20Report%202015-0815.pdf

This report provides an overview of the state of Montana's economy, outlining the statistics on a number of economic indicators. The report describes how the state is facing a worker shortage because of a large aging baby boomer population retiring without enough younger workers available to replace them, argues that labor force participation must increase considerably, and describe efforts currently underway to address Montana's expected worker shortage.

2—Montana Math Pathways Task Force (2015). *Report and recommendations of the Montana Math Pathways Task Force: October 2015 report.* https://mus.edu/board/meetings/2015/Nov2015/TwoYear/MathPathwaysReportforBOR_Nov2015.pdf

This report describes how the state's decision to join the Complete College America (CCA) Math Pathways Initiative and to create a faculty-led Math Pathways Task force comprising faculty across the Montana University System (MUS) to make recommendations to the commissioner and the Montana Board of Regents (BOR) regarding math pathways. Five recommendations from the Math Pathways Task Force summit are described in the report. These include: (a) provide a clear pathway for non-STEM students; (b) evaluate curricular requirements involving college algebra; (c) strengthen advising processes for math; (d) strengthen communication between secondary schools and college; and (e) strengthen communication through MUS system.

1—Montana Secretary of State (2013). *Rule 10.55.905: Graduation requirements.* https://rules.mt.gov/gateway/ruleno.asp?RN=10.55.905

The website shows state and district requirements that students must fulfill to receive a high school diploma. As a minimum, a school district's requirements for graduation shall include a total of 20 units of study that enable all students to meet the content standards and content-specific grade-level learning progressions. In order to meet the content and performance standards, the following 13 units shall be part of the 20 units required for all students to graduate:

(a) 4 units of English language arts;

(b) 2 units of mathematics;

(c) 2 units of social studies;

(d) 2 units of science;

(e) 1 unit of health enhancement, with 1/2 unit each year for two years;

(f) 1 unit of arts; and

(g) 1 unit of career and technical education.

1—Montt, G. (2011). Cross-national differences in educational achievement inequality. *Sociology of Education, 84*(1), 49–68. https://journals.sagepub.com/doi/abs/10.1177/0038040710392717?journalCode=soea

The author argues that school systems are called on not only to instruct and socialize students but also to differentiate among them. Although much research has investigated inequalities in educational outcomes associated with students' family background and other ascriptive traits, little research has examined cross-national differences in the total amount of differentiation that school systems produce—the total achievement inequality. This article evaluates whether two dimensions of educational systems—variations in opportunities to learn and intensity of schooling—are associated with achievement inequality independent of family background. It draws data from the Programme for International Student Assessment for more than 50 school systems and models the variance in achievement. Findings suggest that decreasing the variability in opportunities to learn—in the form of greater homogeneity in teacher quality and the absence of tracking—within the school system might reduce achievement inequality. More intense schooling is also related to lower achievement inequality to the extent that this intensity is homogeneously distributed within the school system, particularly in the form of a more highly qualified teacher workforce.

4—Mortensen, S. (2022, January). College or career: Work-based education provides both. *Vancouver Family Magazine*. https://vancouverfamilymagazine.com/college-or-career-work-based-education-provides-both/

This article describes findings from a recent survey looking at the personal financial and graduation plans for American high school students, showing that 25 percent of respondents were delaying college due to the pandemic. The onset of COVID-19 seemed to increase a trend of young people pushing back against the repeated narrative that college is the best path for a successful future. With frequent decrying of the student debt crisis, it is no wonder that the cost of college is a major concern for high schoolers. Are four more years and tens of thousands of dollars of debt worth the investment that college is supposed to be? The author describes how a financial algebra high school teacher helps students answer that question for themselves, individually. Despite being a math course, the intended outcome of financial algebra is for students to "become responsible and self-reliant consumers." The course dedicates an entire week to each of the main types of debt students will encounter in their future, including student loans, car loans, mortgages, and credit cards. This information provides graduating students with a better grasp of the true cost of college, including opportunity cost.

1—Mosqueda, E. (2010). Compounding inequalities: English proficiency and tracking and their relation to mathematics performance among Latina/o secondary school youth. *Journal of Urban Mathematics Education*, *3*(1), 57–81. https://jume-ojs-tamu.tdl.org/JUME/article/download/47/48

In this article, the author examines whether disparities in mathematics performance might be exacerbated by the track placement of native and non-native Latina/o English speakers in the Education Longitudinal Study of 2002. The effect of track placement on the mathematics performance of English learners (ELs) differed as a function of their level of English proficiency. The scores of Latinas/os with low levels of English proficiency in the general track were similar to the scores of students in the college track with comparable levels of English proficiency. The scores of non-native English speakers in the college track with high levels of English proficiency, however, were higher than those of their peers in the general track and nearly as high as those of native English speakers in the college track. Implications for the potential development of the mathematics language register of ELs are discussed.

5—Moussa, A., Barnett, E. A., Brathwaite, J., Fay, M. P., & Kopko, E. (2020). *A changing paradigm in high school mathematics.* Community College Research Center, Teachers College, Columbia University. https://files.eric.ed.gov/fulltext/ED609225.pdf

Educators and others have raised concerns about the extent to which the high school math sequence of Algebra I–Geometry–Algebra II, which prioritizes the mastery of algebra, is appropriate for the longer-term education and career goals of students who do not intend to pursue STEM degrees in college. These concerns have impelled educators and policymakers to reexamine the prominence of algebra in high school mathematics curricula and to consider new approaches that provide students with more mathematics course options better aligned with their academic and career goals. In this paper, the authors explore existing approaches to high school mathematics curricula as well as new developments in the field. To start, they discuss a range of high school mathematics course sequences that are currently offered across the country and look at some of the systemic challenges embedded within the traditional paradigm. Then they explore federal and state changes to the provision of high school mathematics in the early 21st century, which they follow with a look at the influence of postsecondary institutions on high school math curricula. They then introduce short case studies of innovative high school math reforms that are occurring in five states. They conclude the paper by considering the Charles A. Dana Center's new initiative, Launch Years, and how this project works to reimagine high school mathematics and its relationship to postsecondary education and careers.

1—Mullis, I. V. S., Martin, M. O., Goh, S., & Cotter, K. (Eds.). (2016). *TIMSS 2015 encyclopedia: Education policy and curriculum in mathematics and science.* Boston College, TIMSS & PIRLS International Study Center. http://timssandpirls.bc.edu/timss2015/encyclopedia/

The book is a compendium of how mathematics and science are taught around the world. Each Trends in International Mathematics and Science Study (TIMSS) 2015 country and benchmarking participant prepared a chapter summarizing key aspects of mathematics and science education, and completed the TIMSS 2015 Curriculum Questionnaire. The chapters describe the structure of each education system, the mathematics and science curricula in the primary and lower secondary grades, and overall policies related to mathematics and science instruction. Taken together, the data from the curriculum questionnaire and the information in the chapters present a portrait of mathematics and science education globally.

3—National Academies Press (2016). Barriers and opportunities for 2-year and 4-year STEM degrees: Systemic change to support students' diverse pathways. National Academies Press. https://nap.nationalacademies.org/catalog/21739/barriers-and-opportunities-for-2-year-and-4-year-stem-degrees

Nearly 40 percent of the students entering 2- and 4-year postsecondary institutions indicated their intention to major in science, technology, engineering, and mathematics (STEM) in 2012. But the barriers to students realizing their ambitions are reflected in the fact that about half of those with the intention to earn a STEM bachelor's degree and more than two-thirds intending to earn a STEM associate's degree fail to earn these degrees 4 to 6 years after their initial enrollment. Many of those who do obtain a degree take longer than the advertised length of the programs, thus raising the cost of their education. Are the STEM educational pathways any less efficient than for other fields of study? How might the losses be "stemmed" and greater efficiencies realized? These questions and others are at the heart of this study.

1—National Center for Education Statistics. (2013). *The nation's report card: Trends in academic progress 2012* (NCES 2013-456). Institute of Education Sciences, U.S. Department of Education.
https://nces.ed.gov/nationsreportcard/subject/publications/main2012/pdf/2013456.pdf

The report describes results from the 2012 National Assessment of Educational Progress (NAEP) mathematics and reading assessments. The report states, "Both 9- and 13-year-olds scored higher in reading and mathematics in 2012 than students their age in the early 1970s. Scores were 8 to 25 points higher in 2012 than in the first assessment year. Seventeen-year-olds, however, did not show similar gains. Average reading and mathematics scores in 2012 for 17-year-olds were not significantly different from scores in the first assessment year [1971]" (p. 2).

1, 3—National Center for Education Statistics. (2015). *The nation's report card: 2015 mathematics and reading at grades 4 and 8* (NCES 2015-136). Institute of Education Sciences, U.S. Department of Education.
https://www.nationsreportcard.gov/reading_math_2015/#mathematics?grade=4

The report describes results from the 2015 NAEP mathematics and reading assessments. The mathematics summary finds that 2015 scores at both grades were lower than in 2013 and

higher than in 1990. It also illustrates that scores in K–8 mathematics have risen somewhat over the past decades while those at the high school level have remained flat.

1—National Center for Education Statistics. (2017). *The condition of education: 2017.* Institute of Education Sciences, U.S. Department of Education. https://nces.ed.gov/pubs2017/2017094.pdf

The report describes results from the 2015 NAEP mathematics and reading assessments. The mathematics summary finds that 2015 mathematics scores at all grade levels were slightly lower than in 2013.

2,4—National Center on Education and the Economy. (2013). *What does it really mean to be college and work ready? The mathematics and English literacy required of first-year community college students.*

This report is the result of a two-year study, which examined the skills and knowledge in mathematics and English literacy that high school graduates need to succeed in the first year of their community college programs. Roughly 45 percent of our nation's undergraduates are attending community colleges, according to the American Association of Community Colleges (AACC). About half of those students are training to go directly into the workforce and enter popular fields such as nursing, law enforcement, auto mechanics, or education, while others are working to complete the first two years of a four-year degree program. The study found that students are failing to learn the basic math and English skills and concepts needed for success in community colleges, and students who cannot succeed in the first year of a community college program are surely not ready for success in college or the workplace. The report includes policy recommendations to enable more U.S. high school graduates to succeed in our community colleges.

4—National Center on Education and the Economy. (2018). *Top performing countries: Estonia*. https://ncee.org/country/estonia/

This report describes Estonia's performance on the 2018 Programme for International Student Assessment (PISA) assessment. Estonia emerged as a top performer on PISA 2012, ranking in the top tier in science and close to the top in reading and mathematics among all participating countries and regions. By 2018, Estonia had become a top performer globally in all three subjects and the highest performer in Europe. In addition, Estonian performance is relatively equitable with respect to socioeconomic background. Among OECD nations, Estonia has the highest percentage of resilient students, defined as those in the lowest quartile of socioeconomic status who perform in the highest quartile on PISA within their own country. The share of low-performing Estonian students in reading on PISA 2018 was less than half the OECD average.

4—National Council of Teachers of Mathematics. (2013a). *Matching the Common Core State Standards for Mathematics (CCSSM) and Canadian content expectations, content outcomes, and Essential Knowledges.* https://www.nctm.org/uploadedFiles/Standards_and_Positions/Common_Core_State_Standards/2_Chart_CCSSM_in_Ontario_Quebec_WNCP.pdf

This document provides a map matching the Common Core State Standards for Mathematics (CCSSM) and Canadian content expectations, content outcomes, and Essential Knowledge.

4—National Council of Teachers of Mathematics. (2013b). *Matching the Common Core State Standards for Mathematics (CCSSM) to Canadian mathematics curricula.* https://www.nctm.org/uploadedFiles/Standards_and_Positions/Common_Core_State_Standards/1_Matching_NCTM_%20Standards_and_CCSSM.pdf

This document provides an accompanying document to the resource above, focused on mathematical processes and competencies.

2, 3—National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.*

This book describes principles, actions, and specific teaching practices essential for a high-quality mathematics education for all students to be college- and career-ready. The book offers guidance to teachers, specialists, coaches, administrators, policymakers, and parents:

- It builds on the principles articulated in Principles and Standards for School Mathematics to present six updated Guiding Principles for School Mathematics.
- It supports the first Guiding Principle, teaching and learning, with eight essential, research-based Mathematics Teaching Practices.
- It details the five remaining principles—the essential elements that support teaching and learning as embodied in the Mathematics Teaching Practices.
- It identifies obstacles and unproductive and productive beliefs that all stakeholders must recognize, as well as the teacher and student actions that characterize effective teaching and learning aligned with the Mathematics Teaching Practices.

4—National Council of Teachers of Mathematics. (2015). *Computer science and mathematics education: A position of the National Council of Teachers of Mathematics.* https://www.nctm.org/Standards-and-Positions/Position-Statements/Computer-Science-and-Mathematics-Education

This position statement describes NCTM's position on substituting computer science for mathematics coursework. NCTM asserts that although knowledge of computer science is fundamental, a computer science course should be considered as a substitute for a mathematics course graduation requirement only if the substitution does not interfere with a student's ability to complete core readiness requirements in mathematics. For example, in states requiring four years of mathematics courses for high school graduation, such a substitution would be unlikely to adversely affect readiness. Further, courses designated as mathematics courses should include only those designed explicitly to teach mathematics, with clear mathematical learning goals guiding the content, and taught by professionals certified to teach mathematics, while courses addressing computer science content should be labeled and counted as computer science courses and should be taught by professionals certified to teach such content.

5—National Council of Teachers of Mathematics. (2016). *Providing opportunities for students with exceptional mathematical promise* (NCTM Position Statement). http://www.nctm.org/Standards-and-Positions/Position-Statements/Providing-Opportunities-for-Students-with-Exceptional-Promise/

This position statement describes the NCTM's position on providing opportunities for students with exceptional mathematical promise. The NCTM asserts that students with exceptional mathematical promise must be engaged in enriching learning opportunities during and outside the school day to allow them to pursue their interests, develop their talent, and maintain their passion for mathematics. Such opportunities must be open to a wide range of students who express a higher degree of interest in mathematics, not just to those who are identified through traditional assessment instruments.

1, 2, 3, 4, 5—National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*.

This book highlights the need for change in high school mathematics. The book aims to open up serious discussions among the key stakeholders in high school mathematics education to engage in resolving the vexing barriers that have long impeded meaningful and necessary change in high school mathematics education. Four key recommendations are offered, and Essential Concepts for high school mathematics are presented as the mathematics needed by each student. The key recommendations include: (a) Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics. (b) High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways. (c) Classroom instruction should be consistent with research-informed and equitable teaching practices. (d) High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts, to ensure the highest-quality mathematics education for all students.

4—National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* U.S. Department of Education. https://files.eric.ed.gov/fulltext/ED500486.pdf

During most of the 20th century, the United States possessed peerless mathematical prowess—not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. But without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century. This report is about actions that should be taken to strengthen the American people in this central area of learning.

1—National Research Council (2011a). *Successful K–12 STEM education: Identifying effective approaches in science, technology, engineering, and mathematics*. National Academies Press. https://nap.nationalacademies.org/catalog/13158/successful-k-12-stem-education-identifying-effective-approaches-in-science

In this article, the National Research Council argues that science, technology, engineering, and mathematics (STEM) are cultural achievements that reflect our humanity, power our economy, and constitute fundamental aspects of our lives as citizens, consumers, parents, and members of the workforce. Providing all students with access to quality education in the STEM disciplines is important to our nation's competitiveness. However, it is challenging to identify the most successful schools and approaches in the STEM disciplines because success is defined in many ways and can occur in many different types of schools and settings. In addition, it is difficult to determine whether the success of a school's students is caused by actions the school takes or simply related to the population of students in the school.

1—National Research Council (2011b). *Successful K–12 STEM education: A workshop summary*. National Academies Press. https://nap.nationalacademies.org/catalog/13230/successful-stem-education-a-workshop-summary

The National Research Council workshop summary argues that what students learn about the science disciplines, technology, engineering, and mathematics during their K–12 schooling shapes their intellectual development, opportunities for future study and work, and choices of career, as well as their capacity to make informed decisions about political and civic issues and about their own lives. Most people share the vision that a highly capable STEM workforce and a population that understands and supports the scientific enterprise are key to the future place of the United States in global economics and politics and to the well-being of the nation. Indeed, the solutions to some of the most daunting problems facing the nation will require not only the expertise of top STEM professionals but also the wisdom and understanding of its citizens.

1—National Research Council (2013a). *Monitoring progress toward successful K–12 STEM education: A nation advancing?* National Academies Press.

Following a 2011 report by the National Research Council (NRC) on successful K–12 education in the STEM disciplines, Congress asked the National Science Foundation to identify methods for tracking progress toward the report's recommendations. In response, the NRC convened the Committee on an Evaluation Framework for Successful K–12 STEM Education to take on this assignment. The committee developed 14 indicators linked to the 2011 report's recommendations. By providing a focused set of key indicators related to students' access to quality learning, educators' capacity, and policy and funding initiatives in STEM, the committee addresses the need for research and data that can be used to monitor progress in K–12 STEM education and make informed decisions about improving it.

The recommended indicators provide a framework for Congress and relevant deferral agencies to create and implement a national-level monitoring and reporting system that assesses progress toward key improvements recommended by a previous National Research Council (2011) committee; measures student knowledge, interest, and participation in the STEM disciplines and STEM-related activities; tracks financial, human capital, and material investments in K–12 STEM education at the federal, state, and local levels; provides information about the capabilities of the STEM education workforce, including teachers and principals; and facilitates strategic planning for federal investments in STEM education and workforce development when used with labor force projections. All 14 indicators explained in this report are intended to form the core of this system. *Monitoring Progress Toward Successful K–12 STEM Education: A Nation Advancing?* summarizes the 14 indicators and tracks progress toward the initial report's recommendations.

2—National Research Council. (2013b). *The mathematical sciences in 2025*. National Academies Press. https://nap.nationalacademies.org/catalog/15269/the-mathematical-sciences-in-2025

The mathematical sciences are part of nearly all aspects of everyday life—the discipline has underpinned such beneficial modern capabilities as Internet search, medical imaging, computer animation, numerical weather predictions, and all types of digital communications. *The Mathematical Sciences in 2025* examines the current state of the mathematical sciences and explores the changes needed for the discipline to be in a strong position and able to maximize its contribution to the nation in 2025. The report finds that the vitality of the discipline is excellent and that it contributes in expanding ways to most areas of science and engineering, as well as to the nation as a whole, and recommends that training for future generations of mathematical scientists should be reassessed in light of the increasingly cross-disciplinary nature of the mathematical sciences. In addition, because of the valuable interplay between ideas and people from all parts of the mathematical sciences, the report emphasizes that universities and the government need to continue to invest in the full spectrum of the mathematical sciences in order for the whole enterprise to continue to flourish long-term.

1—National Science Board. (2018). Elementary and secondary mathematics and science education: High school course-taking in mathematics and science. In *Science and Engineering Indicators 2018*. National Science Foundation. https://nsf.gov/statistics/2018/nsb20181/report/sections/elementary-and-secondary-mathematics-and-science-education/high-school-coursetaking-in-mathematics-and-science

This chapter examines high school students' participation in mathematics and science courses using data from HSLS:09, the College Board's AP program, and data collected by the Department of Education's Office for Civil Rights. HSLS:09 is a longitudinal study of a nationally representative sample of approximately 20,000 students who were first surveyed in fall 2009 as 9th graders and were surveyed again in 2012, when most were spring-term 11th graders. The HSLS:09 sample includes students from public and private schools, so it is representative of the overall in-school population. It does not include home-schooled students, who make up about 3 percent of the student population in the United States (Redford et al., 2017). Transcript data were collected for HSLS:09 students in summer 2013, when most would have completed high school (Dalton et al., 2016). Compared with students' self-reports of course-taking, transcript data provide a more accurate account of mathematics and science course-taking for all students in the study for whom transcripts were collected. Transcript data were used to examine the mathematics and science courses taken by students who had completed high school by summer 2013.

1—Ngo, F. J., & Velasquez, D. (2020). *Inside the math trap: Chronic math tracking from high school to community college*. https://journals.sagepub.com/doi/10.1177/0042085920908912

Examining linked academic transcripts from urban community colleges and their feeder high schools, the authors identify math course-taking patterns that span sectors. The authors

highlight stifled mobility and chronic repetition of math coursework in the transition to college, and we identify “math traps” from which students do not escape. Math mobility was limited, math repetition was rampant, and nearly half of students found themselves in math traps. All else being equal, being trapped in math was significantly linked to race/ethnicity, suggesting that these forms of chronic math tracking across sectors expose previously undocumented forms of inequality in educational experiences.

1—Niederle, M., & Vesterlund, L. (2010). Explaining the gender gap in math test scores: The role of competition. *Journal of Economic Perspectives*, *24*(2), 129–144. https://pubs.aeaweb.org/doi/pdf/10.1257%2Fjep.24.2.129

The mean and standard deviation in performance on math test scores are only slightly larger for males than for females. Despite minor differences in mean performance, many more boys than girls perform at the right tail of the distribution. This gender gap has been documented for a series of math tests including the AP calculus test, the mathematics SAT, and the quantitative portion of the Graduate Record Exam (GRE). The objective of this paper is not to discuss whether the mathematical skills of males and females differ, whether as a result of nurture or nature. Rather the authors argue that the reported test scores do not necessarily match the gender differences in math skills. They present results that suggest that the evidence of a large gender gap in mathematics performance at high percentiles in part may be explained by the differential manner in which men and women respond to competitive test-taking environments. The effects in mixed-sex settings range from women failing to perform well in competitions, to women shying away from environments in which they have to compete. The authors find that the response to competition differs for men and women, and in the examined environment, gender difference in competitive performance does not reflect the difference in noncompetitive performance. They argue that the competitive pressures associated with test taking may result in performances that do not reflect those of less competitive settings. Of particular concern is that the distortion is likely to vary by gender and that it may cause gender differences in performance to be particularly large in mathematics and for the right tail of the performance distribution. Thus the gender gap in math test scores may exaggerate the math advantage of males over females.

1—Nirode, W., & Boyd, B. (2023). The prevalence of teacher tracking in high school mathematics departments. *Journal for Research in Mathematics Education*, *54*(1), 7–23.

This study examined the prevalence of teacher tracking in a population of 1,822 mathematics teachers in 184 high schools in a single state. Results showed that 70 percent of teachers were tracked by course level, course track, or both. Three fourths of high schools tracked at least 58 percent of their mathematics teachers. The authors also found significant differences in teaching assignments across quintiles of years of experience at a teacher’s current school. First-quintile teachers were the most likely to be assigned low-track or entry-level courses. In contrast, fifth-quintile teachers were the most likely to be assigned high-track or upper-level courses. These findings indicate that the tracking of mathematics teachers is a prevalent and persistent inequitable structure in most high schools.

4—Noble, S. U. (2018). *Algorithms of oppression: How search engines reinforce racism*. NYU Press.

In this book, the author challenges the idea that search engines like Google offer an equal playing field for all forms of ideas, identities, and activities. They discuss issues of data discrimination as a real social problem. Through an analysis of textual and media searches as well as extensive research on paid online advertising, the author exposes a culture of racism and sexism in the way discoverability is created online.

4—Noble, T., Nemirovsky, R., Dimattia, C., & Wright, T. (2004). Learning to see: Making sense of the mathematics of change in middle school. *International Journal of Computers for Mathematical Learning, 9,* 109–167. https://www.researchgate.net/profile/Ricardo-Nemirovsky/publication/226178871_Learning_to_See_Making_Sense_of_the_Mathematics_of_Change_in_Middle_School/links/56e636eb08ae98445c2173fb/Learning-to-See-Making-Sense-of-the-Mathematics-of-Change-in-Middle-School.pdf

The authors describe the results of a study of 6th-grade students learning about the mathematics of change. The students in this study worked with software environments for the computer and the graphing calculator that included a simulation of a moving elevator, linked to a graph of its velocity versus time. Authors describe how the students and their teacher negotiated the mathematical meanings of these representations, in interaction with the software and other representational tools available in the classroom. The class developed ways of selectively attending to specific features of stacks of centimeter cubes, hand-drawn graphs, and graphs (labeled "velocity versus time") on the computer screen. In addition, the class became adept at imagining the motions that corresponded to various velocity-versus-time graphs. In this article, the authors describe this development as a process of learning to see mathematical representations of motion. The main question this article addresses is: How do students learn to see mathematical representations in ways that are consistent with the discipline of mathematics?

1—Oakes, J. (2005). *Keeping track: How schools structure inequality* (2nd ed.). Yale University Press.

The author briefly discusses the current practice of special education in the United States and presents definitions of divergence versus deficit to build the argument that focusing on inadequacy of student performance as opposed to variation in student performance can lead to inaccurate identification of appropriate strategies for students with exceptionalities. A definition of "deficit model" in conjunction with cultural and social implications with regard to students identified as having emotional and behavior disabilities will be discussed. Emotional intelligence and moral development are introduced as theoretical models that can inform selection and classroom use of preventative mental health curriculum and interventions designed to improve student capacity in emotion-based utility, perspective taking, and decision making.

1—Oakes, J., Ormseth, T., Bell, R., & Camp, P. (1990). *Multiplying inequalities: The effects of race, social class, and tracking on opportunities to learn mathematics and science.* RAND. https://files.eric.ed.gov/fulltext/ED329615.pdf

This study examines the way the nation's educational system distributes opportunities to learn mathematics and science among various groups of students. Participation and achievement in mathematics and science by women, minorities, and the poor is

disproportionately low. Minorities and the poor, especially in inner cities, have considerably fewer opportunities to learn science and math, largely because of the kinds of schools they attend. The section titles of this report are as follows: (a) "The Distribution of Opportunity"; (b) "The Effects of Student Characteristics on Opportunity"; (c) "Access to Science and Mathematics Programs"; (d) "Access to Qualified Science and Mathematics Teachers"; (e) "Access to Resources"; (f) "Classroom Opportunities: Curriculum Goals and Instruction"; and (g) "Implications." An appendix provides a classification of courses offered at the secondary schools included in the sample. A 133-item reference list is included.

1—O'Connor, C., Lewis, A., & Mueller, J. (2007). Researching "Black" educational experiences and outcomes: Theoretical and methodological considerations. *Educational Researcher, 36*(9), 541–552. https://www.researchgate.net/profile/Amanda-Lewis-11/publication/228343219_Researching_Black_Educational_Experiences_and_Outcomes_Theoretical_and_Methodological_Considerations/links/551d36da0cf2000f8f9388db/Researching-Black-Educational-Experiences-and-Outcomes-Theoretical-and-Methodological-Considerations.pdf

This article delineates how race has been undertheorized in research on the educational experiences and outcomes of Blacks. The authors identify two dominant traditions by which researchers have invoked race (i.e., as culture and as a variable) and outline their conceptual limitations. They analyze how these traditions mask the heterogeneity of the Black experience, underanalyze institutionalized productions of race and racial discrimination, and confound causes and effects in estimating when and how race is "significant." The authors acknowledge the contributions of more recent scholarship and discuss how future studies of Black achievement might develop more sophisticated conceptualizations of race to inform more rigorous methodological examinations of how, when, and why Black students perform in school as they do.

5—Ohio Department of Education. (2016). *Career-technical education pathway guidelines for Algebra 2/Mathematics III replacement*. https://education.ohio.gov/getattachment/Topics/Ohios-Learning-Standards/Mathematics/Math-Graduation-Requirements/Career-Technical-Pathway-Algebra-II-Replacement-Guidelines.pdf.aspx

This document describes the most recent guidelines around career and technical education (CTE) for Algebra II/Mathematics III replacement. New legislation allowed students who entered high school on or after July 1, 2015, and who are pursuing a CTE pathway to replace the Algebra II/Mathematics III requirement with a career-based mathematics course. All students are required to have access to a minimum of 10 CTE pathways.

5—Ohio Department of Education. (2019). *Mathematics graduation requirements*. https://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics/Resources-for-Mathematics/Math-Graduation-Requirements

This website describes Ohio's mathematics graduation requirements. Students in the class of 2014 and beyond need four credits of high school mathematics as one requirement to earn an Ohio high school diploma. The only math course required by state law is Algebra 2/Math 3 or its equivalent. The recommendation is that students complete Algebra 2/Math 3 by the end of their third year in high school. Algebra 2/Math 3 or its equivalent is the highest-level math course required for graduation. The other three math credits must be high school–level courses. Students may take any combination of high school and/or college-level mathematics courses to earn the four credits of high school mathematics as long as one of the credits is Algebra 2/Math 3 or its equivalent. For information on courses that are considered equivalent to Algebra 2/Math 3, see the High School Mathematics Pathways web page. The fourth math course can, but does not need to be, higher than Algebra 2/Math 3 or its equivalent. Suggested fourth math courses can be, but are not limited to Precalculus, Discrete Mathematics/Computer Science, Statistics and Probability, Data Science Foundations, Transitions to College Algebra, Computer Science, AP mathematics courses, Trigonometry, Advanced Quantitative Reasoning, College Credit Plus Transfer Module Mathematics (TMM) mathematics courses, and many CTE courses. All courses must be at least high school–level and be based on the content in Ohio's Learning Standards or higher.

1—Ohio Department of Education. (2022). *High school math pathways*. https://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics/Resources-for-Mathematics/Math-Pathways

This web page gives an overview of Ohio's current high school mathematics pathways, including a description of challenges with older pathways models, explanations for the changes that have been made recently, and answers to frequently asked questions.

2—Olsen, B., & Kirtman, L. (2002). Teacher as mediator of school reform: An examination of teacher practice in 36 California restructuring schools. *Teachers College Record*, *104*(2), 301-324. https://journals.sagepub.com/doi/abs/10.1177/016146810210400205?journalCode=tcza

The authors' analysis investigates variations among intended reforms as demonstrated by observed teacher practice in 36 California restructuring schools. They identify a series of individual and school-wide influences that shape any teacher's relationship to the particular reform(s), therefore leading each teacher to mediate the reform(s) in individual ways. This paper posits a theoretical model of the teacher-as-mediator process, which we use to shed analytic light on the "black box" of the teacher-as-mediator role in the reform process. The authors use data collected over three years in 36 schools to highlight a process whereby three concurrent strands of "mediating influences" (the formal implementation process, school-wide influences shaping climate, and individual influences on the teacher) interrelate to mold each teacher's disposition to implement the particular reform. This disposition, which they call "individual's mediating responses," determines the shape, color, and tenor of the reform as it unfolds through teacher practice in the classroom. This produces the variation between teachers in a given school, between departments, and between schools adopting similar reforms, and the discrepancy between intended reform consequences on the one hand and actual classroom practices on the other. The essay illuminates the mediation process by identifying and illustrating lines of influences on teachers enacting reform and by exploring how those influences interrelated in practice. The authors' conclusion offers a series of questions researchers and policymakers may wish to take up as they consider how to better align school-wide reform efforts with actual practices of classroom teachers.

4—O'Neil, C. (2016). *Weapons of math destruction: How big data increases inequality and threatens democracy*. Broadway Books.

In this book, the author discusses issues related to the use of algorithmic models in modern society that reinforce discrimination. The author exposes the "black box" models that shape our future, both as individuals and as a society. The author calls on modelers to take more responsibility for their algorithms and on policymakers to regulate their use. She also suggests that individuals must become more savvy about the models that govern their lives.

4—Oregon Department of Education (2020). *High school core math guidance—Publication version 4.1*. https://www.oregon.gov/ode/educator-resources/standards/mathematics/Documents/High%20School%20Core%20Mathematics%20Guidance.pdf

This report provides guidance to districts and schools for planning courses that give the opportunity for students to have access to the adopted high school standards by the end of a three-credit sequence or sooner.

4—Oregon Department of Education (n.d.). *Oregon Math Project: Meaningful math for every student.* https://www.oregon.gov/ode/educator-resources/standards/mathematics/Pages/Oregon-Math-Project.aspx

The website gives an overview of the Oregon Math Project (OMP), which works to advance mathematics education in the state by cultivating a network of educators who promote equitable math experiences for all students through guidance and support of policies, standards, curricula, assessments, and instructional best practices. The vision on math education in Oregon is to ensure that all students attain mathematics proficiency by having access to high-quality instruction that includes challenging and coherent content in a learning environment where each student receives the support they need to succeed in mathematics.

1—Organisation for Economic Co-operation and Development. (2016a). *Equations and inequalities: Making mathematics accessible to all.* http://dx.doi.org/10.1787/9789264258495-en.

This report, based on results from the OECD's Programme for International Student Assessment (PISA) 2012, argues that one way to improve quantitative reasoning is to ensure that all students spend more "engaged" time learning core mathematics concepts and solving challenging mathematics tasks. The opportunity to learn mathematics content—the time students spend learning mathematics topics and practicing math tasks at school—can accurately predict mathematics literacy. Differences in students' familiarity with mathematics concepts explain a substantial share of performance disparities in PISA between socioeconomically advantaged and disadvantaged students. Widening access to mathematics content can raise average levels of achievement and, at the same time, reduce inequalities in education and in society at large.

1, 4—Organisation for Economic Co-operation and Development. (2016b). *PISA 2015 results (vol. I): Excellence and equity in education.*

The OECD’s PISA examines not just what students know in science, reading, and mathematics, but what they can do with what they know. Results from PISA show educators and policymakers the quality and equity of learning outcomes achieved elsewhere and allow them to learn from the policies and practices applied in other countries. *PISA 2015 Results* (vol. I): *Excellence and Equity in Education*, is one of five volumes that present the results of the PISA 2015 survey, the sixth round of the triennial assessment. It summarizes student performance in science, reading, and mathematics, and defines and measures equity in education. It focuses on students’ attitudes toward learning science, including their expectations of working in science-related careers. The volume also discusses how performance and equity have evolved across PISA-participating countries and economies over recent years.

1—Organisation for Economic Co-operation and Development. (2018). *PISA 2018 results: United States country note.* https://www.oecd.org/pisa/publications/PISA2018_CN_USA.pdf

PISA is a triennial survey of 15-year-old students around the world that assesses the extent to which they have acquired the key knowledge and skills essential for full participation in society. The assessment focuses on the core school subjects of reading, mathematics, and science. Students’ proficiency in an innovative domain is also assessed; in 2018, this domain was global competence. This report focuses on key findings for students in the United States.

1—Organisation for Economic Co-operation and Development. (2018). *PISA 2018 results.* https://nces.ed.gov/surveys/pisa/pisa2018/pdf/PISA2018_compiled.pdf

PISA is a triennial survey of 15-year-old students around the world that assesses the extent to which they have acquired the key knowledge and skills essential for full participation in society. The assessment focuses on the core school subjects of reading, mathematics, and science. Students’ proficiency in an innovative domain is also assessed; in 2018, this domain was global competence.

1—Packer, A. (2003). Making mathematics meaningful. In *Quantitative literacy: Why numeracy matters for schools and colleges* (pp. 171–173). National Council on Education and the Disciplines. https://www.maa.org/sites/default/files/pdf/QL/WhyNumeracyMatters.pdf

Despite every person’s need for Quantitative Literacy (QL), in the discipline-dominated K–16 education system in the United States there is neither an academic home nor an administrative promoter for this crucial competency. Needs for QL extend across the traditional American guarantees of life, liberty, and the pursuit of happiness. Health concerns increasingly are immersed in risk analysis and probabilities; government decisions and political arguments are steeped in uses and misuses of quantitative data; and consumer issues, sports, and investments frequently are reported in terms of averages, rates of change, and changes in rates of change. To better understand quantitative literacy and the educational challenge it presents, the National Council on Education and the Disciplines (NCED) initiated a national examination of issues surrounding QL education, especially in the context of school and college studies. As a starting point, NCED published a case statement on numeracy in contemporary society and 12 responses. To expand the conversation about QL, NCED subsequently sponsored a national forum, “Quantitative Literacy: Why Numeracy Matters for Schools and Colleges,” held at the National Academy of Sciences in Washington, DC, on December1–2, 2001. This volume represents the proceedings of this forum and includes papers and selected reactions to the forum.

1—Palmer, R. T., Maramba, D. C., & Dancy, T. E., II. (2011). A qualitative investigation of factors promoting the retention and persistence of students of color in STEM. *The Journal of Negro Education*, *80*(4), 491–504. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=a14de1f625aec0801ee864fd5d131837d6ce159b

The literature on science, technology, engineering, and mathematics (STEM) is abounding with the importance of increasing college access, retention, and persistence among students because of implications for America's global competitiveness. Particular emphasis has been placed on college students of color who remain underrepresented in STEM education. Therefore, increasing college access, retention, and persistence for students of color in STEM is not merely a matter of U.S. economic competitiveness, but also a matter of equity. Using in-depth interview methods, this article delineates factors facilitating the retention and persistence of students of color in STEM education at a predominantly white institution. Implications for institutional practice and research are provided.

1, 4—Patchnowski, L., & Cannelongo, A. (2021). Advocacy corner: High school math pathways—Ohio's road to equity in mathematics. *Ohio Journal of School Mathematics 89*(1). https://library.osu.edu/ojs/index.php/OJSM/article/view/8489

In this report, the authors summarize changes underway in Ohio's high school mathematics curriculum that provide alternatives to high school Algebra II and beyond. The authors share historical background of the current secondary-level mathematics curriculum, the rationale for alternative pathways for Ohio's mathematics students, and the plan for implementing these pathways in Ohio schools.

1—Pennsylvania Department of Education (2018). *Statewide high school graduation requirement.* https://www.education.pa.gov/K-12/Assessment%20and%20Accountability/GraduationRequirements/Pages/default.aspx

The website describes Senate Bill 1095, signed into law by the state governor, to shift Pennsylvania's reliance on high-stakes testing as a graduation requirement to provide alternatives for high school students to demonstrate readiness for postsecondary success. The bill expanded the options for students to demonstrate postsecondary readiness using four additional pathways that more fully illustrate college, career, and community readiness. Historical context for the bill is provided, as well as a link to current graduation requirements.

4—Pheatt, L., Trimble, M.J., & Barnett, E.A. (2016). *Improving the transition to college: Estimating the impact of high school transition courses on short-term college outcomes* (CCRC Working Paper No. 86). Community College Research Center, Teachers College, Columbia University. https://files.eric.ed.gov/fulltext/ED622904.pdf

Many recent high school graduates remain inadequately prepared for college and are required to enroll in remedial or developmental education courses in mathematics or English upon enrollment in college. High rates of college remediation are associated with lower

progression and college completion rates. To address this problem, some states, districts, and individual high schools have introduced "transition courses" to prepare students for college-level math and English coursework. Transition courses are typically offered to high school seniors who have been assessed as being underprepared for college math or English. This study uses a regression discontinuity design to estimate the effect of participation in a mathematics transition course on college-level math outcomes in West Virginia for the 2011–12 and 2012–13 high school senior cohorts. The authors' findings suggest that, among students who scored very close to the cutoff score on an assessment used to decide what students took the course, the math transition course had no statistically significant effect on improving college readiness (as measured by exemption from remedial education upon college entry due to a passing score on a placement test) and in fact had a negative impact on students' likelihood of passing a college gatekeeper math course. Possible explanations for these outcomes include that (a) the transition course may have displaced traditional senior-year courses that were in practice more rigorous than the transition course or that provided positive impacts from inclusion of higher-performing peers, and that (b) the transition course curricula may not have been well aligned to the skills required for success on the COMPASS placement test. Most students who took the transition course did not pass the COMPASS, which was taken at the conclusion of the course. The specific math course studied is no longer offered; math transition courses in West Virginia now use a different curriculum.

1, 2—President's Council of Advisors on Science and Technology. (2012). *Reporting to the President—Engage to excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics.* https://files.eric.ed.gov/fulltext/ED541511.pdf

Economic projections point to a need for approximately one million more STEM professionals than the U.S. will produce at the current rate over the next decade if the country is to retain its historical preeminence in science and technology. To meet this goal, the United States will need to increase the number of students who receive undergraduate STEM degrees by about 34 percent annually over current rates. The title of this report, *Engage to Excel*, applies to students, faculty, and leaders in academia, industry, and government. Students must be engaged to excel in STEM fields. To excel as teachers, faculty must engage in methods of teaching grounded in research about why students excel and persist in college. Moreover, success depends on engagement by great leadership. Leaders—including the president of the United States; college, university, and business leadership; and others—must encourage and support the creation of well-aligned incentives for transforming and sustaining STEM learning. They also must encourage and support the establishment of broad-based reliable metrics to measure outcomes in an ongoing cycle of improvement. Transforming STEM education in U.S. colleges and universities is a daunting challenge. The key barriers involve faculty awareness and performance, reward and incentive systems, and traditions in higher education. The recommendations in this report address the most significant barriers and use both tangible resources and persuasion to inspire and catalyze change. Attacking the issue from numerous angles and with various tools is aimed at reaching a point at which the movement will take on a momentum of its own and produce sweeping change that is sustainable without further federal intervention. The President's Council of Advisors on Science and Technology (PCAST) proposes five overarching recommendations to transform undergraduate STEM education during the transition from high school to college and during

the first two years of undergraduate STEM education: (a) Catalyze widespread adoption of empirically validated teaching practices. (b) Advocate and provide support for replacing standard laboratory courses with discovery-based research courses. (c) Launch a national experiment in postsecondary mathematics education to address the math preparation gap. (d) Encourage partnerships among stakeholders to diversify pathways to STEM careers. (e) Create a Presidential Council on STEM Education with leadership from the academic and business communities to provide strategic leadership for transformative and sustainable change in STEM undergraduate education.

2—Ran, F. X., & Lin, Y. (2019). *The effects of corequisite remediation: Evidence from a statewide reform in Tennessee* (CCRC Working Paper No. 115). Community College Research Center, Teachers College, Columbia University. https://ccrc.tc.columbia.edu/publications/effects-corequisite-remediation-tennessee.html

Corequisite remediation, which mainstreams students deemed academically underprepared into college-level courses with additional learning support, is rapidly being adopted by colleges across the nation. This paper provides the first causal evidence on a system-wide corequisite reform, using data from all 13 community colleges affiliated with the Tennessee Board of Regents. Using regression discontinuity and difference-in-regression-discontinuity designs, the authors estimated the causal effects of placement into corequisite remediation compared with placement into traditional prerequisite remediation and direct placement into college-level courses. For students on the margin of the college readiness threshold, those placed into corequisite remediation were 15 percentage points more likely to pass gateway math and 13 percentage points more likely to pass gateway English within one year of enrollment than similar students placed into prerequisite remediation. Compared with their counterparts placed directly into college-level courses, students placed into corequisite remediation had similar gateway course completion rates and were about 8 percentage points more likely to enroll in and pass a subsequent college-level math course after completing gateway math. The positive effects of corequisite remediation compared with prerequisite remediation in math were largely driven by efforts to guide students to take math courses aligned with the requirements for their program rather than placing most students into the algebra–calculus track by default, as has been the standard practice. The authors found no significant impacts of placement into corequisite remediation on enrollment persistence, transfer to a four-year college, or degree completion. This suggests that corequisite reforms, though effective in helping students pass college-level math and English, are not sufficient to improve college completion rates overall.

1—Reis, S. M., & Renzulli, J. S. (2010). Is there still a need for gifted education? An examination of current research. *Learning and Individual Differences*, *20*(4), 308–317. https://www.researchgate.net/publication/223686993_Is_there_still_a_need_for_gifted_education_An_examination_of_current_research#fullTextFileContent

What recent research has been conducted about gifted and talented students and their learning experiences in school? As we complete the first decade of the new century, we are entering a time when much attention is focused on remediation and test preparation; it only seems appropriate to reflect upon what has been learned about gifted education during the last few decades and consider the compelling evidence that may or may not support special services for gifted and talented students. Consensus on which research themes and studies should be included in this type of examination would be difficult to reach, but the authors identify six important themes, which are discussed in the article. The authors suggest that the need for gifted education programs remains critical during the current time period in American education when our nation's creative productivity is being challenged by European and Asian nations.

1—Rennie Center for Education Research and Policy. (2009). *Policy perspectives: Raise the age, lower the dropout rate? Considerations for policymakers*. https://www.renniecenter.org/sites/default/files/2017-01/RaiseTheAge.pdf

This policy brief focuses on the question: Is there empirical evidence to support Massachusetts raising its compulsory school attendance age to 18? Through an examination of research and analysis of other states' policies, the Rennie Center examines the arguments for and against raising the compulsory age of school attendance to 18 and concludes that there is no credible empirical evidence to support this policy alone as an effective strategy to combat the dropout crisis. The Center argues that prior to considering a raise in the compulsory age of attendance, the Commonwealth should focus its energy and resources on developing policies and programs that research has shown to be successful in helping at-risk students stay in school and persist to earning a diploma.

1—Riegle-Crumb, C., & Grodsky, E. (2010). Racial-ethnic differences at the intersection of math course-taking and achievement. *Sociology of Education, 83*, 248–270. https://www.soc.umn.edu/assets/doc/crcgrodsky2010final.pdf

Despite increases in the representation of African American and Hispanic youth in advanced math courses in high school over the past two decades, recent national reports indicate that substantial inequality in achievement remains. These inequalities can temper one's optimism about the degree to which the United States has made real progress toward educational equity. Using data from the Education Longitudinal Study (ELS) of 2002, the authors find that the math achievement gap is most pronounced among those students who take the most demanding high school math classes, such as Precalculus and Calculus. The authors explore the roles of family socioeconomic status and school composition in explaining this pattern. Findings suggest that among those students reaching the advanced math high school stratum, Hispanic youth from low-income families and African American youth from segregated schools fare the worst in terms of closing the achievement gap with their white peers. The authors discuss potential explanations for the achievement differences observed and stress the need for more research that focuses explicitly on the factors that inhibit minority/majority parity at the top of the secondary curricular structure.

2—Rieley. M. (2018 June). Big data adds up to opportunities in math careers. *Beyond the Numbers: Employment & Unemployment, 7*(8). U.S. Bureau of Labor Statistics, U.S. Department of Labor. https://www.bls.gov/opub/btn/volume-7/big-data-adds-up.htm

The article describes occupations in the field of mathematical science, projected to grow 27.9 percent from 2016 to 2026, much faster than the average for all occupations, resulting in about 50,400 new jobs. The article takes a detailed look at four occupations and explores the main reason for this projected growth: business and government use of "big data," which individuals in mathematical science occupations are particularly trained to process and analyze. The occupations include actuaries, mathematicians, operations research analysts, and statisticians.

1—Rittle-Johnson, B., Farran, D. C., & Durkin, K. L. (2021). Marginalized students' perspectives on instructional strategies in middle-school mathematics classrooms. *The Journal of Experimental Education*, *89*(4), 569–586. https://www.researchgate.net/publication/335795428_Compare_and_Discuss_to_Promote_Deeper_Learning#fullTextFileContent

Marginalized students face a range of gaps in experience, highlighting the importance of understanding these students' perspectives on their opportunities to learn. The current study contributes to this effort by reporting on marginalized students' experiences and liking of mathematics instructional strategies in middle school mathematics classrooms in a large metropolitan school district in the southern United States. Middle school students ($N = 466$), many of whom attended racially segregated schools, sorted instructional strategies and discussed their experiences with the strategies in small groups or interviews. Most students reported that traditional and student-focused instructional strategies happened in their mathematics class, but fewer student-focused strategies were experienced in racially segregated schools than in racially balanced schools. Most students reported liking all but one of the student-focused strategies and not liking the traditional strategies. Common reasons that emerged during discussions of why students liked particular instructional strategies were that the strategies provided opportunities to learn, built the students' confidence, or increased their interest. Overall, marginalized students' experiences and views should inform efforts to increase the instructional opportunities for all students.

3—Rose, H., & Betts, J. R. (2004). The effect of high school courses on earnings. *Review of Economics and Statistics, 86*(2), 497–513. https://direct.mit.edu/rest/article-abstract/86/2/497/57469/The-Effect-of-High-School-Courses-on-Earnings?redirectedFrom=fulltext

The authors estimate the effect that six types of high school math courses have on students' earnings nearly a decade after graduation. They use High School and Beyond transcript data to differentiate courses at a more detailed level than in previous research. This enables them to show that more advanced courses have larger effects than less advanced ones. The authors also provide evidence that math courses can help close the earnings gap between students from low-income families and those from middle-income families. Finally, by incorporating other academic subjects, the authors demonstrate how specific course combinations can explain the earnings premium related to an additional year of school.

1—Rosenstein, J. G., & Ahluwalia, A. (2017). Putting the brakes on the rush to AP Calculus. In D. Bressoud (Ed.), *The Role of Calculus in the Transition from High School to College Mathematics* (pp. 27–40). MAA Press. https://www.maa.org/sites/default/files/RoleOfCalc_rev.pdf

The report summarized the discussion from a three-day meeting in March 2016 including high school teachers; mathematicians; mathematics and science education researchers; state and district supervisors of mathematics; and representatives of organizations with a stake in the issues, which included the College Board and the National Academy of Sciences. The meeting focused on clarifying what is known and what needs to be known about the role of calculus in the transition from high school to college mathematics.

1—Russell, C. (2019). Connecting mathematics with world heritage. *The Mathematics Teacher 112*(4), 274–279. https://pubs.nctm.org/view/journals/mt/112/4/article-p274.xml

Although social studies or language arts classes may seem to offer more natural settings for "global studies," the students in this article engage in projects demonstrating that mathematical topics can work well at furthering the goal of increasing students' cultural literacy. Mathematics learning experiences that tie together art, culture, technology, geography, and history provide a rich environment for students to push themselves beyond basic skills. The author describes one project here.

2—Saxe, K., & Braddy, L. (2015). *A Common Vision for undergraduate mathematical sciences programs in 2025.* Mathematical Association of America. https://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf

This report summarizes a two-and-a-half-day workshop held in May 2015. Participants represented all of the professional associations that have as one of their primary objectives the increase or diffusion of knowledge in one or more of the mathematical sciences, as well partner disciplines in science, technology, and engineering. Each of the five professional associations has published curricular guides, and some were written without intense collaboration with the other associations. The ultimate goal of Common Vision was to galvanize the mathematical sciences community around a modern vision for undergraduate programs and to spur grassroots efforts within the community as a foundation for addressing the collective challenges we face and for capitalizing on the opportunities outlined in two prior reports by the National Research Council and the President's Council of Advisors on Science and Technology.

1—Schmidt, W. H. (2009). *Exploring the relationship between content coverage and achievement: Unpacking the meaning of tracking in eighth grade mathematics.* Education Policy Center at Michigan State University. https://files.eric.ed.gov/fulltext/ED537158.pdf

Many studies analyzing the effect of tracking on achievement, however, have had a number of problematic limitations. In this paper, the authors address these study limitations by examining the effect of tracking on 8th-grade mathematics achievement and by defining track location in terms of the actual mathematics content that students covered in that track. The authors have previously documented the extent of the variation that passes for 8th-grade mathematics in terms of course titles, textbooks used, amount of instructional time devoted to specific topics, and the relative difficulty of courses from an international perspective. Here they report on a set of analyses designed explicitly to explore the effect of tracking on achievement. Using a unique nationally representative sample of 7th- and 8th-grade students that allows us to overcome, to some extent, the limitations due to within-school sampling (by providing a full characterization of tracking within the sampled schools), they explore the relationship of tracking in 8th grade to what mathematics topics are studied during 8th grade (content exposure) and to what is learned during the year as well as to what is achieved by the end of 8th grade.

1—Schmidt, W. H. (2012). Measuring content through textbooks: The cumulative effect of middle-school tracking. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to "lived" resources: Mathematics curriculum materials and teacher development*. Springer. https://link.springer.com/chapter/10.1007/978-94-007-1966-8_8

This chapter demonstrates a method to summarize instructional materials that can be used to distinguish different types of curriculum. Using a U.S. national longitudinal sample, indices derived from textbook content coverage were used in quantitative analyses to illustrate curricular differences. Textbook data collected from mathematics teachers of public school students from grades 7–12 were coded to capture content and performance expectations intended by the textbooks. Different approaches were examined to quantitatively summarize the data to characterize students' exposure to mathematics content. The results were tested against the expectation of the long-term effects of tracking in American middle schools. By grouping students by the type of courses they took in 7th and 8th grades, different approaches to defining the "amount" of mathematics exposure in the subsequent years were compared, each capturing different characteristics of the mathematics contents represented in the textbooks. The results showed that students who enrolled in Algebra prior to 9th grade were exposed to almost three times as much mathematics content as students who did not take algebra in secondary school. The gap in exposure persisted when contrasting these students with other students who had algebra in high school, though to a smaller extent. As it is broadly accepted that textbooks are a good reflection of the implemented curriculum in most countries and a particularly accurate reflection in the United States, the chapter concluded with a path-analytic model showing that exposure to demanding curricular coverage in mathematics strongly predicted mathematics achievement.

3—Schmidt, W., Houang, R., & Cogan, L. (2002, Summer). A coherent curriculum: The case of mathematics. *American Federation of Teachers*, 1-18. https://www.aft.org/sites/default/files/media/2014/curriculum.pdf

This article discusses findings from the first TIMSS study in 1999, with a particular focus on conceptual coherence of mathematics curricula in different countries. The U.S. mathematics curriculum is characterized as "a mile wide and an inch deep," with different branches of mathematics taught in such a way that highly related big ideas are siloed from one another in separate courses (for example, Algebra I and Geometry), whereas in countries rated more highly in the TIMSS study, these ideas are taught in a coherent, integrated fashion.

1—Schmidt, W. H., Cogan, L. S., & McKnight, C. C. (2010). Equality of educational opportunity: Myth or reality in U.S. schooling? *The Education Policy Center at Michigan State University*. http://files.eric.ed.gov/fulltext/EJ909927.pdf

In this article, the authors explore the extent to which students in different schools and districts have an equal opportunity to learn mathematics. Specifically, the authors discuss research on (a) the amount of variability in content coverage in 8th grade across 13 districts (or consortia of districts) and nine states, and (b) the variation in mathematics courses offered by high schools in 18 districts spread across two states. The authors explain why all the different mathematics content roads do not fairly and equally lead to the same high-quality educational outcomes.

4—Seago, N., Jacobs, J., Driscoll, M., Nikula, J., Matassa, M., & Callahan, P. (2013). Developing teachers' knowledge of a transformations-based approach to geometric similarity. *Mathematics Teacher Educator, 2*(1), 74–85. https://www.researchgate.net/publication/259749701_Developing_Teachers'_Knowledge_of_a_Transformations-Based_Approach_to_Geometric_Similarity#fullTextFileContent

U.S. students' poor performance in the domain of geometric transformations is well documented, as are their difficulties applying transformations to similarity tasks. At the same time, a transformations-based approach to similarity underlies the Common Core State Standards for middle and high school geometry. The authors argue that engaging teachers in this topic represents an urgent but largely unmet need. The article considers what a transformations-based approach to similarity looks like by contrasting it with a traditional, static approach and by providing classroom examples of students using these different methods. In addition, the authors highlight existing professional development opportunities for teachers in this area.

3—Senge, P. M. (2006). *The fifth discipline: The art and practice of the learning organization.* Broadway Business.

Senge discusses the need for organizational learning as a way toward sustainable competitive advantage. The book includes leadership stories intended to demonstrate the core ideas of organizational learning and their integration into how people see the world and manage their organizations. The author describes how companies can rid themselves of the learning blocks that threaten their productivity and success by adopting the strategies of learning organizations.

3—Senge, P. M., Cambron-McCabe, N., Lucas, T., Smith, B., & Dutton, J. (2012). *Schools that learn (updated and revised): A fifth discipline fieldbook for educators, parents, and everyone who cares about education*. Currency.

A new edition of the groundbreaking book that brings the principles of organizational learning to today s schools and classrooms. A unique collaboration between the celebrated management thinker and *Fifth Discipline* author Peter Senge and a team of renowned educators and organizational change leaders, the revised edition of *Schools That Learn* addresses the new and unique pressures on our educational system that have emerged since the book's initial publication in 2000. The new revised and updated edition offers practical advice for overcoming the many challenges that face our communities and educational systems today. It shows teachers, administrators, students, parents and community members how to successfully use principles of organizational learning, including systems thinking and shared vision, to address the challenges that face our nation's schools. In a fast-changing world where school populations are increasingly diverse, children live in ever-more-complex social and media environments, standardized tests are applied as overly simplistic "quick fixes," and advances in science and technology continue to accelerate, the pressures on our educational system are inescapable. *Schools That Learn* offers a much-needed way to open dialogue about these problems and provides pragmatic opportunities to transform school systems into learning organizations.

4—Shaghaghian, Z., Yan, W., & Song, D. (2021). *Towards learning geometric transformations through play: An AR-powered approach.* https://arxiv.org/pdf/2106.03988

Despite the excessive developments of architectural parametric platforms, parametric design is often interpreted as an architectural style rather than a computational method. Also, the problem is still a lack of knowledge and skill about the technical application of parametric design in architectural modeling. Students often dive into utilizing complex digital modeling without having a competent pedagogical context to learn algorithmic thinking and the corresponding logic behind digital and parametric modeling. The insufficient skills and superficial knowledge often result in utilizing the modeling software through trial and error, not taking full advantage of what it has to offer. Geometric transformations as the fundamental functions of parametric modeling are explored in this study to anchor learning essential components in parametric modeling. Students need to understand the differences between variables, parameters, functions, and their relations. Fologram, an augmented reality tool, is utilized in this study to learn geometric transformation and its components in an intuitive way. A LEGO set is used as an editable physical model to improve spatial skill through hand movement beside instant feedback in the physical environment.

2, 4—Shaughnessy, J.M. (2011). *NCTM Summing Up: An opportune time to consider integrated mathematics*. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/J_-Michael-Shaughnessy/An-Opportune-Time-to-Consider-Integrated-Mathematics/

Then-NCTM President Shaughnessy discusses his belief that the “layer cake” approach to high school mathematics that dominates so many secondary school mathematics programs—built on course sequences such as algebra I, geometry, algebra II, or algebra I, algebra II, geometry—is an outmoded approach in a 21st-century educational system, and what types of pathways might serve students more effectively.

1—Siegle, D., Gubbins, E. J., O’Rourke, P., Langley, S. D., Mun, R. U., Luria, S. R., Little, C. A., McCoach, D. B., & Plucker, J. A. (2016). Barriers to underserved students’ participation in gifted programs and possible solutions. *Journal for the Education of the Gifted, 39*, 103–131. https://files.eric.ed.gov/fulltext/ED566253.pdf

Gifted students’ learning gains result from complex, advanced, and meaningful content provided by a knowledgeable teacher through high-quality curriculum and instruction at an appropriate pace with scaffolding and feedback. These elements exert influence that increases with dosage and within structures that facilitate student engagement in rigorous experiences, including interactions with one another. Talent development is a two-part process. First, educators and parents must provide opportunities for talent to surface, and then they must recognize the talent and provide educational opportunities that engage the emerging talent and move it to exceptional levels. Unfortunately, a variety of barriers exist that limit underserved students’ participation in this process. The authors discuss these barriers within a proposed model of talent development.

4—Silva, E., & White, T. (2013). *Pathways to improvement: Using psychological strategies to help college students master developmental math.* Carnegie Foundation for the Advancement of

Teaching. https://carnegiemathpathways.org/wp-content/uploads/2021/03/pathways_to_improvement.pdf

This report describes features and the evolution of the Carnegie Math Pathways developmental mathematics program to better prepare math-taking community college students.

5—Silvernail, D. Batista, I., Sloan, J., Stump, E., Johnson, A. (2014). *Pathways to mathematics college readiness in Maine.* University of Southern Maine: Maine Education Policy Research Institute (MEPRI). https://digitalcommons.usm.maine.edu/cgi/viewcontent.cgi?article=1000&context=cepare_readiness

The goal of this study was to examine the pathways to being college-ready in mathematics. Students who enter high school already having demonstrated mathematics proficiency on a standardized test in the 8th grade have already taken a significant step toward being college-ready. The best scenario is to enter high school proficient in mathematics and having already completed Algebra I, then to complete at least Algebra II and Calculus before graduating from high school. Students completing this pathway are virtually guaranteed to be college-ready in mathematics. There also is an alternative path to being college-ready. Being proficient entering high school and then completing a course sequence that includes at least Algebra I, Algebra II, and Precalculus significantly increased students' chances of being college-ready in mathematics. Thus, it appears 8th-grade proficiency is key to becoming college-ready in mathematics. It affords opportunities for students to complete Algebra I before entering high school and then take higher-level mathematics courses in high school. Alternatively, even if students wait to take Algebra I in high school, if they are proficient and complete at least Precalculus, they have a high likelihood of being college-ready. The key is 8th grade mathematics proficiency. It opens the gate to a successful high school and college experience in mathematics. The typical sequence of courses completed by most high school students is Algebra I, Geometry, and Algebra II. The Common Core State Standards Initiative (2012) has endorsed this three-course sequence as preparing students for college. However, the evidence from this study does not support this endorsement. Completing Geometry does not substantially ensure college readiness, nor does completing Algebra II ensure college readiness. Students also need to successfully complete either a Precalculus or Calculus course in high school to be college-ready.

4—Slovin, H. (2000). Take time for action: Moving to proportional reasoning. *Mathematics Teaching in the Middle School, 6*(1), 58-60. https://pubs.nctm.org/view/journals/mtms/6/1/article-p58.xml

Ratio and proportion are essential topics in the middle-grades mathematics curriculum. The ability to reason proportionally is important in students' transition from elementary school mathematics to high school mathematics and science. The study of ratio and proportion lays a foundation for first-year Algebra, and subsequent mathematics and science study assumes that students can reason proportionally. The author argue that many textbooks, however, limit the problems involving proportions to those that emphasize procedural competence rather than conceptual understanding. These problems usually require students to use ratios and set

up equations. Students may solve such problems "correctly" using the traditional algorithm of cross-multiplying and dividing, but their solutions do not necessarily give us much information about their understanding of the multiplicative relationships involved in proportions.

2—Smith, W. M., & Funk, R. (2021). The Student Engagement in Mathematics through an Institutional Network for Active Learning (SEMINAL) Project: An overview. In In W. M. Smith, M. Voigt, A. Ström, D. C. Webb, & W. G. Margin (Eds.), *Transformational Change efforts: Student Engagement in Mathematics through an Institutional Network for Active Learning* (p. 138). American Mathematical Society & Conference Board of Mathematical Scientists. https://bookstore.ams.org/mbk-138/

The purpose of this handbook is to help launch institutional transformations in mathematics departments to improve student success. Authors report findings from the Student Engagement in Mathematics through an Institutional Network for Active Learning (SEMINAL) study. SEMINAL's purpose is to help change agents, those looking to (or currently attempting to) enact change within mathematics departments and beyond by trying to reform the instruction of their lower-division mathematics courses in order to promote high achievement for all students. SEMINAL specifically studies the change mechanisms that allow postsecondary institutions to incorporate and sustain active learning in Precalculus to Calculus 2 learning environments. Out of the approximately 2.5 million students enrolled in collegiate mathematics courses each year, over 90 percent are enrolled in Precalculus to Calculus 2 courses. Forty-four percent of mathematics departments think active learning mathematics strategies are important for Precalculus to Calculus 2 courses, but only 15 percent state that they are very successful at implementing these strategies. Therefore, insights into the following research question will help with institutional transformations: What conditions, strategies, interventions and actions at the departmental and classroom levels contribute to the initiation, implementation, and institutional sustainability of active learning in the undergraduate calculus sequence (Precalculus to Calculus 2) across varied institutions?

2—Society for Human Resource Management. (2019). *The global skills shortage: Bridging the talent gap with education, training, and sourcing*. https://www.shrm.org/hr-today/trends-and-forecasting/research-and-surveys/Documents/SHRM Skills Gap 2019.pdf

This report argues that the United States is facing a growing skills gap that threatens the nation's long-term economic prosperity. The workforce simply does not have enough workers and skilled candidates to fill an ever-increasing number of high skilled jobs. Seven million jobs were open in December 2018, but only 6.3 million unemployed people were looking for work. As the country nears full employment, businesses face an even greater talent shortage that will have a stifling impact on the economy and global innovation. Business and HR leaders view the skills shortage as a top concern that needs to be addressed. Among HR professionals, 75 percent of those having recruiting difficulty say there is a shortage of skills in candidates for job openings. Until now, little has been known about how businesses are addressing the skills gap or whether there are effective remedies. The authors' research shows that employers are making efforts. They are investing in employee training, hiring from more diverse pools of talent and collaborating with educational institutions. But they also say more progress is urgently needed. The skills gap presents HR professionals with an opportunity to better understand the needs of their organizations and strategize remedies. This will require increasing worker training and education, closely collaborating with educational institutions to improve graduate employability, and supplementing the existing workforce with foreign-born talent. The study described in this report begins the exploration of the skills gap. To supplement these preliminary findings, SHRM will be conducting robust additional studies in 2019 and beyond.

2—Society for Industrial and Applied Mathematics. (2019, February). *Guidelines for assessment and instruction in mathematical modeling education (GAIMME), second edition* (S. Garfunkel & M. Montgomery, Eds.). Consortium for Mathematics and Its Applications & Society for Industrial and Applied Mathematics. https://www.siam.org/Publications/Reports/Detail/guidelines-for-assessment-and-instruction-in-mathematical-modeling-education

This report enables the modeling process to be understood as part of STEM studies and research, and taught as a basic tool for problem solving and logical thinking. GAIMME helps define core competencies to include student experiences and provides direction to enhance mathematical modeling education at all levels. The second edition includes changes primarily to the "Early and Middle Grades (K–8)" chapter.

1—Soni, A., & Kumari, S. (2015). The role of parental math attitude. *International Journal of Applied Sociology, 5*(4), 159–163. http://article.sapub.org/10.5923.j.ijas.20150504.01.html

Mathematics as a subject is imperative for excelling in any field of study, and it acts as glue that connects various disciplines together. The crucial variable determining achievement in mathematics is the attitude toward mathematics. The purpose of the study was to investigate whether children's math attitude serves as an underlying pathway between parental math attitude and their children's math achievement. A total of 482 students (251 females and 231 males) ranging from 10 to 15 years in age (5th to 10th grade) and one of their parents (mother/father) participated in the study. Parents' and their wards' math attitudes were measured, and a mathematics achievement test was constructed for each grade level based on the current curriculum. It was found that the father's math attitude contributes positively to his son's math attitude and math achievement as compared to the mother's math attitude. In contrast, the mother's math attitude positively influences her daughter's math attitude and math achievement in comparison with the father's math attitude.

3,4—Southern Regional Education Board. (2016). *Math Ready.* https://www.sreb.org/math-ready

This website describes the Math Ready course, which emphasizes understanding of mathematics concepts rather than just memorizing procedures. Math Ready students learn the context behind procedures and understand why to use a certain formula or method to solve a problem. By engaging students in real-world applications, Math Ready develops critical thinking skills that students will use in college and their careers. Math Ready is not designed to prepare students for advanced mathematics in STEM majors.

4—Southern Regional Education Board. (2019). *Math Ready: Three years of evidence*. Authors. https://www.sreb.org/sites/main/files/file-attachments/19v07w_math_ready_final_report_1.pdf?1553099090

This study examines three years of data collected from 35 schools in five states that implemented Math Ready. Two of these states implemented the course in 2015–16, and three states implemented the course in 2016–17. The study examines data collected for 1,080 Math Ready students in the five states, 366 of whom retook the ACT and 127 of whom retook the SAT after taking Math Ready. In addition to reporting Math Ready students' performance on the ACT and SAT, schools also reported Math Ready course performance data, high school graduation data, and college entrance data for all students enrolled in Math Ready. This study examines the impact of Math Ready on the following student outcomes: (a) Math Ready course performance; (b) performance on the ACT exam before and after taking the course; (c) Math Ready students' performance compared with a comparison group that did not take math during their senior year; (d) Math Ready students' ACT performance in schools that implemented the course for three years; (e) achievement gaps on the ACT exam by gender, race, and income; (f) performance on the SAT exam before and after taking the course; (g) high school graduation rates; (h) college entrance rates; and (i) results from Math Ready student and teacher surveys administered nationwide at participating schools.

4—Sowder, J., Armstrong, B., Lamon, S., Simon, M., Sowder, L., & Thompson, A. (1998). Educating teachers to teach multiplicative structures in the middle grades. *Journal of Mathematics Teacher Education, 1*(2), 127–155. https://par.nsf.gov/servlets/purl/10316737

This paper identifies the types of reasoning involved in middle grade mathematics with regard to multiplicative structures and the difficulties students have with the concepts, why these difficulties might occur, and the interconnections within this content area. The author offers four recommendations for the preparation and professional development of teachers.

2—State of Montana. (2013). *Bullock announces Complete College Montana initiative.*

This press release describes the Complete College Montana initiative, which aims to increase the number Montanans who have completed their college degrees and certificates. Several key components of this initiative include creating a postsecondary education funding system focused on results, changing developmental education for students who enter postsecondary institutions academically underprepared, increasing access to dual-credit opportunities, simplifying the maze of academic courses and student requirements by creating easy-to-follow academic maps, and improving data collection to ensure Montana is achieving its goals. The press release also discusses the role of K–12 schools to prepare students. Montana joins 33 other states in the Complete College America effort. This partnership will provide Montana with technical assistance, best practices, and other guidance for implementing policies that will help to increase the number of Montanans with completed college degrees and certificates.

1—Steele, C. M., & Aronson, J. (1995). Stereotype threat and the intellectual test performance of African Americans. *Journal of Personality and Social Psychology*, *69*(5), 797. https://pubmed.ncbi.nlm.nih.gov/7473032/

Studies in this paper varied the stereotype vulnerability of Black participants taking a difficult verbal test by varying whether or not their performance was ostensibly diagnostic of ability, and thus, whether or not they were at risk of fulfilling the racial stereotype about their intellectual ability. Reflecting the pressure of this vulnerability, Blacks underperformed in relation to whites in the ability-diagnostic condition but not in the nondiagnostic condition. Study 3 validated that ability-diagnosticity cognitively activated the racial stereotype in these participants and motivated them not to conform to it or to be judged by it. Study 4 showed that mere salience of the stereotype could impair Blacks' performance even when the test was not ability-diagnostic. The role of stereotype vulnerability in the standardized test performance of ability-stigmatized groups is discussed.

4—Steketee, S., & Scher, D. (2016). Connecting functions in geometry and algebra. *The Mathematics Teacher, 109*(6), 448–455. https://www.jstor.org/stable/10.5951/mathteacher.109.6.0448

This article focuses on the critical importance of geometric transformations and the key connections between geometric transformations and functions, as well as on supporting students to understand the concept of a function in ways that go beyond mapping one set of numbers onto another. The authors describe a way of helping students forge strong connections between geometric and algebraic functions, a connection that can deepen students' concept of function and develop their appreciation for the interconnectedness of geometry and algebra. The activities described (and additional geometric functions activities) are freely available in classroom-ready form, including student worksheets and teacher notes. Each activity is supported by an interactive web page (http://geometricfunctions.org/links/connecting-functions), powered by The Geometer's Sketchpad®, which runs in any modern web browser.

1—Stiff, L. V., & Johnson, J. L. (2011). Mathematical reasoning and sense making begins with the opportunity to learn. In M. E. Strutchens & J. R. Quander (Eds.), *Focus in high school mathematics: Fostering reasoning and sense making for all students*. NCTM. https://www.nctm.org/Store/Products/Focus-in-High-School-Mathematics--Fostering-Reasoning-and-Sense-Making-for-All-Students-(Download)/

In this chapter, authors discuss how access is a necessary condition for mathematical reasoning and sense making. They emphasis how none of the benefits of NCTM's *Principles and Standards* matters if students are denied access to the curricula and instruction created to achieve the vision of the standards. Moreover, if assessments are not aligned with standards-based curricula and instruction, then a coherent and cohesive mathematics program that promotes mathematical reasoning and sense making will probably not be achieved. The authors discuss challenges to equitable access to rigorous mathematics and recommendations for addressing such challenges.

1—Stiff, L. V., Johnson, J. L., & Akos, P. (2011). Examining what we know for sure: Tracking in middle grades mathematics. In W. Tate, K. King, C. Rousseau Anderson (Eds.), *Disrupting tradition: Research and practice pathways in mathematics education* (pp. 63–76). https://www.nctm.org/Handlers/AttachmentHandler.ashx?attachmentID=GVFqvrpylSE%3D

This chapter chronicles one large school district's challenges and successes in connecting research, practice, and policy to meet the mathematics education needs of its students. Lessons learned—or failed—should help other school districts better understand what they know and don't know about access to high-quality mathematics.

1, 4—Stigler, S. (1990). *The history of statistics: The measurement of uncertainty before 1900*. Belknap Press of Harvard University Press. http://www.med.mcgill.ca/epidemiology/hanley/bios601/GaussianModel/CombiningObservationsStiglerCh1.pdf

This book is the first comprehensive history of statistics from its beginnings around 1700 to its emergence as a distinct and mature discipline around 1900. The author discusses how statistics arose from the interplay of mathematical concepts and the needs of several applied sciences, including astronomy, geodesy, experimental psychology, genetics, and sociology. He addresses many intriguing questions, such as these: How did scientists learn to combine measurements made under different conditions? And how were they led to use probability theory to measure the accuracy of the result? Why were statistical methods used successfully in astronomy long before they began to play a significant role in the social sciences? How could the introduction of least squares predate the discovery of regression by more than 80 years? Stigler's emphasis is upon how, when, and where the methods of probability theory were developed for measuring uncertainty in experimental and observational science, for reducing uncertainty, and as a conceptual framework for quantitative studies in the social sciences. He describes the scientific context in which the different methods evolved and identifies the problems (conceptual or mathematical) that retarded the growth of mathematical statistics and the conceptual developments that permitted major breakthroughs.

2—Stigler, J. W., & Heibert, J. (2004). Improving mathematics teaching. *Educational leadership, 61*(5), 12–17. https://www.researchgate.net/profile/James-Stigler/publication/228731157_Improving_mathematics_teaching/links/02e7e529e9b1081f6f000000/Improving-mathematics-teaching.pdf

The authors discuss the Trends in International Mathematics and Science Study (TIMSS), in which typical teaching practices in various countries are documented via video and studied in order to better understand how teachers around the world teach mathematics and science and what the implications of different instructional practices might be for students. Researchers concluded that teaching is a cultural activity: learned implicitly, hard to see from within the culture, and hard to change. They were struck by the homogeneity of teaching methods observed within each country and by the striking differences in methods they observed across Germany, Japan, and the United States. Even in the United States, a country with great diversity in language, ethnicity, and economic conditions and an education system controlled by local governing boards, the nationwide variation in 8th grade mathematics teaching was much smaller than we had expected. Overwhelmingly, in most countries (including the United States), mathematics teachers emphasized rote procedures, with little attention given to making connections between concepts or between concepts and procedures.

1—Stinson, D. W., & Wager, A. (2012). A sojourn into the empowering uncertainties of teaching and learning mathematics for social change. *Teaching mathematics for social justice:*

Conversations with educators, *1*(1), 3–18. https://www.academia.edu/download/63403548/Stinson___Wager_TMFSJ_BC_2012.pdf

In this introductory chapter, the authors give an overview of the topic of social justice in mathematics education and why it matters. In particular they discuss the development of critical pedagogy and how the NCTM standards connect to social justice, as well as giving a general introduction to the book.

1—Stone, C. B., & Turba, R. (1999). School counselors using technology for advocacy. *Journal of Technology in Counseling, 1*(1). https://eric.ed.gov/?id=EJ657105

The authors argue that the use of computer technology in the school counselor's advocacy role is a powerful yet underutilized skill. Counselors who understand how to access student data and advocate with facts in hand are equipped to more fully participate in social advocacy as they look to eradicate institutional and environmental barriers that impede students' academic success.

4—Strother, S., & Klipple, K. (2019). *Corequisite remediation in mathematics: A review of first-year implementation and outcomes of Quantway and Statway.* WestEd. https://www.wested.org/wp-content/uploads/2019/10/Corequisite-Remediation-in-Mathematics.pdf

Carnegie Math Pathways (CMP) corequisite offerings were piloted at two institutions in spring 2018 and were made fully available in fall 2018, for the 2018–19 school year. Counting the institutions that piloted the courses and those that offered the courses starting in the fall, six institutions across the nation implemented one of the new offerings during this extended period. Between them, the two offerings were delivered by 15 faculty across 21 sections, reaching 410 students. Of the 410 students, 65.1 percent earned a grade of C or better, thereby gaining college-level math credit in a single term. This brief examines the experiences of faculty and students across the six institutions; explores the elements of successful implementation, such as number of contact hours, use of cohort models, and faculty preparedness; and discusses implementation challenges, including managing class time and supporting students with varied levels of reading and of foundational math knowledge. Finally, the brief provides suggestions for improvement in the ongoing enhancement of these offerings, such as streamlining materials and building mechanisms for faculty support, and identifies how CMP plans to respond to those suggestions.

1—Strutchens, M. E., Quander, J. R., & Gutiérrez, R. (2011). Mathematics learning communities that foster reasoning and sense-making for all high school students. In M. E. Strutchens & J. R. Quander (Eds.), *Focus in high school mathematics: Fostering reasoning and sense making for all students* (pp. 101–113). NCTM.

Authors discuss the many factors that contribute to the opportunity gap between white and nonwhite students, with particular attention to the systemic inequities created by the structures that characterize many high schools. They provide department- and school-level recommendations for fostering mathematical success for all students, thereby decreasing the opportunity gap.

4—Stump, S. (1999). Secondary mathematics teachers' knowledge of slope. *Mathematics Education Research Journal, 11*(2), 124–144. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=89cf64bc137f6ffdc0537070b89bc96c53c419ef

This study, conducted in the United States, investigated secondary mathematics teachers' concept definitions, mathematical understanding, and pedagogical content knowledge of slope. Surveys were collected from 18 preservice and 21 in-service teachers; 8 teachers from each group were also interviewed. Geometric ratios dominated teachers' concept definitions of slope. Problems involving the recognition of parameters, the interpretation of graphs, and rate of change challenged teachers' thinking. Teachers' descriptions of classroom instruction included physical situations more often than functional situations. Results suggest that mathematics teacher education programs need to specifically address slope as a fundamental concept, emphasizing its connection to the concept of function.

1—Su, F. E. (2017). Mathematics for human flourishing. *The American Mathematical Monthly*, *124*(6), 483–493. https://www.semanticscholar.org/paper/Mathematics-for-Human-Flourishing-Su/534718b0c7d318d1e07a7bb37b772c956eded8f6

This essay asks readers to reflect on the question: Why do mathematics? How we answer affects who we think should be doing mathematics and how we will teach it. Su draws on his own experiences, those of his students, and the writings of Simone Weil to illustrate how the pursuit of mathematics can meet basic human desires for play, beauty, truth, justice, and love. He shows us how the fundamentally human drives that motivate us to do mathematics can be channeled to build a world in which all can truly flourish.

4—Tanton, J., Coe, T., Ström, A., and Pearce, K. (2020). *Proportional relationships decluttered—at last!* [White paper]. NWEA. https://www.nwea.org/research/publication/proportional-relationships-decluttered-at-last/

The beauty of the mathematical idea of proportionality is too easily lost in a clutter of procedures and rules. As such, it may seem mysterious to students despite the best efforts of teachers. Yet, proportional reasoning is the capstone of arithmetic and the cornerstone of advanced mathematics (p. 94). Given this importance, it is essential to examine how we might build strong ways of thinking to help students better understand proportional relationships. In this article, the authors attempt to declutter proportionality. They want students to think about varying quantities and to investigate patterns so they can describe quantities in terms of proportional relationships. Students encounter proportional relationships in their everyday world, and the authors provide examples of where these relationships surface and how we can approach building strong ways of thinking.

4—Tarr, J. E., Grouws, D. A., Chávez, Ó., & Soria, V. M. (2013). The effects of content organization and curriculum implementation on students' mathematics learning in second-year high school courses. *Journal for Research in Mathematics Education*, *44*(4), 683–729. https://www.researchgate.net/publication/259749669_The_Effects_of_Content_Organization_and_Curriculum_Implementation_on_Students'_Mathematics_Learning_in_Second-Year_High_School_Courses#fullTextFileContent

The authors examined curricular effectiveness in high schools that offered parallel paths in which students were free to study mathematics using one of two content organizational structures, an integrated approach or a (traditional) subject-specific approach. The study involved 3,258 high school students, enrolled in either Year 2 Mathematics or Geometry, in 11 schools in five geographically dispersed states. Students in the integrated curriculum scored significantly higher than those in the subject-specific curriculum on the standardized achievement test. Significant student-level predictors included prior achievement, gender, and ethnicity.

1—Tate, M. L. (2013). *Reading and language arts worksheets don't grow dendrites: 20 literacy strategies that engage the brain*. Corwin Press. https://us.corwin.com/books

This book contains instructional activities and brain-compatible literacy. Newly consistent with Common Core State Standards, this resource offers hands-on techniques to help teach reading in relevant, motivating, and engaging ways. A range of literacy activities are described.

5—Texas Education Agency. (2016). House Bill 5: Foundation High School Program. https://tea.texas.gov/academics/graduation-information/house-bill-5-foundation-high-school-program

In 2013, the 83rd Texas Legislature established the new Foundation High School Program as the default graduation program for all students entering high school beginning in 2014–15. The State Board of Education adopted rules related to the new Foundation High School Program in January 2014. House Bill (HB) 5 required the commissioner of education to adopt a transition plan to implement HB 5 and replace the Minimum High School Program (MHSP), Recommended High School Program (RHSP), and Distinguished Achievement Program (DAP) with the Foundation High School Program beginning with the 2014–15 school year. These rules allow students who entered high school before the 2014–15 school year the option to graduate under the new Foundation High School Program. The Texas Education Agency, in collaboration with the Texas Higher Education Coordinating Board and the Texas Workforce Commission, developed the Graduation Toolkit to provide essential information to students, parents, and counselors about the new Foundation High School Program. The Graduation Toolkit is available for download.

4, 5—Texas Legislature. (2013). Acts 2013, Tex. Leg. 83rd R. S., Ch. 211, General and Special Laws (H.B. 5). https://statutes.capitol.texas.gov/Docs/ED/htm/ED.28.htm

The law states:

Sec. 28.014. COLLEGE PREPARATORY COURSES.

(a) Each school district shall partner with at least one institution of higher education to develop and provide courses in college preparatory mathematics and English language arts. The courses must be designed:

(1) for students at the 12th grade level whose performance on:

(A) an end-of-course assessment instrument required under Section 39.023(c) does not meet college readiness standards; or

(B) coursework, a college entrance examination, or an assessment instrument designated under Section 51.334 indicates that the student is not ready to perform entry-level college coursework; and

(2) to prepare students for success in entry-level college courses.

(b) A course developed under this section must be provided:

(1) on the campus of the high school offering the course; or

(2) through distance learning or as an online course provided through an institution of higher education with which the school district partners as provided by Subsection (a).

(c) Appropriate faculty of each high school offering courses under this section and appropriate faculty of each institution of higher education with which the school district partners shall meet regularly as necessary to ensure that each course is aligned with college readiness expectations. The commissioner of education, in coordination with the commissioner of higher education, may adopt rules to administer this subsection.

(d) Each school district shall provide a notice to each district student to whom Subsection (a) applies and the student's parent or guardian regarding the benefits of enrolling in a course under this section.

(e) A student who successfully completes an English language arts course developed under this section may use the credit earned in the course toward satisfying the advanced English language arts curriculum requirement for the foundation high school program under Section 28.025(b-1)(1). A student who successfully completes a mathematics course developed under this section may use the credit earned in the course toward satisfying an advanced mathematics curriculum requirement under Section 28.025 after completion of the mathematics curriculum requirements for the foundation high school program under Section 28.025(b-1)(2).

(f) A course provided under this section may be offered for dual credit at the discretion of the institution of higher education with which a school district partners under this section.

(g) Each school district, in consultation with each institution of higher education with which the district partners, shall develop or purchase instructional materials for a course developed under this section consistent with Chapter 31. The instructional materials must include technology resources that enhance the effectiveness of the course and draw on established best practices.

4—Teuscher, D., Tran, D., & Reys, B. J. (2015). Common Core State Standards in the middle grades: What's new in the geometry domain and how can teachers support student learning? *School Science and Mathematics, 115*(1), 4–13. https://www.researchgate.net/publication/270456020_Common_Core_State_Standards_in_the_Middle_Grades_What's_New_in_the_Geometry_Domain_and_How_Can_Teachers_Support_Student_Learning#fullTextFileContent

The Common Core State Standards for Mathematics (CCSSM) is a primary focus of attention for many stakeholders (e.g., teachers, district mathematics leaders, and curriculum developers) intent on improving mathematics education. This article reports on specific content shifts related to the geometry domain in the middle grades (6–8) mathematics

curriculum. The methodology employed allows for comparisons of content across multiple standards documents. The authors report on some dramatic changes with regard to the geometry content taught in the middle grades. They found 52 percent of the middle grades geometry CCSSM learning expectations will be new to the respective grade level at which they are taught in at least six of the eight states analyzed in this study (57 percent in grade 6, 50 percent in grade 7, and 50 percent in grade 8). The authors also highlight three areas that represent "new" geometry content in the middle grades based on their analysis of CCSSM and pre-CCSSM state standards.

4—Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel and J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 179–234). SUNY Press. http://pat-thompson.net/PDFversions/1994ConceptSpeedRate.pdf

One crucial part of sound mathematical development is students' construction of powerful and generative concepts of rate. What that means is a source of controversy. The author discusses controversies surrounding the concept of rate as well as a teaching experiment designed to investigate individual children's construction of the concept of speed as a rate and to investigate the components of a conception of average rate.

1—Thompson, C. L., & O'Quinn, S. D. (2001). *Eliminating the Black-White achievement gap: A summary of research*. North Carolina Education Research Council. https://files.eric.ed.gov/fulltext/ED457250.pdf

The gap between the academic achievement of minority group and white majority group students is at the forefront of educators' and policymakers' agenda in North Carolina and nationwide. The North Carolina Education Cabinet has set the target of eliminating the gap by the year 2010. This report reviews the evidence for each of several steps intended to help reduce the gap. These include extending high-quality, academically focused early childhood education to all children at risk of school failure; ensuring that African American children are taught by able, well-prepared, experienced teachers; reducing class size in the early grades; adopting sound and equitable grouping practices in elementary schools; ensuring that African American students are equitably represented across curriculum tracks in high schools; bridging home and school cultures by adapting teaching and discipline practices to suit students' backgrounds; demanding success by holding both schools and students accountable; supporting students with individual tutoring, more comprehensive reforms, summer programs, and follow-up assistance; and desegregating schools and programs within schools. Appended are the 16 comprehensive school reform models with the soundest basis of research and strongest evidence of effectiveness.

4—Tierney, C., & Monk, S. (2007). Children's reasoning about change over time. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 185–200). Lawrence Erlbaum Associates. https://doi.org/10.4324/9781315097435-9; https://www.taylorfrancis.com/chapters/edit/10.4324/9781315097435-9/children-reasoning-change-time-cornelia-tierney-stephen-monk

This chapter describes episodes from three classroom conversations and one individual interview in which children ages 8–10 interpret and create representations to tell a story about change. The chapter elaborates on the varied means children have of carrying out the given problems and the particular thinking processes the authors believe are indicated by their solutions. By providing these descriptions and discussions, the authors hope to support the view that algebra can be construed in ways that not only are possible for children to learn, but can be experienced as exciting and challenging during the learning process.

1—Thompson, K. D. (2017). What blocks the gate? Exploring current and former English learners' math course-taking in secondary school. *American Educational Research Journal*, *54*(4), 757–798. https://journals.sagepub.com/doi/pdf/10.3102/0002831217706687

This mixed-methods study couples large-scale analyses of student course-taking with case study data to explore what blocks the gate to enrollment in and successful completion of secondary math courses for students ever classified as English learners (ever ELs). Initial quantitative findings indicate that half of all students across six California districts, including ever ELs, repeated a math course between 8th and 10th grades, with limited evidence of additional learning during students' second time in the course. Ever EL case study findings indicate that interactions between institutional (course placement policies), classroom (ways of knowing), and individual (student motivation) factors shaped students' math course-taking trajectories, suggesting that opportunity to learn is necessary but not sufficient for educational success.

2—TPSEMath: Transforming Post-Secondary Education in Mathematics. [Website]. (2021). https://www.tpsemath.org/

TPSEMath has the goal of making available to all students postsecondary education in mathematics that enables them to develop the modern mathematical knowledge and skills they need for productive engagement in society and in the workplace, regardless of their identity, background, or chosen program of study. The organization's mission is to facilitate an inclusive movement to strengthen postsecondary education in mathematics by working closely with and mobilizing faculty leaders, university administrations, membership associations, and relevant disciplinary societies in the pursuit of mathematically rich and relevant education for all students. The organization works to identify innovative practices where they exist, advocate for innovation where they do not, and work with and through partners to implement and scale effective practices, striving throughout to ensure that students and society are enriched by the power and beauty of mathematics.

4—Trimble, M. J., Pheatt, L., Papikyan, T., & Barnett, E. A. (2017). *Can high school transition courses help students avoid college remediation? Estimating the impact of a transition program in a large urban district* (CCRC Working Paper No. 99). Columbia University, Teachers College, Community College Research Center. https://academiccommons.columbia.edu/doi/10.7916/D8FR146C/download

This study examines the effectiveness of math and English transition courses with added supports through the At Home in College program in New York City, which was developed by the City University of New York. The study asks: What is the impact of the availability of the transition course program on students' attainment of college readiness benchmarks upon initial college enrollment and on students' likelihood of passing a first college-level (gatekeeper) course in the related subject in the first year of college? Taking advantage of staggered program implementation, the authors employ a difference-in-differences (DID) methodology to compare the difference in student outcomes between cohorts of students in schools that continuously implemented the transition program during a given time frame to the difference in outcomes between cohorts that had not yet implemented the program. Findings in relation to English suggest a small negative impact (three percentage points) on college readiness and no impact on passing an English gatekeeper course within the first-year of college entry. In math, the authors find no impact on college readiness in math and a small positive and significant effect (one percentage point) on passing a math gatekeeper course within one year of college entry. In both subjects, authors find a small, positive impact (one credit) on the number of college course credits earned in the first year. However, these results are somewhat sensitive to alternate sample specifications. Taken together, the findings suggest that offering the program is likely neutral to mildly beneficial and at least not harmful to high school seniors. Yet because the counterfactual circumstance typically includes a college-preparatory course of some kind that is displaced in favor of the treatment, it is important for policymakers and educators implementing transition courses to carefully consider the unintended consequences of removing students from alternative courses. If the alternative courses are already rigorous, well-taught, and packed with content that is useful for college success, the transformative impact of a transition course may be limited.

1—Trefil, J. (2008). Science education for everyone: Why and what? *Liberal Education, 94*(2), 6–11. https://files.eric.ed.gov/fulltext/EJ822735.pdf

What the author explores in this essay is not so much the "whether" of general science education, but the "why." What exactly constitutes good science education, and how can one recognize when students have received it? Once this question has been answered, the answer to the "what" question—the actual content of the curriculum—is relatively easy to find. There are (at least) two different kinds of things that go under the name of "science education." One involves the education of future scientists and engineers—an endeavor that is in pretty good shape (although improvements are always possible). The other involves the education of what the author calls "the other 98 percent"—the students who will not go on to careers in science and technology. It is this latter sort of education that the author wants to discuss. In particular, he wants to ask what sort of education the other 98 percent should get in the sciences.

1—Tyson, K. (2006). The making of a burden: Tracing the development of a "burden of acting White" in schools. In E. M. Horvat & C. O'Connor (Eds.), *Beyond Acting White: Reframing the debate on Black student achievement* (pp. 57–88). Rowman & Littlefield. https://rowman.com/ISBN/9780742542723/Beyond-Acting-White-Reframing-the-Debate-on-Black-Student-Achievement

Why do Black students underperform in school? Researchers continue to pursue this question with vigor not only because Blacks currently lag behind Whites on a wide variety of educational indices but because the closing of the Black-white achievement gap slowed and by some measures reversed during the last quarter of the 20th century. The social implications of the persistent educational "gap" between Blacks and whites are substantial.

Black people's experience with poor school achievement and equally poor access to postsecondary education reduces their probability for achieving competitive economic and social rewards and is inconsistent with repeated evidence that Black people articulate high aspirations for their own educational and social mobility. Despite the social needs that press us toward making better sense of the "gap," we are, nevertheless, limited in our understanding of how race operates to affect Black students' educational experiences and outcomes. In *Beyond Acting White*, the authors contend with one of the most often cited explanations for Black underachievement: the notion that Blacks are culturally opposed to "acting white" and, therefore, culturally opposed to succeeding in school. The book uses the "acting white" hypothesis as the point of departure in order to explore and evaluate how and under what conditions Black culture and identity are implicated in our understanding of why Black students continue to lag behind their white peers in educational achievement and attainment.

4—UCLA Center X. (2019 May 14). Introduction to Data Science [Website resources]. University of California, Los Angeles. https://centerx.gseis.ucla.edu/category/center-x-projects/introduction-to-data-science

This website resource describes the UCLA Center X and provides information about the Introduction to Data Science (IDS) Project, a leading national provider of high school data science education materials, professional development, and technological support. More information is available on the website.

2—University System of Maryland. (n.d.). *First in the world Maryland Mathematics Reform Initiative*. https://www.usmd.edu/BORPortal/Materials/2016/FB/20161209/5e.P20_FITW.pdf

This document describes how the University System of Maryland (USM), in collaboration with the Maryland Community Colleges and the other private and public institutions of higher education in Maryland, are working to address the mathematics "pipeline" issues that have created a significant bottleneck for postsecondary students. The Maryland Mathematics Reform Initiative (MMRI) is a collaborative effort currently underway between the public four-year USM institutions and the two-year community colleges in Maryland to develop and implement multiple high-quality mathematics pathways for students that are relevant for their chosen career path while also ensuring that the new courses have sufficient mathematical integrity and rigor to be deemed "college-level."

4—Utah State Board of Education. (2021). *Utah high school mathematics graduation pathways*. https://www.schools.utah.gov/file/08a2d9be-d56d-4109-a1b7-0c88092eda38

This document outlines Utah's high school mathematics graduation pathways, including course options, graduation requirements for different pathways, and frequently asked questions.

1—Vanfossen, B. E., Jones, J. D., & Spade, J. Z. (1987). Curriculum tracking and status maintenance. *Sociology of Education*, 104–122. https://www.jstor.org/stable/2112586

Prior studies of the role of curriculum tracking in status maintenance have offered contradictory results, suggesting either that (a) tracking sorts children from different backgrounds into different curricular programs where they receive differential educational treatments; (b) tracking sorts children on the basis of ability rather than class background, thus facilitating social mobility; or (c) while tracking may sort children, it has little effect upon educational outcomes and thus has no role in status maintenance. This paper uses 1980s data from the High School and Beyond Study to estimate the effects of curriculum tracking over a two-year period on a number of dependent variables for students who have experienced only one track placement. The results show that there are substantial differences among students from different socioeconomic origins in ultimate track destination. Track location is modestly to moderately related to number of courses taken, academic performance, educational and occupational aspirations, satisfaction with school, perceived values of friends, self-esteem, extracurricular participation and leadership, enrollment in postsecondary education, disciplinary climate, and teacher treatment. A reopening of the status maintenance hypothesis is suggested.

1—Waggener, J. F. (1996). *A brief history of mathematics education in America* [Unpublished manuscript]. University of Georgia. https://libres.uncg.edu/ir/uncp/f/Taylor%20Cooper%20Senior%20Project.pdf

This paper provides a historical overview of how mathematics curricula have evolved from colonial times to modern-day America. This paper offers a historical perspective of how math has been taught and how historical events, such as the Great Depression and World War II, affected mathematics education in the United States. The paper also discusses how the value of math education has changed and how different views of mathematics affected what was taught in the classroom. Government legislation and its effects, both positive and negative, are also addressed. The purpose of this paper is not to critique the way that math has been taught, but rather to provide insight into the development of mathematics education and its dynamic nature.

4—Washington State Board for Community & Technical Colleges. (2015). *Bridge to College courses in Washington: Project overview, spring 2015*. https://app.leg.wa.gov/committeeschedules/Home/Document/173875

This document describes how the Bridge Course State Partnership Team (Office of Superintendent of Public Instruction and the State Board for Community and Technical Colleges) worked to offer a Bridge to College Course for English language arts and/or mathematics starting in the 2015–16 school year. The document includes descriptions of the Bridge courses and how they are intended to function. With regard to mathematics, the document states: “The Bridge to College Mathematics course is grounded in the Southern Regional Education Board’s Math Ready course. Intended for students heading for college pathways not requiring calculus, the curriculum emphasizes modeling with mathematics and the Common Core Standards for Mathematical Practice, and a variety of essential standards from Algebra I, statistics and geometry, plus the Algebra II standards agreed to as essential college- and career-readiness standards for most students. The course emphasizes student engagement based heavily on conceptual teaching and learning” (p. 1).

4—Washington State Board for Community & Technical Colleges. (2023). *Bridge to College transition courses: Questions and answers*.

https://www.k12.wa.us/sites/default/files/public/ela/pubdocs/Bridge%20to%20College%20QA%20Feb%202023%20FINAL.pdf

This document further discusses Washington's Bridge to College transition courses, in particular providing answers to common questions in four areas: (a) background and application process, (b) placement agreement and how the courses "count," (c) who can/should enroll in the courses, and (d) course content and general information. The document states that, as of February 2023, participants were "currently, Eastern Washington University and all 34 of the state's community and technical colleges (through a system-wide agreement). For participating institutions, high school students who earn a B or better in the Bridge courses, and then begin college the year following graduation, will be placed directly into a college-level math (not on a calculus or STEM pathway) or English composition course."

4—Weiland, T., Orrill, C. H., Nagar, G. G., Brown, R. E., & Burke, J. (2021). Framing a robust understanding of proportional reasoning for teachers. *Journal of Mathematics Teacher Education, 24*(2), 179–202. https://web.s.ebscohost.com/ehost/pdfviewer/pdfviewer?vid=0&sid=f04ba2a3-38b7-44a8-b454-212e48ac63c4%40redis

Proportional reasoning is foundational in school mathematics. Despite the importance of proportional reasoning, little has been written about the knowledge needed for teachers to teach proportional reasoning relative to its importance. In this paper, the authors present an initial definition for a robust understanding of proportional reasoning for teaching based on synthesizing past scholarship in conjunction with an empirical study of the knowledge resources practicing teachers use to make sense of proportional reasoning tasks. As a result of this effort, the authors present an initial framing of operationalized knowledge resources based upon their definition for a robust understanding that could be used as an analytic tool. They also describe how such a framework of knowledge resources is useful in the context of teacher education, and they provide an example of how the framework was used to analyze the knowledge resources a teacher employed to make sense of a proportional reasoning task. The authors end by discussing possible implications their framework has for designing teacher education experiences and further research that is needed.

1—Welner, K. G., & Carter, P. (2013). Achievement gaps arise from opportunity gaps. In P. L. Carter & K. G. Welner (Eds.), *Closing the opportunity gap: What America must do to give every child an even chance* (pp. 1-10). Oxford Academic. https://academic.oup.com/book/9921/chapter-abstract/157242600?redirectedFrom=fulltext

The authors explain why it is important to shift the nation's focus back toward an "opportunity gap" framing of educational inequity. Thinking about inputs helps us to focus on the deficiencies in the foundational components of societies, schools, and communities that produce significant differences in educational—and ultimately socioeconomic—outcomes. Thinking in terms of "achievement gaps" emphasizes the symptoms; thinking about unequal opportunity highlights the causes. Importantly, discussions of both achievement and opportunity gaps sensibly begin with the premise that we as a nation must act to redress the serious inequities that exist between and within schools, as well as among

different people, groups, and communities across the country. Both discussions include an understanding that outcomes should be measured, analyzed, and addressed. Welner and Carter argue that test-score and attainment differences will not disappear until policy is dedicated to changing the conditions that shape and impede achievement.

1—Wheelock, A. (1992). *Crossing the tracks: How "untracking" can save America's schools*. New Press. https://files.eric.ed.gov/fulltext/ED353349.pdf

This book examines the effects of tracking on public education, makes a case for "untracking" as a key education reform, and describes successful untracking efforts. Tracking is an educational practice that groups children of similar abilities and achievement levels together into homogeneous groups. "Untracking" is the process of eliminating a track system in schools and reworking the education process to serve heterogeneous groups. Untracking schools create academic communities founded on a moral vision of all students learning together at high levels with the understanding that they are engaged in the first stages of a process of lifelong learning. Part 1, an introduction, looks at the current status of tracking in public schools, problems with tracking, and reasons for detracking. Part 2 summarizes components of successful untracking in schools and districts. In Part 3, seven chapters explore these components through the experiences of specific schools and cover the following topics: (a) involving parents and the community in the untracking effort; (b) establishing a culture of high expectations within an untracked school; (c) organizing and grouping students for diversity, including grouping practices, compensatory education, language-diverse students, and multi-age grouping; (d) fashioning high-level curricula for heterogeneous groups; (e) untracking instruction and assessment in heterogeneous classrooms; (f) teaching mathematics; and (g) the effect of untracking on student aspirations. Included are a list of untracked schools and extensive references for each chapter or section.

4—Wiles, P., Lemon, T., & King, A. (2019). Transforming middle school geometry instruction. *Mathematics Teaching in the Middle School 24*(7), 414–421. https://doi.org/10.5951/mathteacmiddscho.24.7.0414

Although the shifts toward proportional and algebraic reasoning are often emphasized in middle school curricula, changes in the nature of geometric reasoning are not always given the same weight. As highlighted in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM 2018), transformations have taken a more prominent role in secondary geometry instruction, a dramatic change from the way geometry has been previously taught. Moreover, transformations are often not adequately represented in traditional U.S. curriculum materials, and many teachers may not be fluent with these ideas. This article discusses tools and strategies for helping students to understand similarity and congruence formally via transformational geometry.

4—Will, M. (2014). In transition to Common Core, some high schools turn to "Integrated" Math. *Education Week, 34*(12), 18–19. https://www.edweek.org/teaching-learning/in-transition-to-common-core-some-high-schools-turn-to-integrated-math/2014/11

This article describes how the mathematics course sequence of Algebra 1, Geometry, and Algebra 2 faced several challenges as schools transitioned to the Common Core State Standards and how a small but growing number of high schools across the country were

moving toward an integrated mathematics pathway. It describes how in the United States, integrated math had been in use sporadically since the 1990s but was beginning to gain ground. The article describes the approach of a number of different states with regard to siloed or integrated mathematics curriculum.

4—Wineburg, S., & McGrew, S. (2016). *Evaluating information: The cornerstone of civic online reasoning.* Analysis & Policy Observatory. https://apo.org.au/node/70888

Between January 2015 and June 2016, the Stanford History Education Group has prototyped, field-tested, and validated a bank of assessments that tap civic online reasoning—the ability to judge the credibility of information that floods young people's smartphones, tablets, and computers. The authors administered 56 tasks to students across 12 states. In total, they collected and analyzed 7,804 student responses. Sites for field-testing included underresourced inner-city schools in Los Angeles and well-resourced schools in suburbs outside of Minneapolis. College assessments, which focused on open web searches, were administered online at six different universities that ranged from Stanford, an institution that rejects 94 percent of its applicants, to large state universities that admit the majority of students who apply. In this paper, the authors provide an overview of what they learned and sketch paths their future work might take. They end by providing samples of their assessments of civic online reasoning.

4—Wu, H. (2011). *The mathematics K–12 teachers need to know.* http://math.berkeley.edu/~wu/Schoolmathematics1.pdf

This paper begins with a general survey of the basic characteristics of mathematics (pp. 2–7). Some examples are then given to illustrate the general discussion (Part I). The bulk of the paper is devoted to a description of the mathematics that teachers of K–8 should know (Part II, pp. 22–69). The omission of what high school teachers should know is partly explained by the fact that the author was at the time in the process of writing a series of textbooks covering the mathematics of grades 8–12 for prospective teachers.

4—Wu, H.-H. (2017). The content knowledge mathematics teachers need. In Y. Li, W. James Lewis, & J. Madden (Eds.), *Mathematics matters in education.* Springer. https://www.researchgate.net/profile/Hung-Hsi-Wu/publication/320235745_The_Content_Knowledge_Mathematics_Teachers_Need/links/5a2133fdaca27229a06eb4c6/The-Content-Knowledge-Mathematics-Teachers-Need.pdf

The authors describe the mathematical content knowledge a teacher needs in order to achieve a basic level of competence in mathematics teaching. They also explain why content knowledge is essential for this purpose, how Textbook School Mathematics (TSM) stands in the way of providing teachers with this knowledge, and the relationship of this concept of content knowledge with pedagogical content knowledge (PCK).

1—Xu, D., & Dadgar, M. (2017). How effective are community college remedial math courses for students with the lowest math skills? *Community College Review, 46*(1) 62–81. https://journals.sagepub.com/doi/abs/10.1177/0091552117743789?journalCode=crwa

This article examines the effectiveness of remediation for community college students who are identified as having the lowest skills in math. The authors use transcript data from a state community college system and take advantage of a regression discontinuity design that compares statistically identical students who are assigned to the lowest level of the math sequence, which consists of three remedial courses, versus the next lowest level, which consists of two courses. The results suggest that for the students with the lowest preparation in math, the longest developmental sequence offers little benefit and may even reduce the likelihood of earning a degree or certificate within four years. This study is one of the first attempts to compare the academic outcomes of students assigned to a longer sequence of developmental math education to the outcomes of students with similar academic skills but assigned to a shorter developmental math sequence. Results from this study can therefore help inform the national effort in reforming remedial education, especially in terms of whether shortening the long remedial sequence would either benefit or harm the academic outcomes of students who are least prepared for college-level coursework.

4—Zachry Rutschow, E., Diamond, J., & Serna-Wallender, E. (2017). *Math in the real world: Early Findings from a study of the Dana Center Mathematics Pathways*. Center for the Analysis of Postsecondary Readiness. https://files.eric.ed.gov/fulltext/ED583571.pdf

In 2012, the Charles A. Dana Center at the University of Texas at Austin introduced the Dana Center Mathematics Pathways (DCMP, formerly known as the New Mathways Project), aiming to revise the structure, content, and pedagogy of developmental and college-level math classes in an effort to improve students' outcomes. In 2014, the Center for the Analysis of Postsecondary Readiness, with support from the U.S. Department of Education's Institute of Education Sciences, partnered with the Dana Center to launch a rigorous evaluation of the DCMP. Overall, the findings are encouraging; DCMP students are having qualitatively different classroom experiences from those of students in traditional developmental math courses and enrolling in and passing these courses at higher rates. However, work still needs to be done to ensure that all eligible students are correctly advised into these new pathways and that their math credits will transfer seamlessly to four-year college partners.

4—Zucker, A., Noyce, P., & McCullough, A. (2020). JUST SAY NO! Teaching students to resist scientific misinformation. *The Science Teacher, 87*(5), 24–29. https://www.nsta.org/science-teacher/science-teacher-january-2020/just-say-no

Authors describe a free research-based one-week curriculum unit they developed for use in grades 6–12. They expand upon the reasons for creating the unit, describe the materials, discuss the pilot test conducted in six schools, and then indicate how the unit aligns with internal education standards. This unit is aligned with both the CCSSM and the Next Generation Science Standards.