# Illustrating Student Achievement
# Using National Assessment of Educational Progress Questions:
# Grade 4
# Number and Operations—Fractions Domain

The Montana Office of Public Instruction (OPI) adopted new standards for language arts and mathematics in November 2011. The new standards will be implemented in the 2013-2014 school year with the Smarter Balanced (SBAC) assessment taking place in the spring of 2014.

This document uses National Assessment of Education Progress (NAEP) questions that seem to have a close alignment with the new standards to illustrate or suggest current levels of student achievement for the new standards. It is not intended to make any predictions about how students will do on a new assessment but may have instructional implications in terms of showing students' strengths and weaknesses. NAEP releases some items after each NAEP administration; performance data is given for the nation and states for each released item. Since 2003, every state has participated in the grade 4 and grade 8 NAEP mathematics and language arts assessments, which are given every other year. SBAC released practice tests matching the Operations and Algebraic Thinking domain have been included in this document as another example to illustrate the standards. There are no NAEP 2013 released questions as examples but these questions may be accessed via the **NAEP Questions Tool (NQT).**

This work has been made available through the **National NAEP Year Projects** (NNYP). This document parallels the work of Alaska's NAEP state coordinator. The following jurisdictions have made this information possible: Alaska, Iowa, New York, Florida, Oregon and the District of Columbia. For more information and resources, please visit:

- **Alaska Department of Education**
- **Iowa Department of Education**
- **NYC Department of Education**
- **Florida Department of Education**
- **Oregon Department of Education**
- **District of Columbia**
- **AIR: Examining the Content and Context of the Common Core State Standards: A First Look at Implications for the National Assessment of Educational Progress**

National NAEP Year Projects
Common Core

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

**A note about NAEP performance:** NAEP rates multiple-choice or constructed-response questions scored either right or wrong as "easy" if answered correctly by 60% or more of students, "medium" is answered correctly by 40 to 59%, or "hard" if answered correctly by fewer than 40%.

## Montana Common Core Standards (MCCS):

**Develop understanding of fractions as numbers.**

- **3.NF.2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram.
  a. Represent a fraction 1/b (e.g., 1/4) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b (e.g., 4) equal parts. Recognize that each part has size 1/b (e.g., 1/4) and that the endpoint of the part based at 0 locates the number 1/b (e.g., 1/4) on the number line.
  b. Represent a fraction a/b (e.g., 2/8) on a number line diagram or ruler by marking off a lengths 1/b (e.g., 1/8) from 0. Recognize that the resulting interval has size a/b (e.g., 2/8) and that its endpoint locates the number a/b (e.g., 2/8) on the number line.
- **3.NF.3.**

**Extend understanding of fraction equivalence and ordering. (limited in this grade to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100)**

- **4.NF.1**. Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- **4.NF.2.** Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).

**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

- **4.NF.3.** Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$.
  a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
  b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model). *For example: 3/8 = 1/8 + 1/8 + 1/8 ;3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
  c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction).
  d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).
- **4.NF.4.** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
  a. Understand a fraction $a/b$ as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
  b. Understand a multiple of $a/b$ as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
  c. Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). Check for the reasonableness of the answer. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

**Understand decimal notation for fractions, and compare decimal fractions.**

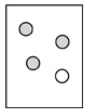
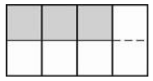
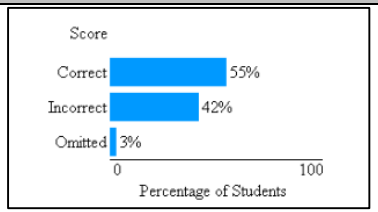
- **4.NF.5.** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
- **4.NF.6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

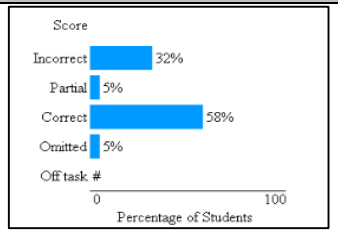
**Perform operations with multi-digit whole numbers and with decimals to hundredths**

- **5.NBT.7.** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**For more information on the MCCS- Grade Level Standards by Domain and Cluster, please visit:**
http://opi.mt.gov/Curriculum/montCAS/MCCS/index.php?gpm=1_4

| Year | Grade | Block | # | Type | Difficulty | Content Area | % Correct | Item | Description | Iowa CCSS Code | Alaska CCSS Code |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2003 | 4 | 6 | 16 | SCR | Hard | Number sense, properties, and operations | 36.77 | Item1 | Show where 3/4 is on a number line | 3.NF.2 | 4.NF.2. |
| 2003 | 4 | 7 | 15 | SCR | Hard | Number sense, properties, and operations | 20.18 | Item2 | Justify two correct interpretations (calculator available) | 4.NF.1 | 4.NF.2 |
| 2003 | 4 | 10 | 18 | ECR | Medium | Number sense, properties, and operations | 46.43 | Item3 | Analyze a situation involving equivalent fractions | 3.NF.3 | 4.NF.1. |
| 2005 | 4 | 4 | 5 | SCR | Easy | Number properties and operations | 65.26 | Item4 | Determine the value of a point on a number line | 4.NF.5 & 4.NF.6 | 4.NF.5 & 4.NF.6 |
| 2005 | 4 | 12 | 2 | MC | Medium | Number properties and operations | 53.03 | Item5 | Subtract fractions with common denominators | 4.NF.3 | 4.NF.3 |
| 2005 | 4 | 12 | 8 | MC | Medium | Number properties and operations | 53.39 | Item6 | Determine number of pieces from cutting wholes into fifths | 4.NF.3. | 4.NF.3. |
| 2007 | 4 | 9 | 11 | SCR | Easy | Number properties and operations | 60.21 | Item7 | List fractions equivalent to given fractions | 3.NF.3 | 4.NF.1. |
| 2007 | 4 | 9 | 18 | SCR | Medium | Number properties and operations | 40.85 | Item8 | Compare unit fractions to solve a problem | 3.NF.3 | 4.NF.2. |
| 2009 | 4 | 5 | 11 | MC | Medium | Number properties and operations | 55.1 | Item9 | Identify pictorial representation of equivalent fractions | XXXX | 4.NF.1. |
| 2009 | 4 | 5 | 15 | SCR | Medium | Number properties and operations | 49.11 | Item10 | Determine missing numbers on a number line | XXXX | XXXX |
| 2009 | 4 | 10 | 7 | MC | Medium | Number properties and operations | 46.43 | Item11 | Identify the value of a point on a number line | 3.NF.2 | 3.NF.2 |
| 2009 | 4 | 10 | 15 | MC | Hard | Number properties and operations | 25.38 | Item12 | Identify fraction closest to given value | 4.NF.2 | 4.NF.2 |
| 2011 | 4 | 12 | 12 | SCR | Hard | Number properties and operations | 30.47 | Item13 | Identify point on path using decimals | 5.NBT.7 | 4.NF.5. & 4.NF.6. |

| **NAEP Content Area:** Number properties and operations<br>**Question**: Identify pictorial representation of equivalent fractions. Gr.4. 2009. Item9.<br>**Iowa CCSS classification**: xxxx; **Alaska CCSS classification**: 4.NF.1. | **National Data:** | **MT Data:** |
|---|---|---|
| Which picture shows that $\frac{3}{4}$ is the same as $\frac{6}{8}$ ?<br>A. <br>B. <br>C.  <br>D.  |  | **53% correct**<br><br>Answer: A |

| **NAEP Content Area:** Number properties and operations<br>**Question**: List fractions equivalent to given fractions. Gr. 4. 2007. Item7.<br>**Iowa CCSS classification**: 3.NF.3; **Alaska CCSS classification**: 4.NF.1. | **National Data:** | **MT Data:** |
|---|---|---|
| <br>These three fractions are equivalent. Give two more fractions that are equivalent to these. |  | **61% correct** |

| **NAEP Content Area:** Number sense, properties, and operations<br>**Question**: Analyze a situation involving equivalent fractions. Gr.4. 2003. Item3.<br>**Iowa CCSS classification**: 3.NF.3; **Alaska CCSS classification**: 4.NF.1. | **Key/Scoring Guide:** | **National Data:** | **MT Data:** |
|---|---|---|---|

The shaded part of each strip below shows a fraction.

A.

This fraction strip shows 3/6.

B.

What fraction does this fraction strip show? _____

C.

What fraction does this fraction strip show? _____

What do the fractions shown in A, B, and C have in common?

_____

_____

_____

Shade in the fraction strips below to show two different fractions that are equivalent to the ones shown in A, B, and C.

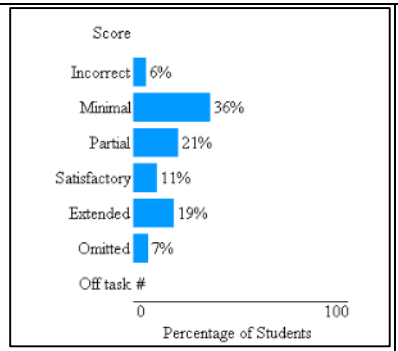
Question | Key/Scoring Guide | Sample Responses | National Data | Jurisdiction D

**Solution:**

There are a total of 5 responses required for this problem:
Response 1:
Part 1 (B) 1/2

Acceptable forms of 1/2: 1 out of 2, one half, 1:2, 1 ÷ 2, 50%, 1\2, $\frac{1}{2}$
Note: If student draws a model for B or C and names it correctly, it will be accepted.
Response 2:
Part 1 (C) 5/10 or 1/2
Response 3:
Part 2. There are many possible answers, e.g.,
They are equivalent fractions.
They all equal 1/2.
They are all equal.
They are all the same size.
They all end at the same place.
They are all 3 centimeters long.
The bottom number is twice the top number.

Unacceptable responses:
They are all rectangles.
They are the same.
They are all long and skinny.
The top number is odd and the bottom number is even.
They are all shaded.
They are all 6 cm. long.
They are even.
They are alike.



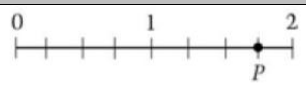

**16% extended**
**9% satisfactory**
**22% partial**

**Note:** Two NAEP questions that align with 3.NF.2 are included to show the base knowledge students would expected to have with the new fourth grade standards. These questions were part of the fourth grade NAEP assessment and performance data is for fourth graders.

**<u>NAEP Content Area:</u>** Number properties and operations
**Question**: Identify the value of a point on a number line. Gr. 4. 2009. Item11.
**Iowa CCSS classification**: 3.NF.2; **Alaska CCSS classification**: 3.NF.2.

**National Data:**

**MT Data:**



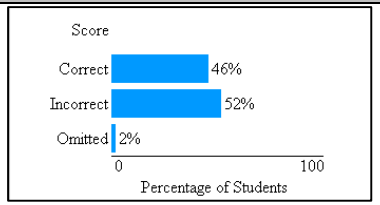

On the number line, what number does *P* represent?

A. $\frac{2}{3}$

B. $\frac{3}{4}$

C. $1\frac{2}{3}$

D. $1\frac{3}{4}$




**47% correct**

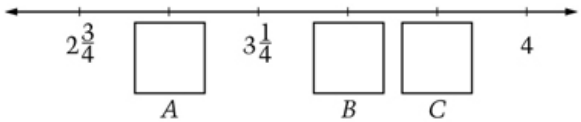
Answer: D

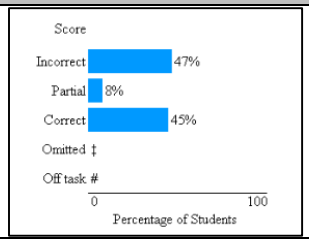
**<u>NAEP Content Area</u>:** Number properties and operations
**Question**: Determine missing numbers on a number line. Gr. 4. 2009. Item10.
**Iowa CCSS classification**: xxxx; **Alaska CCSS classification**: 3.NF.2.

**National Data:**

**MT Data:**

Jorge left some numbers off the number line below. Fill in the numbers that should in *A*, *B*, and *C*.




**46% correct**

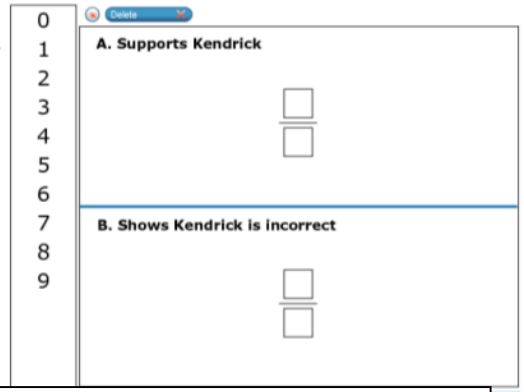
## SBAC Practice Test Item

**1**

Kendrick says that the only way to create a fraction greater than $\frac{3}{7}$ is to make the denominator less than 7.

A. Drag one number into each box to create a fraction that supports Kendrick.

B. Drag one number into each box to create a fraction that shows Kendrick is incorrect.




For this item, a full-credit response (2 points) includes:

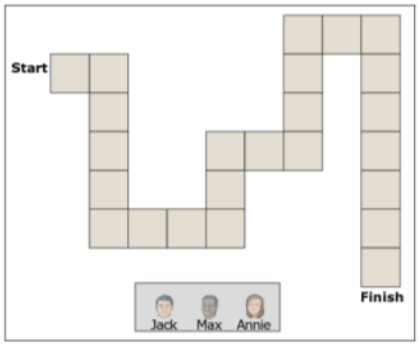
- a fraction with a denominator less than 7 that is greater than 3/7 AND
- a fraction with a denominator greater than 7 that is greater than 3/7

**13**

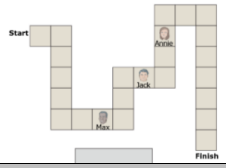
Jack, Max, and Annie are hiking on the same trail.

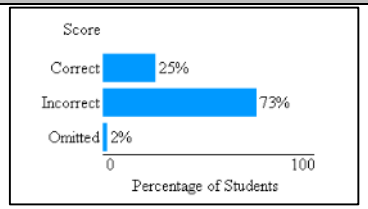
- Jack is $\frac{1}{2}$ finished.
- Max is $\frac{1}{3}$ finished.
- Annie is $\frac{5}{8}$ finished.

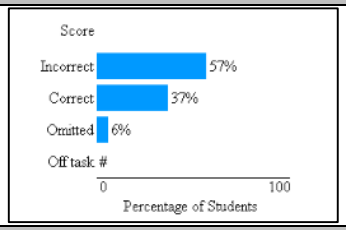
Drag each person to his or her location on the trail.




For this item, a full-credit response (2 points) includes the correct placement of Max, Jack, and Annie



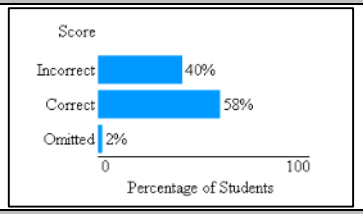

| **NAEP Content Area:** Number properties and operations<br>**Question**: Identify fraction closest to given value. Gr.4. 2009. Item12.<br>**Iowa CCSS classification**: 4.NF.2; **Alaska CCSS classification**: 4.NF.2. | **National Data:** | **MT Data:** |
|---|---|---|
| Which fraction has a value closest to $\frac{1}{2}$?<br>A. $\frac{5}{8}$<br>B. $\frac{1}{6}$<br>C. $\frac{2}{2}$<br>D. $\frac{1}{5}$ |  | **24% correct**<br>Answer: A |

| **NAEP Content Area:** Number sense, properties, and operations<br>**Question**: Show where 3/4 is on a number line. Gr. 4. 2003. Item1<br>**Iowa CCSS classification**: 3.NF.2; **Alaska CCSS classification**: 4.NF.2. | **National Data:** | **MT Data:** |
|---|---|---|
| On the portion of the number line below, a dot shows where ½ is. Use another dot to show where ¾ is.<br> |  | **37% correct** |

| **NAEP Content Area:** Number properties and operations<br>**Question**: Compare unit fractions to solve a problem. Gr.4. 2007. Item8. | **National Data:** | **MT Data:** |
|---|---|---|

**Iowa CCSS classification**: 3.NF.3; **Alaska CCSS classification**: 4.NF.2.

Mark says $\frac{1}{4}$ of his candy bar is smaller than $\frac{1}{5}$ of the same candy bar.
Is Mark right? ⬭ Yes ⬭ No
Draw a picture or use words to explain why you think Mark is right or wrong.




**53% correct**

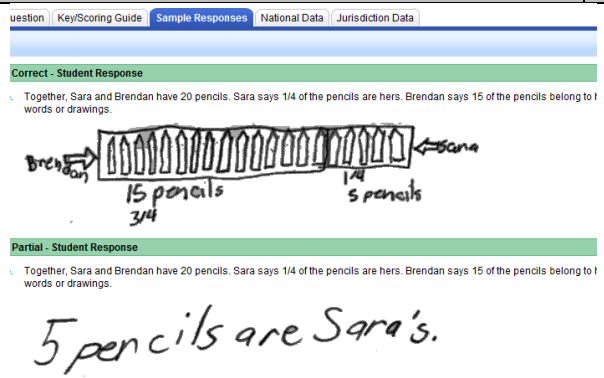
**NAEP Content Area:** Number sense, properties, and operations
**Question**: Justify two correct interpretations (calculator available). Gr.4. 2003. Item2.
**Iowa CCSS classification**: 4.NF.1; **Alaska CCSS classification**: 4.NF.2.

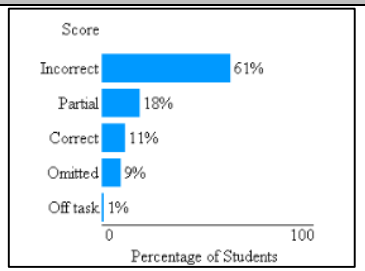
**Key/Scoring Guide:**

**National Data:**

**MT Data:**

Together, Sara and Brendan have 20 pencils. Sara says 1/4 of the pencils are hers. Brendan says 15 of the pencils belong to him. Explain how they both could be right. Use words or drawings.

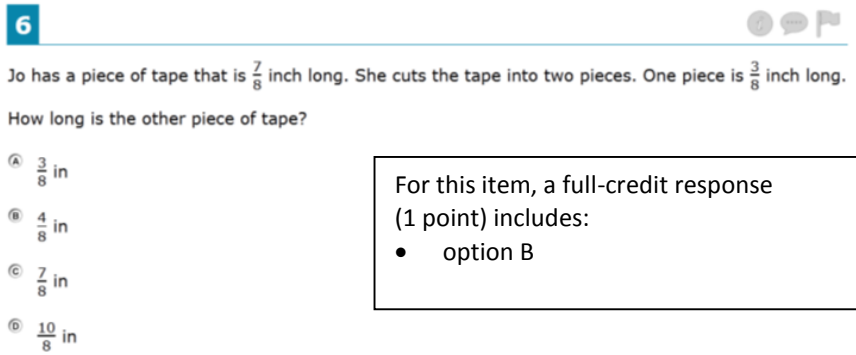



**10% correct**
**19% partial**

**SBAC Practice Test Item**

**Alaska CCSS classification:** 4.NF.3



6

Jo has a piece of tape that is $\frac{7}{8}$ inch long. She cuts the tape into two pieces. One piece is $\frac{3}{8}$ inch long.

How long is the other piece of tape?

Ⓐ $\frac{3}{8}$ in

Ⓑ $\frac{4}{8}$ in

Ⓒ $\frac{7}{8}$ in

Ⓓ $\frac{10}{8}$ in

For this item, a full-credit response (1 point) includes:

- option B

**NAEP Content Area:** Number properties and operations
**Question**: Subtract fractions with common denominators. Gr.4. 2005. Item5.
**Iowa CCSS classification**: 4.NF.3; **Alaska CCSS classification**: 4.NF.3.

**National Data:**

**MT Data:**

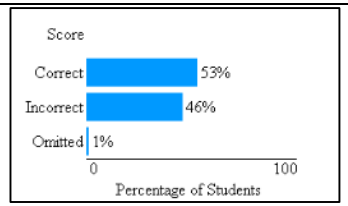
$\frac{4}{6} - \frac{1}{6} =$

A. 3

B. $\frac{3}{6}$

C. $\frac{3}{0}$

D. $\frac{5}{6}$




**42% correct**

Answer: B

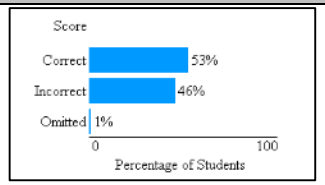
**NAEP Content Area:** Number properties and operations
**Question**: Determine number of pieces from cutting wholes into fifths. Gr. 4. 2005. Item6.
**Iowa CCSS classification**: 4.NF.3; **Alaska CCSS classification**: 4.NF.3.

**National Data:**

**MT Data:**

Luis had two apples and he cut each apple into fifths. How many pieces of apple did he have?

A. $\frac{2}{5}$
B. 2
C. 5
D. 10




**55% correct**

Answer: D

**SBAC Practice Test Item**

**Alaska CCSS classification**: 4.NF.5. & 4.NF.6.

**8**

Select all equations that are true.

- ☐ $\frac{4}{10} = 0.04$
- ☐ $\frac{17}{100} = 0.17$
- ☐ $\frac{9}{100} = 0.09$
- ☐ $\frac{6}{100} = 0.60$

For this item, a full-credit response (1 point) includes:

- option B
  AND
- option C

**14**

Which of the following expressions have a sum equal to $\frac{91}{100}$?

- ☐ $\frac{6}{10} + \frac{31}{100}$
- ☐ $\frac{7}{10} + \frac{19}{100} + \frac{2}{10}$
- ☐ $\frac{17}{100} + \frac{3}{10} + \frac{44}{100}$
- ☐ $\frac{33}{100} + \frac{28}{100} + \frac{4}{10}$
- ☐ $\frac{86}{100} + \frac{5}{10}$

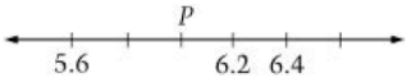
For this item, a full-credit response (1 point) includes:

- option A
  AND
- option C

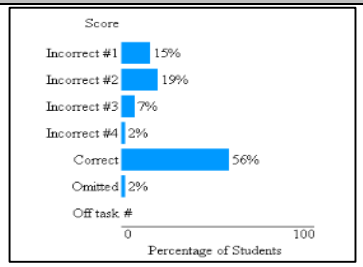
**NAEP Content Area:** Number properties and operations
**Question**: Determine the value of a point on a number line. Gr. 4. 2005. Item4.
**Iowa CCSS classification**: 4.NF.5 & 4.NF.6; **Alaska CCSS classification**: 4.NF.5. & 4.NF.6.

**National Data:**

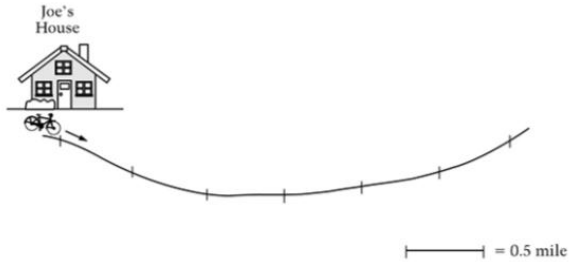
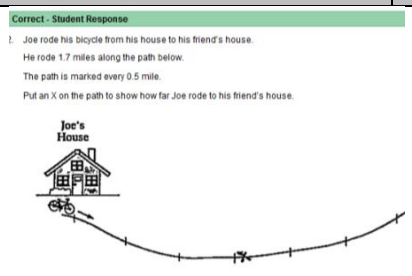
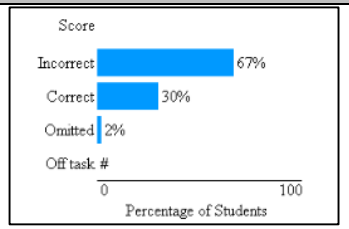
**MT Data:**

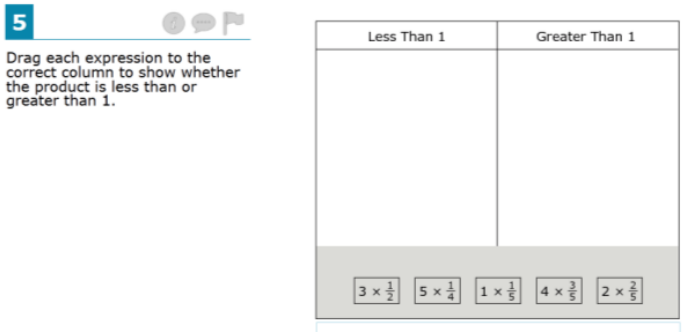



On the number line above, what number would be located at point *P*?




**55% correct**

| **<u>NAEP Content Area</u>:** Number properties and operations<br>**Question**: Identify point on path using decimals. Gr. 4. 2011. Item13.<br>**Iowa CCSS classification**: 5.NBT.7; **Alaska CCSS classification**: 4.NF.5. & 4.NF.6. | **Key/Scoring Guide:** | **National Data:** | **MT Data:** |
|---|---|---|---|
| Joe rode his bicycle from his house to his friend's house.<br>He rode 1.7 miles along the path below.<br>The path is marked every 0.5 mile.<br>Put an X on the path to show how far Joe rode to his friend's house.<br>  <br> |  <br> |  <br> | **31% correct** |

| **SBAC Practice Test Item** | |
|---|---|
| **Alaska:** 4.NF.4. | |
|  <br> | There is no released NAEP question that matches this standard, but the SBAC practice test item is included as an illustration of the standard.<br><br>For this item, a full-credit (1 point) includes:<br>• $1 \times \frac{1}{5}$ and $2 \times \frac{2}{5}$ in the "Less Than 1" column<br>AND<br>• $3 \times \frac{1}{2}$, $5 \times \frac{1}{4}$, and $4 \times \frac{3}{5}$ in the "Greater Than 1" column |