Ko'ko'hasenestôtse:

"The Art of Clicking Things Together"

Fast Facts

Curriculum Area: Mathematics
Grade Level: Grade 7
Suggested Duration: Three to four 50-minute class periods

Overview

This unit incorporates a traditional Cheyenne game called Ko'ko'hasenestôtse or "The Art of Clicking Things Together" into a probability lesson. Students will learn that probabilities are represented by fractions derived from modeling real-world experiments and simulations of chance.

This lesson incorporates cultural information about the Northern Cheyenne people of Montana. Students will learn that, historically and today, American Indians use games and stories to connect to their unique cultural heritage. These traditions vary between tribes and sometimes within tribes. Traditional games like Ko'ko'hasenestôtse, which is also called the Cheyenne Basket Game, provide social and educational opportunities for the Cheyenne. During this game people tell stories, socialize, and build relationships while they practice strategy, math, and communication skills. In this unit, students will use Ko'ko'hasenestôtse in order to improve their understanding of statistics and probability.

Strategies & Practices

Critical Area 4 *for instructional focus*

Drawing inferences about populations based on samples. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematical Practices *for student focus*

1. **Make sense of problems and persevere in solving them.** Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.
2. **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
3. **Model with mathematics.** They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions.

INDIAN EDUCATION FOR ALL • Montana Office of Public Instruction

4. **Attend to Precision.** Mathematically proficient students try to communicate precisely to others.
5. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure.

Standards

Math Clusters for Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Math Standards for Statistics and Probability

7.SP.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.5: Understand that the probability of a chance event is a number between 0 and 1 that express the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

7.SP.7b: Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Montana Content Standards for Reading and Writing

College and Career Readiness Anchor Standards for Reading (Grades 6-12):

R.CCR.2: Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.

College and Career Readiness Anchor Standards for Writing (Grades 6-12):

W.CCR.4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Indian Education for All Essential Understandings Regarding Montana Indians

Essential Understanding 1: There is great diversity among the twelve sovereign tribes of Montana in their languages, cultures, histories, and governments. Each tribe has a distinct and unique cultural heritage that contributes to modern Montana.

Essential Understanding 3: The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.

Learning Objectives

In this lesson, students will . . .

- understand that cultural heritage is important to all peoples, including American Indians;
- learn that Ko'ko'hasenestôtse and other Cheyenne traditional games are still played by Cheyenne people today;
- define and apply Statistics and Probability terms, such as: trial, experimental probability, sample space, Law of Large Numbers, and theoretical probability;
- collect and organize data into a table while simulating games of Ko'ko'hasenestôtse;
- determine the sample space of the game (by listing or tree diagramming) in order to show all possible combinations;
- determine the experimental probability of each throw;
- use the Law of Large Numbers to predict theoretical probability;
- learn the difference between experimental and theoretical probability and how each is determined;
- use probabilities to make predictions for future throws in the game of Ko'ko'hasenestôtse.

Assessment

Pre-assessment to check prior knowledge, skill levels, and potential misconceptions

- Review concepts from 6th grade Statistics and Probability standards:
  - ✓ Students should be able to summarize numerical data sets in relation to their context.
  - ✓ Students should recognize that a statistical question anticipates variability in the data related to the question and accounts for it in the answers.
- Review concepts from 6th grade Number System:
  - ✓ Apply and extend previous understandings of numbers to the system of rational numbers including fraction and decimal concepts.

Formative Assessment

- Learn and practice the Cheyenne game as well as the meaning behind its symbols.
- Complete the worksheet with data collection, sample space, and probabilities.
- Answer conceptual questions on the worksheet, engaging in meaningful dialogue with partners.
- Participate in class discussions by communicating ideas, concepts, and terms precisely.

Summative Assessment

The following Essential Questions may be used as a written assessment or for a class discussion at the end of this unit:

- In what way does Essential Understanding 3 explain the tradition of teaching Ko'ko'hasenestôtse?
- What does the persistence of traditional games like Ko'ko'hasenestôtse suggest about Northern Cheyenne culture in Montana? How is culture important in our lives?
- Can you explain the difference between experimental and theoretical probability and how each is determined?
- How is the Law of Large Numbers used to predict the theoretical probability?
- Can you identify and explain a technique that would increase your chances of winning this game?
- How are trial, experimental probability, sample space, Law of Large Numbers, and theoretical probability defined and used?

Materials and Resources

Each student will need a copy the following documents, which are provided in this lesson plan:

- "Ko'ko'hasenestôtse: The Art of Clicking Things Together" (game description)
- "Ko'ko'hasenestôtse Game – Worksheet"

For playing the game:

- For each player: Eight "sticks" (straws, Q-tips, or toothpicks)
- For each pair (or pair of teams): A "basket" (paper bowl)
- For each pair (or pair of teams): Five tiles or chips (roughly the size of a poker chip). Two should have "bear paw" markings on one side and be blank on the other. Three should have "crosses" on one side and be blank on the other side. The chips or tiles should be of sturdy material (wood, plastic, ceramic, etc.), not cardboard, so they will flip easily.

For the teacher:

- "Who Are the Northern Cheyenne?" (article to be shared with the class)
- Northern Cheyenne identity video (to be shared with the class)
- Math Goodies Website
- Math is Fun Probability Tree Diagrams Website
- Answer key for the worksheet is provided at the end of the lesson.

Optional Resources to be used as needed:

- Northern Cheyenne people today (video, 3:32 minutes long):
- Khan Academy Law of Large Numbers
- Montana Content Standards for Mathematics

Teacher Preparation

Make copies of the game description and worksheet for each student. Obtain game materials and prepare the tiles. Review the Northern Cheyenne background article, game description & rules, worksheet, sample answers, and the Essential Questions in the Summative Assessment.

Instructional Plan

Learning Activity One

1. Introduce the lesson by sharing aloud the "Essential Questions" this lesson aims to address. (These are listed in the Summative Assessment).
2. Explore student understanding of the foundational math concepts listed in the Pre-Assessment. As needed, review these skills and understandings.
3. Using a Montana map or the map provided in the background article, show students the location of the Northern Cheyenne Indian Reservation and inform students reservations are lands reserved by and for tribes for their continued existence.
4. Provide students with additional background information on the Northern Cheyenne Nation in Montana using the article "Who Are the Northern Cheyenne?" and the brief video on contemporary Northern Cheyenne identity.
5. Review aloud the IEFA Essential Understandings that pertain to this lesson, so students understand why they will be learning the Northern Cheyenne game Ko'ko'hasenestôtse.
6. Lead a class discussion that explores the importance of games and traditions to various cultures.

Learning Activity Two

1. Put the following on the board and discuss:
    a. Write 3 out of 4 as a ratio. *(3/4, 3 to 4, or 3:4)* Discuss the concept of **ratios** with the class. (*A ratio is a comparison of two quantities or measures.)*
    b. Define **probability**: *how likely it is that an event will happen; the ratio of the number of favorably outcomes out of the total number of possible outcomes.* Discuss the difference between **theoretical probability** and **experimental probability**.
    c. Define **sample space** and **tree diagram.** *(A branching diagram to show all possible outcomes.)* Demonstrate a tree diagram.
    d. Review the **Law of Large Numbers**.
2. Inform the students they will be playing a Cheyenne game called Ko'ko'hasenestôtse**,** which means "The Art of Clicking Things Together." This game is also called Monshimout, but according to Dr. Richard Little Bear, President of Chief Dull Knife College, that name is an Anglicized version of Móhenėšemahtôtse, a Northern Cheyenne word meaning "card game." It is also sometimes referred to as the Cheyenne Basket Game. This game is not unique to the Cheyenne, but is played by several different indigenous cultures; however, not all of the tribes who play this game use the cross and bear paw markings the Northern Cheyenne version uses.

3. Distribute the game description provided in this lesson and read it aloud.
4. With a student, model a game for the class, thinking aloud and answering students' questions as you complete the game. Display recording requirements on the board for students to refer to as they complete their worksheets.
5. Hand out the Ko'ko'hasenestôtse Game worksheet found at the end of this lesson.
   - Assign student partners and set expectations for discussions and sportsmanship.
   - Ask students to complete question 1 on the worksheet.
   - Distribute the materials needed for playing the game.
   - Have partners play the game as many times as they can in 20 minutes and record their data in Tables 1 and 2.
   - Ask students to complete questions 2 – 5 on the worksheet.
6. Have a class discussion about questions 2 – 5.

Learning Activity 3

1. Discuss how to determine the **sample space** and possible methods (list or tree diagram) for listing all possible outcomes.
   - Use Math Goodies to review sample space and how to format probability equations.
   - Use Math is Fun to discuss tree diagrams.
2. Ask students to complete question 6 on the worksheet.
3. As a class, discuss the number of outcomes in the sample space (32 possible outcomes).
4. Compile the class data by putting three columns on the board or in a projected spreadsheet.
   - The first column will be a copy of the first column from Table 1.
   - Ask pairs to report how many tallies they had for each throw. Record the data for each pair, and then find the total for the whole class for each throw. Write the total for the whole class in the second column.
   - In the third column, write the experimental class probability.
   - Ask students to copy this information into Table 3. (This is question 7.)

Throw	Total Number of Class Tallies	Class Experimental Probability
2B, 2P, 1C…		

5. Ask students to complete Table 3 and discuss the **theoretical probabilities** and how they were determined.
6. Have students answer questions 8 – 10.
7. As a class, discuss the answers to questions 8 – 10 and review the definition of the **Law of Large Numbers**.
8. Ask students to complete questions 11 – 13 with their partner.

9. As a class, review the unit through question 12.
10. If necessary, pull small group remediation, while proficient students choose one of the following activities:
    - Aid in teaching math concepts and the game to their fellow students;
    - Continue playing this game and complete their charts to compile more data;
    - Review concept lesson (Khan Academy, Math is Fun, etc.);
    - Research other games where probability might be useful.

Learning Activity 4

1. As a class, review the math concepts and the learning objectives of this unit, using the Essential Questions listed in the Summative Assessment.
2. Together, discuss students' responses to question 13.
3. Brainstorm other games where probability might or might not be useful, and have students explain why/why not. (Example: Probability might be useful in certain card games like cribbage or poker, but not as useful in a game like Monopoly.)

Extension Activities and Additional Resources

To learn more about the Northern Cheyenne Tribal Nation, or to learn more traditional indigenous games, see the following resources:

❖ Macfarlan, Allan and Paulette. *Handbook of American Indian Games*. Mineola, NY: Dover Publications, 1985.

❖ Montana Tribes

❖ Northern Cheyenne Tribal Website

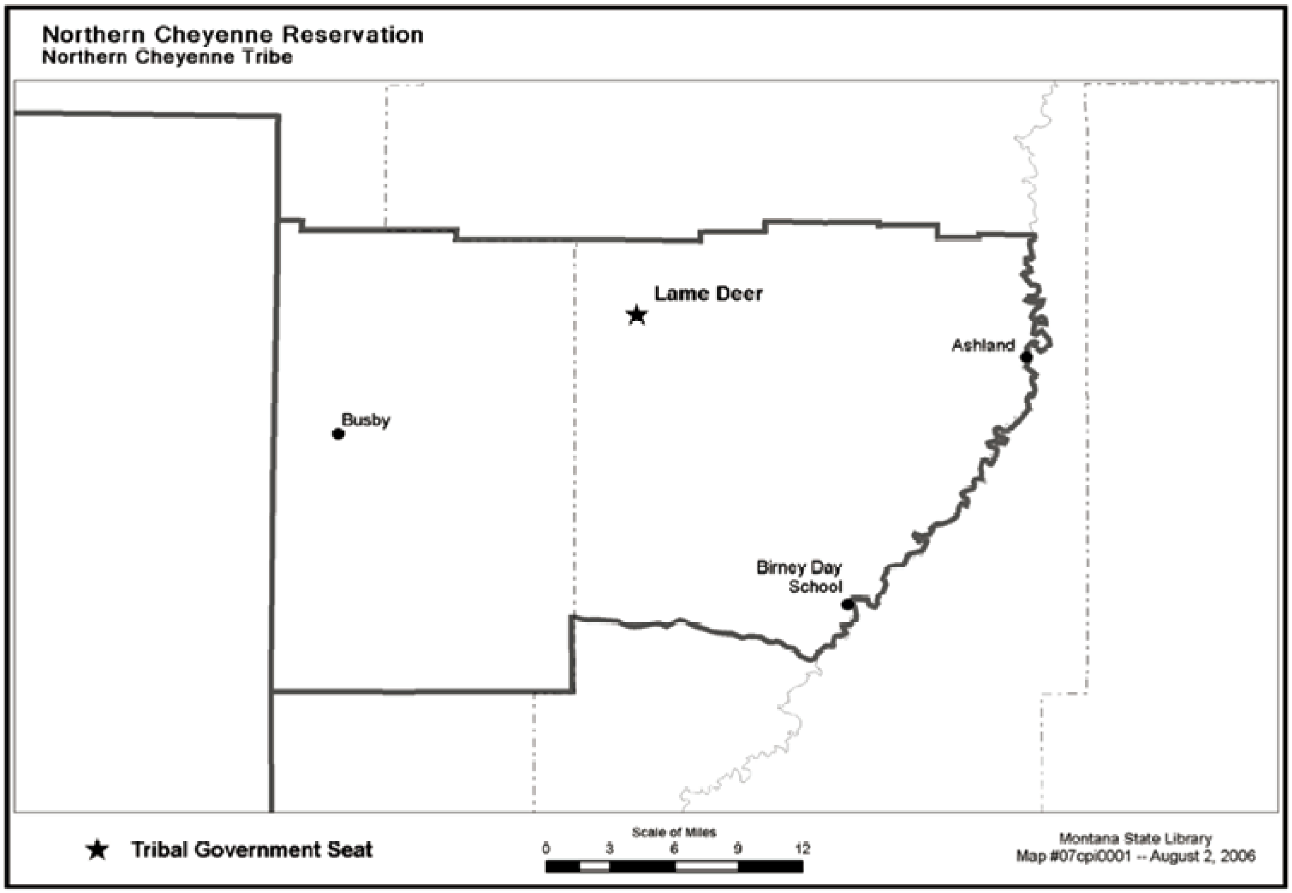
❖ Stekler, Paul and James Welch. *Last Stand at Little Big Horn: Sitting Bull and Crazy Horse Battle Custer.* (DVD) WGBH Boston,MA: 2004.

❖ *We, the Northern Cheyenne People: Our Land, Our History, Our Culture*. Lame Deer, MT: Chief Dull Knife College, 2008.

❖ Weist, Tom. *History of the Cheyenne People.* Billings, MT: Montana Council for Indian Education, 1977.

Who are the Northern Cheyenne?

Adapted from "The Northern Cheyenne Reservation" in *Montana Indians: Their History and Location.* Helena, MT: Office of Public Instruction, 2016.



Location

The Northern Cheyenne Reservation, situated in southeastern Montana, lies within Big Horn and Rosebud Counties. The Crow Reservation borders it on the west. The reservation consists of open ponderosa pine plateau and valley country with an annual rainfall of approximately 16 inches. The topography ranges from about 4,800 feet to just under 3,000 feet. The reservation headquarters and the center for businesses are located in Lame Deer. The reservation is divided into five districts; Busby, Lame Deer, Ashland, Birney, and Muddy.

Historical Background

The Cheyenne Indians are part of a linguistic group of the Algonquian language stock. Originally, it is believed that the ancestors of the Cheyenne lived south of the Hudson Bay and James Bay areas and slowly moved west into what is now northwestern Minnesota where the Red River forms a border between Minnesota and the Dakotas. During the late 1600s, they settled among the tribes of the upper Missouri River and began farming rather than subsisting as small game hunters and fishermen. During the early 1700s, they were still primarily farmers growing corn, but they also hunted buffalo. The Cheyenne acquired horses around 1750, and made a gradual transition from a horticultural existence to a horse culture. Hunting buffalo became a way of life as they migrated as far south as New Mexico and Texas.

The Cheyenne participated in the treaty making in 1825 near what is now Fort Pierre, South Dakota. A few years later, the larger part of the tribe (now the southern Cheyenne) moved southward to occupy much of the Arkansas River in Colorado and Kansas. The remainder continued to inhabit the plains from the headwaters of the North Platte up on to the Yellowstone River in Montana. The division of the tribe was recognized by the Fort Laramie Treaty of 1851.

[In 1868, General Custer led a massacre against a large campe of Cheyenne people, killing over 100 Cheyenne men, women, and children in their camp on the Washita River. Those killed were members of Black Kettle's peaceful band. Six years later, Custer led a prospecting expedition into the Black Hills, searching for gold. Once gold was discovered, the United States pressured the Lakota Sioux and their Cheyenne allies to relinquish the Black Hills.]

In the Battle of the Little Bighorn in 1876, the Northern Cheyenne joined the Sioux in what the Cheyenne call “Where Long Hair was Wiped Away Forever.” Cheyenne oral history recalls a time when George Custer smoked a Cheyenne pipe . . . and the ashes from the pipe dropped on his boot and scattered on the ground. These ashes were wiped away signaling Custer’s commitment never to fight the Cheyenne again.

Although the Cheyenne won the Battle of the Little Big Horn, it was the beginning of the end for them, as they were exiled to Indian Territory in Oklahoma to be imprisoned with the Southern Cheyenne. A small band escaped in a desperate effort led by Chief Dull Knife (Morning Star) and Chief Littlewolf. These two chiefs, in one of the most heroic episodes of western history, bravely fought against overwhelming odds, leading a small band of men, women, and children back to their homelands. The Northern Cheyenne call themselves “The Morning Star People.” The name is taken from and used in respect for Chief Dull Knife, who was also known as Morning Star.

Chief Littlewolf and Chief Dull Knife are buried side by side in the Lame Deer cemetery. By Executive Order of November 26, 1884, a tract of country east of the Crow Reservation was set apart as a reservation for the Northern Cheyenne. The reservation was expanded by another Executive Order in 1900 to its present boundaries.

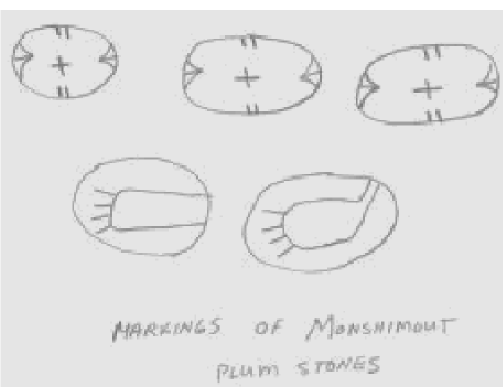
Ko'ko'hasenestôtse, "The Art of Clicking Things Together"

Ko'ko'hasenestôtse (The Art of Clicking Things Together, or the Cheyenne Basket Game) is a game from the Cheyenne people and is traditionally played by women. It may be played by two or more players. When played by more than two people, the players divide into two equal teams.

Game equipment traditionally consists of five plum stones and a basket made of woven grass or willow twigs. The basket measures 3-4 inches deep, 8 inches across at the top, and almost 1/2 inch thick. The plum stones are plain on one side and marked on the opposite side. Three have mark using a pattern similar to markings the women used when painting their faces: a cross (which was on the bridge of the nose) and side marks (on cheeks, forehead, and chin). The other two stones are marked with a representation of a bear's foot.

The players sit opposite each other (in two rows if more than one player on each side). Each player has eight sticks that represent the points she must score to win. When a player has won all the sticks belonging to her opponent she has won the game and the stakes wagered. If teams are playing, each member wins or loses according to the wins or losses of the teammate in control of the basket and stones.

A throw of the stones is done in the following manner. The stones are placed into the basket. Then, the basket is raised slightly and the stones tossed only a few inches and caught in the basket. The basket is then brought down firmly onto the ground so it strikes the ground making a slight noise. How the stones land determines the points scored or lost, as well the control of the throw. A player continues to throw so long as she throws a scoring toss. If she throws a toss with no score, the throw passes to her opponent or to the next player on her team. (If teams are playing, the throw passes to the thrower's teammate and continues as such until the last teammate throws. If the last member of a team throws a zero score the throw passes to the opposing team.)



The scoring is as follows:

2 blanks, 2 bears, 1 cross = 0
4 blanks, 1 bear = 0
5 blanks = 1 (thrower takes 1 stick)
3 blanks, 2 bears = 1 (thrower takes 1 stick)
2 bears, 2 crosses, 1 blank = 1 (thrower takes 1 stick)
2 blanks, 3 crosses = 3 (thrower takes 3 sticks)
2 bears, 3 crosses = 8 (thrower takes 8 sticks and wins the game!)

As with many games, wagers were made and placed prior to beginning the play. During this game the players traditionally did not sing, but did chat and joke quite heavily.

Game directions adapted from Manataka American Indian Council.

Ko'ko'hasenestôtse Game – Worksheet

Name: ___________________________________ **Period:** ______

1. Predict how many times you think you would need to throw the stones before you would see the winning throw of 2 bears and 3 crosses. Write your prediction as a ratio of 1 win to the number of throws.

2. Play the game Ko'ko'hasenestôtse with a partner. Tally each trial (the number of times you obtain sticks from your partner) for the following throws. Play the game one, two, or three times, continuing to tally the throws on **Table 1** *and* record how many throws it took to win each game into **Table 2**. Determine the **experimental probability** for each throw using all the data from each game combined. (Experimental probability is the number of successes out of the number of trials). **B = blank, P = bear paw, and C = cros**s.

Table 1

Throw	# of Sticks	Tally	Experimental Probability
2 blanks, 2 bear paws, 1cross	**0**		
4 blanks, 1 bear paws, 0 crosses	**0**		
5 blanks, 0 bear paws, 0 crosses	**1**		
3 blanks, 2 bear paws, 0 crosses	**1**		
1 blank, 2 bear paws, 2 crosses	**1**		
2 blanks, 0 bear paws, 3 crosses	**3**		
0 blanks, 2 bear paws, 3 crosses	**8**		
Other	**0**		

Table 2

Game Number	Number of throws to win the game
1	
2	
3	

3. Which throw(s) from Table 1 seemed the most likely to happen? Explain.

4. Which throw(s) from Table 1 seemed the least likely to happen? Explain.

5. Using Table 2, how many times did you have to throw until someone won the game? Do you think this would be the same or close to the same if you were to play again? Why or why not?

6. Determine the **sample space** (every possible way the five stones may land) by listing or tree diagramming. Show all of your work.

7. Copy the **experimental probabilities** from Table 1 into the appropriate column in **Table 3**. Your teacher will help you compile the class data. Determine the experimental probability for each throw using the combined class data. Determine the **theoretical probability** of each throw and write those in Table 3. (Theoretical probability is the number of successes out of the total number of outcomes in the sample space.) ***B = blank, P = bear paw, and C = cross.***

Table 3

Throw	# of Sticks	Game Experimental Probability (from Table 1)	Class Experimental Probability	Theoretical Probability
2 blanks, 2 bear paws, 1cross	0			
4 blanks, 1 bear paws, 0 crosses	0			
5 blanks, 0 bear paws, 0 crosses	1			
3 blanks, 2 bear paws, 0 crosses	1			
1 blank, 2 bear paws, 2 crosses	1			
2 blanks, 0 bear paws, 3 crosses	3			
0 blanks, 2 bear paws, 3 crosses	8			
Other	0			

8. How do the game experimental probability and the class experimental probability compare? Which do you think more accurately reflects the probability of each throw if you were to play multiple times? Explain why.

9. How do the experimental probabilities and theoretical probability compare? Should they be close in value? Why or why not? Which probability is a more accurate prediction of this game if you were to play it multiple times? Why?

10. Which experimental probability is closest to the theoretical probability? If you were to continue to throw the stones, how many times do you think you would need to throw and accurately predict the theoretical probability? Why do you think this? (**Law of Large Numbers** states that if you repeat a random experiment a large number of times, your outcomes should, on average, equal the theoretical average.)

11. Compare the theoretical probability to the prediction you made in question 1. How close was your prediction? Use the theoretical probability to predict how many times you should expect to get 2 bears and 3 crosses if you were to throw the stones 500 times? Show your work.

12. Do you think the Cheyenne people used the theoretical probability of each throw to determine the number of sticks you would obtain? Why or why not? Use the probabilities in your explanation.

13. If the Cheyenne did *not* use probability to determine the value to each possible throw, what other reasons might they have had for assigning sticks (value) for each possible combination? Examine the combinations of markings and the number of sticks assigned to each possible combination to arrive at a possible answer.

Ko'ko'hasenestôtse Answer Key

1. Predict how many times you think you would need to throw the stones before you would see the winning throw of 2 bears and 3 crosses. Write your prediction as a ratio of 1 success or win to the number of throws.

Any answer is acceptable. (Approx. 1/32)

2. Play the game Ko'ko'hasenestôtse with a partner. Tally each trial (the number of times you obtain “sticks” from your partner) for the following throws. Play the game one, two, or three times, continuing to tally the throws on Table 1 and record how many throws it took to win each game into Table 2. Determine the experimental probability for each throw using all the data from each game combined. (Experimental probability is the number of successes out of the number of trials). *B = blank, P = bear paw, and C = cross.*

Sample Game:

Table 1

Throw	# of Sticks	Tally	Experimental Probability
2 blanks, 2 bears, 1cross	**0**	IIII	$\frac{4}{25}$
4 blanks, 1 bear, 0 crosses	**0**	II	$\frac{2}{25}$
5 blanks, 0 bears, 0 crosses	**1**	0	$\frac{0}{25}$
3 blanks, 2 bears, 0 crosses	**1**	I	$\frac{1}{25}$
1 blank, 2 bears, 2 crosses	**1**	II	$\frac{2}{25}$
2 blanks, 0 bears, 3 crosses	**3**	I	$\frac{1}{25}$
0 blanks, 2 bears, 3 crosses	**8**	0	$\frac{0}{25}$
Other	**0**	IIIIIIIIIIIIIII	$\frac{15}{25}$

Table 2

Game Number	Number of throws to win the game
1	*29*
2	*32*
3	*35*

3. Which throw(s) from Table 1 seemed the most likely to happen? Explain.

   Sample response: *The "other" and 2B, 2P, 1C. The "other" had 15 tallies out of 25. These two seemed most likely. The 2B, 2P, 1C occurred 4 out of 25. It was the second most likely to occur.*

4. Which throw(s) from Table 1 seemed the least likely to happen? Explain.

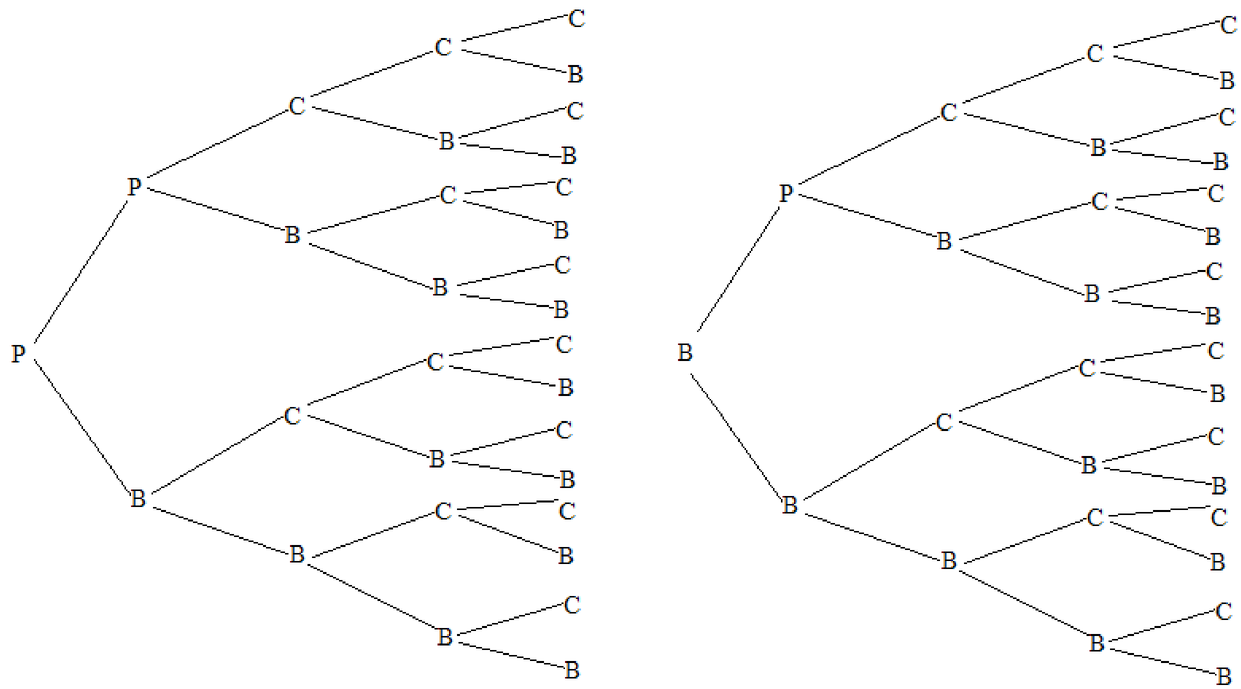
   Sample response: *The 5B and 2P, 3C happened zero times each. <u>3P, 2C</u> and <u>2B, 3C</u> only happened one time each. None of these seem very likely to occur.*

5. Using Table 2, how many times did you have to throw until someone won the game? Do you think this would be the same or close to the same if you were to play again? Why or why not?

   Sample response: *I threw 25 times and only collected 6 of my opponent's sticks. So, I know I would have to throw more than 25 to win. I think the results would be similar. The chances of each throw shouldn't change. So, the results should be similar.*

6. Determine the sample space (every possible way the five stones may land) by listing or tree diagramming. Show all of your work.

Tree diagram:



Sample space listing 32 outcomes:

PPCCC PPCCB PPCBC PPCBB PPBCC PPBCB PPBBC PPBBB PBCCC PBCCB PBCBC PBCBB PBBCC PBBCB PBBBC PBBBB BPCCC BPCCB BPCBC BPCBB BPBCC BPBCB BPBBC BPBBB BBCCC BBCCB BBCBC BBCBB BBBCC BBBCB BBBBC BBBBB

7. Copy the experimental probabilities from Table 1 into the appropriate column in Table 3.Your teacher will help you compile the class data. Determine the experimental probability for each throw using the combined class data. Determine the theoretical probability of each throw and write those in Table 3. (Theoretical probability is the number of successes out of the total number of outcomes in the sample space.)

Table 3 ***(Sample Class Response)***

Throw	# of Sticks	Game Experimental Probability (from Table 1)	Class Experimental Probability	Theoretical Probability
2 blanks, 2 bear paws, 1cross	**0**	$\frac{4}{25}$	$\frac{3}{32}$	$\frac{3}{32}$
4 blanks, 1 bear paws, 0 crosses	**0**	$\frac{2}{25}$	$\frac{2}{32}$	$\frac{2}{32}$
5 blanks, 0 bear paws, 0 crosses	**1**	$\frac{0}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
3 blanks, 2 bear paws, 0 crosses	**1**	$\frac{1}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
1 blank, 2 bear paws, 2 crosses	**1**	$\frac{2}{25}$	$\frac{3}{32}$	$\frac{3}{32}$
2 blanks, 0 bear paws, 3 crosses	**3**	$\frac{1}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
0 blanks, 2 bear paws, 3 crosses	**8**	$\frac{0}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
Other	**0**	$\frac{15}{25}$	$\frac{20}{32}$	$\frac{20}{32}$

8. How do the game experimental probability and the class experimental probability compare? Which do you think more accurately reflects the probability of each throw if you were to play multiple times? Explain why.

Sample response: *The probabilities are close in value. They would probably get closer the more times I throw the stones. They are close because the chances of each throw should stay the same, as the tree diagram shows.*

9. How do the experimental probabilities and theoretical probability compare? Should they be close in value? Why or why not? Which probability is a more accurate prediction of this game if you were to play it multiple times? Why?

> ***Sample response:*** *The class probability and the theoretical probability are somewhat close in value. I think they should be. They would probably get even closer in value the more times I throw the stones. I think the class probability begins to predict what should happen for anytime I play. The theoretical is what should happen. I think they should be close because the chances of each throw shouldn't change very much, as the tree diagram shows.*

10. Which experimental probability is closest to the theoretical probability? If you were to continue to throw the stones, how many times do you think you would need to throw and accurately predict the theoretical probability? Why do you think this? (Law of large numbers states that if you repeat a random experiment a large number of times, your outcomes should on average equal the theoretical average.)

> ***Sample response:*** *The class probability is closer to the theoretical probability. I think the more times I throw the stones the closer the probability will become to the theoretical probability.*

11. Compare the theoretical probability to the prediction you made in question 1. How close was your prediction? Use the theoretical probability to predict how many times you should expect to get 2 bears and 3 crosses if you were to throw the stones 500 times? Show your work.

> ***Sample response:*** *My prediction was fairly close. I would take the theoretical probability of 1:32 and multiply it to the 500 times I would throw the stones. So, I think I would get 2 bears and 3 crosses about 15 or 16 times in 500 throws.*

12. Do you think the Cheyenne people used the theoretical probability of each throw to determine the number of sticks you would obtain? Why or why not? Use the probabilities in your explanation.

> ***Sample response:*** *I don't think they took the probabilities into consideration when assigning the number of sticks you would take from your opponent, because: The theoretical probability of getting <u>2B, 2P, 1C</u> is 3:32, and you don't get to take any sticks if you throw this combination. The probability of <u>1B, 2P, 2C</u> is also 3:32, but you get to take 1 stick. While the probability of <u>4B, 1P</u> is 2:32, you don't get to take any sticks. Combinations <u>5B</u> and <u>3B, 2P</u> both have a 1:32 probability, and you get 1 stick for each. But <u>2B, 3C</u> is 1:32**,** and you get to take 3 sticks. Finally, <u>2P, 3C</u> is 1:32**,** and you take all eight sticks. So, probability does **not** seem to be a factor for assigning value to the possible combinations.*
>
> *I expected that the combinations with the same probabilities would earn the same number of sticks, and that the combinations with the lowest probability would be worth the most sticks while the combinations with the higher probabilities would be worth the least number of sticks, but that is not how it turned out.*

13. If the Cheyenne did not use probability to determine the value to each possible throw, what other reasons might they have had for assigning sticks (value) for each possible combination? Examine the combinations of markings and the number of sticks assigned to each possible combination to arrive at a possible answer.

Look at the combinations. Those with blanks get fewer sticks, whereas those with a combination that includes 3 crosses with something else get more sticks. Crosses are weighted most, then bear paws, then blanks; combinations also have value. The best (most heavily weighted) combination has all bear paws and crosses, but no blanks. Blanks only add value if you get all 5 of them. Clearly, the number of sticks assigned (value) to each possible throw is not based on probability, but is based on the markings themselves.