Montana Content Standards for Mathematics–Model Pathways for High School

Montana Content Standards for Mathematical Practices and Mathematics Content

Adopted November 2011

Montana
Office of Public Instruction
opi.mt.gov Elsie Arntzen, Superintendent

Montana Mathematics Content Standards

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Contents

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Introduction

The high school model pathway document is available for high schools to provide ideas for how they may choose to separate standards into course-specific groupings. This document includes both the traditional model course pathway of Algebra I, followed by Geometry and Algebra II as well as the integrated pathway of model mathematics I followed by model mathematics II and III. The pathways act as models not mandates.

Each model high school course is presented in three sections. First are the course overview which outlines the conceptual categories, domains and clusters included in that course. Next are critical areas for learning in the course. Last are the Montana content standards for that course, organized by conceptual category, domains, and clusters. The models do not dictate curriculum, pedagogy or assessments, but rather are what drives curriculum creation.

The document was adapted from the Massachusetts Standards for Mathematical Content High School: Model Pathways and Model Courses and the Common Core State Standards for Mathematics Appendix: A.

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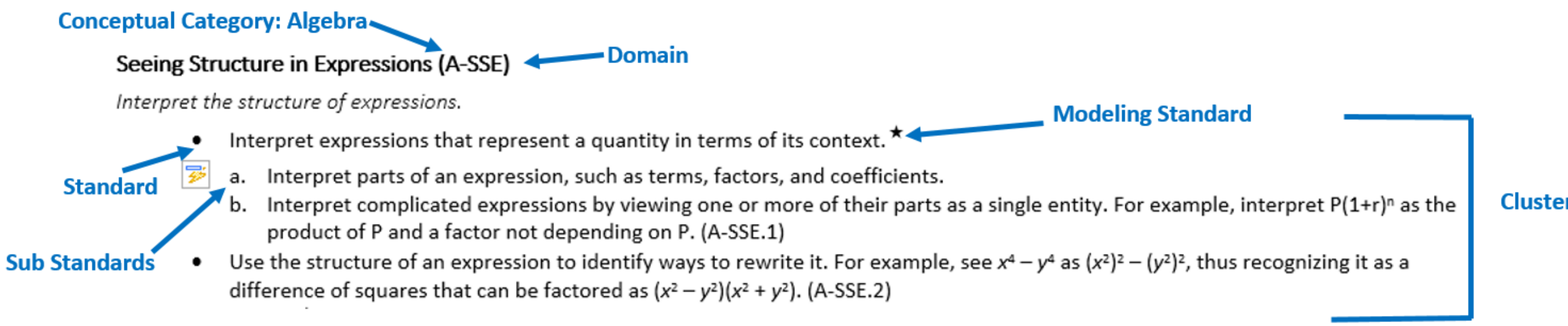
How to read the Model Pathways for High School

The high school Model Pathways are listed by conceptual categories, rather than by grade.

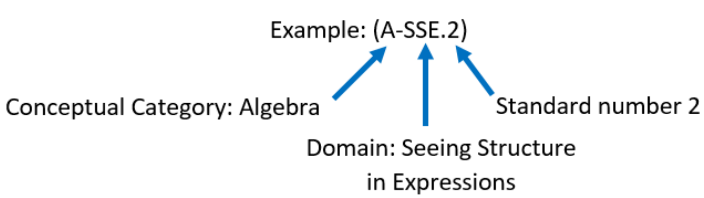
- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Modeling (★)
- Geometry (G)
- Statistics and Probability (S)

Conceptual categories portray a coherent view of high school mathematics.

- Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.
- The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).



Code is written in the following way: (Conceptual Category **–** Domain**.** Standard).



Standards for Mathematical Practice: Grades 9-12 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
HS.MP.1. Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2. Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3. Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4. Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Standards for Mathematical Practice: Grades 9-12 Explanations and Examples

HS.MP.5. Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6. Attend to precision.	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
HS.MP.7. Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
HS.MP.8. Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Adapted from Explanations and Examples Grades 9-12 produced by the Arizona Department of Education Standards and Assessment Division

Montana Mathematics Content Standards

(Adopted November 2011)

Model Traditional Pathway: Model Algebra I Overview

Domains	Number and Quantity	Algebra	Functions	Statistics and Probability
Clusters	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers. • Reason quantitatively and use units to solve problems.	• Interpret the structure of linear, quadratic, and exponential expressions with integer exponents. • Write expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Create equations that describe numbers or relationships. • Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Solve systems of equations. • Represent and solve equations and inequalities graphically.	• Understand the concept of a function and use function notation. • Interpret linear, quadratic, and exponential functions with integer exponents that arise in applications in terms of the context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	• Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. • Summarize, represent, and interpret data on two categorical and quantitative variables. • Interpret linear models.
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

The fundamental purpose of the Model Algebra I course is to formalize and extend the mathematics that students learned in the middle grades. For the high school Model Algebra I course, instructional time should focus on four critical areas:

1. **Deepen and extend understanding of linear and exponential relationships**
   - By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra I, students analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.
2. **Contrast linear and exponential relationships with each other and engage in methods for analyzing, solving, and using quadratic functions**
   - In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In Algebra I, students learn function notation and develop the concepts of domain and range. They focus on linear, quadratic, and exponential functions, including sequences, and also explore absolute value, step, and piecewise-defined functions; they interpret functions given

graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. Students build on and extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

3. **Extend the laws of exponents to square and cube roots**
   - Students extend the laws of exponents to rational exponents involving square and cube roots and apply this new understanding of number; they strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions. Students become facile with algebraic manipulation, including rearranging and collecting terms and factoring. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.
4. **Apply linear models to data that exhibit a linear trend**
   - Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Model Traditional Pathway: Algebra I Content Standards

Number and Quantity

The Real Number System (N-RN)

Extend the properties of exponents to rational exponents.

- Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. (N-RN.1)
- Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N-RN.2)

Use properties of rational and irrational numbers.

- Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. (N-RN.3)

Quantities (N-Q)

Reason quantitatively and use units to solve problems.

- Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (N-Q.1)
- Define appropriate quantities for the purpose of descriptive modeling. (N-Q.2)
- Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (N-Q.3)

Algebra

Seeing Structure in Expressions (A-SSE)

Interpret the structure of linear, quadratic, and exponential expressions with integer exponents.

- Interpret expressions that represent a quantity in terms of its context.★
  a. Interpret parts of an expression, such as terms, factors, and coefficients.
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. (A-SSE.1)
- Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (A-SSE.2)

Write expressions in equivalent forms to solve problems.

- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
  a. Factor a quadratic expression to reveal the zeros of the function it defines.
  b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (A-SEE.3)

Arithmetic with Polynomials and Rational Expressions (A-APR)

Perform arithmetic operations on polynomials.

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.1)

Creating Equations (A-CED)

Create equations that describe numbers or relationships.

- Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1)
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2)
- Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A-CED.3)
- Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R. (A-CED.4)

Reasoning with Equations and Inequalities (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning.

- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A-REI.1)

Solve equations and inequalities in one variable.

- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A-REI.3)
- Solve quadratic equations in one variable.
  a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. (A-REI.4)

Solve systems of equations.

- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (A-REI.5)

- Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A-REI.6)
- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. (A-REI.7)

Represent and solve equations and inequalities graphically.

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A-REI.10)
- Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ (A-REI.11)
- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A-REI.12)

Functions

Interpreting Functions (IF)

Understand the concept of a function and use function notation.

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (F-IF.1)
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F-IF.2)
- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. (F-IF.3)

Interpret linear, quadratic, and exponential functions with integer exponents that arise in applications in terms of the context.

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ (F-IF.4)
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★ (F-IF.5)
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ (F-IF.6)

Analyze functions using different representations.

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  e. Graph exponential functions showing intercepts and end behavior. (F-IF.7)
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. (F-IF.8)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F-IF.9)

Building Functions (F-BF)

Build a function that models a relationship between two quantities.

- Write linear, quadratic, and exponential functions that describe a relationship between two quantities.★
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (F-BF.1)
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.★ (F-BF.2)

Build new functions form existing functions.

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3)
- Find inverse functions.
  a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) =2\,x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$. (F-BF.4)

Linear, Quadratic, and Exponential Models (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

- Distinguish between situations that can be modeled with linear functions and with exponential functions.

   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (F-LE.1)
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (F-LE.2)
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (F-LE.3)

Interpret expressions for functions in terms of the situation they model.

- Interpret the parameters in a linear or exponential function in terms of a context. (F-LE.5)

Statistics and Probability

Interpreting Categorical and Quantitative Data (ID)

Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.

- Represent data with plots on the real number line (dot plots, histograms, and box plots). (S-ID.1)
- Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (S-ID.2)
- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S-ID.3)

Summarize, represent, and interpret data on two categorical and quantitative variables.

- Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S-ID.5)
- Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association. (S-ID.6)

Interpret linear models.

- Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S-ID.7)
- Compute (using technology) and interpret the correlation coefficient of a linear fit. (S-ID.8)
- Distinguish between correlation and causation. (S-ID.9)

Model Traditional Pathway: Model Geometry Overview

Domains	Number and Quantity	Geometry	Statistics and Probability
	• Reason quantitatively and use units to solve problems.	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Prove geometric theorems and, when appropriate, the converse of theorems. • Make geometric constructions. • Understand similarity in terms of transformations. • Prove theorems involving similarity. • Define trigonometric ratios and solve problems involving right triangles. • Apply trigonometry to general triangles. • Understand and apply theorems about circles. • Find arc lengths and area of sectors of circles. • Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically. • Explain volume formulas and use them to solve problems. • Visualize relationships between two-dimensional and three-dimensional objects. • Apply geometric concepts in modeling situations.	• Understand independence and conditional probability and use them to interpret data from simulations or experiments. • Use the rules of probability to compute probabilities of compound events in a uniform probability model.
	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

The fundamental purpose of the Model Geometry course is to formalize and extend students' geometric experiences from the middle grades. In this high school Model Geometry course, students explore more complex geometric situations and deepen their explanations of geometric relationships by presenting and hearing formal mathematical arguments. Important differences exist between this course and the historical approach taken in geometry classes. For example, transformations are emphasized in this course. Close attention should be paid to the introductory content for the Geometry conceptual category.

For the high school Model Geometry course, instructional time should focus on six critical areas:

a. Establish criteria for congruence of triangles based on rigid motions

- Students have prior experience with drawing triangles based on given measurements and performing rigid motions including translations, reflections, and rotations. They have used these to develop notions about what it means for two objects to be congruent. In this course, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats including deductive and inductive reasoning and proof by contradiction—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

b. Establish criteria for similarity of triangles based on dilations and proportional reasoning

- Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students derive the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on their work with quadratic equations done in Model Algebra I. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

c. Informally develop explanations of circumference, area, and volume formulas.

- Students' experience with three-dimensional objects is extended to include informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

d. Apply the Pythagorean Theorem to the coordinate plane

- Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use the rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals, and slopes of parallel and perpendicular lines, which relates back to work done in the Model Algebra I course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

e. Prove basic geometric theorems

- Students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations—which relates back to work done in the Model Algebra I course—to determine intersections between lines and circles or parabolas and between two circles.

f. Extend work with probability

- Building on probability concepts that began in the middle grades, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Model Traditional Pathway: Geometry Content Standards

Number and Quantity

Quantities (N-Q)

Reason quantitatively and use units to solve problems.

- Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (N-Q.3)

Geometry

Congruence (G-CO)

Experiment with transformations in the plane.

- Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G-CO.1)
- Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (G-CO.2)
- Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (G-CO.3)
- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (G-CO.4)
- Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G-CO.5)

Understand congruence in terms of rigid motions.

- Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G-CO.6)
- Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (G-CO.7)
- Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (G-CO.8)

Prove geometric theorems.

- Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G-CO.9)
- Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G-CO.10)

- Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G-CO.11)

Make geometric constructions.

- Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G-CO.12)
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (G-CO.13)

Similarity, Right Triangles, and Trigonometry (G-SRT)

Understand similarity in terms of similarity transformations.

- Verify experimentally the properties of dilations given by a center and a scale factor:
  a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G-SRT.1)
- Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G-SRT.2)
- Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (G-SRT.3)

Prove theorems involving similarity.

- Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G-SRT.4)
- Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G-SRT.5)

Define trigonometric ratios and solve problems involving right triangles.

- Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G-SRT.6)
- Explain and use the relationship between the sine and cosine of complementary angles. (G-SRT.7)
- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (G-SRT.8)

Apply trigonometry to general triangles.

- (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (G-SRT.9)
- (+) Prove the Laws of Sines and Cosines and use them to solve problems. (G-SRT.10)
- (+) Understand and apply the Laws of Sines and Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (G-SRT.11)

Circles (G-C)

Understand and apply theorems about circles.

- Prove that all circles are similar. (G-C.1)
- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G-C.2)
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (G-C.3)
- (+) Construct a tangent line from a point outside a given circle to the circle. (G-C.4)

Find arc lengths and area of sectors of circles.

- Derive using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G-C.5)

Expressing Geometric Properties with Equations (G-GPE)

Translate between the geometric description and the equation for a conic section.

- Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G-GPE.1)
- Derive the equation of a parabola given a focus and directrix. (G-GPE.2)

Use coordinates to prove simple geometric theorems algebraically.

- Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2). (G-GPE.4)
- Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (G-GPE.5)
- Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G-GPE.6)
- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ (G-GPE.7)

Geometric Measurement and Dimension (G-GMD)

Explain volume formulas and use them to solve problems.

- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G-GMD.1)
- (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. (G-GMD.2)
- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ (G-GMD.3)

Visualize relationships between two-dimensional and three-dimensional objects.

- Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G-GMD.4)

Modeling with Geometry (G-MG)

Apply geometric concepts in modeling situations.

- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).★ (G-MG.1)
- Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ (G-MG.2)
- Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ (G-MG.3)

Statistics and Probability

Conditional Probability and the Rules of Probability (S-CP)

Understand independence and conditional probability and use them to interpret data from simulations or experiments.

- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (S-CP.1)
- Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (S-CP.2)
- Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*) and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. (S-CP.3)
- Construct and interpret two-way frequency tables of data, including information from Montana American Indian data sources, when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S-CP.4)
- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S-CP.5)

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

- Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (S-CP.6)
- Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. (S-CP.7)
- (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. (S-CP.8)
- (+) Use permutations and combinations to compute probabilities of compound events and solve problems. (S-CP.9)

Montana Mathematics Content Standards

(Adopted November 2011)

Model Traditional Pathway: Model Algebra II Overview

Domains	Number and Quantity	Algebra	Functions	Statistics and Probability
Clusters	• Perform arithmetic operations with complex numbers. • Use complex numbers in polynomial identities and equations. • Represent and model with vector quantities. • Perform operations on matrices and use matrices in applications.	• Interpret the structure of exponential, polynomial, and rational expressions. • Write expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions. • Create equations that describe numbers or relationships. • Understand solving equations as a process of reasoning and explain the reasoning. • Represent and solve	• Interpret functions that arise in applications in terms of the context (polynomial, rational, square root and cube root, trigonometric, and logarithmic functions). • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems. • Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.	• Summarize, represent and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. • Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies. • Use probability to evaluate outcomes of decisions.
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include logarithmic, polynomial, rational, and radical functions in the Model Algebra II course. Students work closely with the expressions that define the functions, are facile with algebraic manipulations of expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

For the high school Model Algebra II course, instructional time should focus on four critical areas:

1. Relate arithmetic of rational expressions to arithmetic of rational numbers

- A central theme of this Model Algebra II course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students explore the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are made between multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The Fundamental Theorem of Algebra is examined.

2. Expand understandings of functions and graphing to include trigonometric functions

- Building on their previous work with functions and on their work with trigonometric ratios and circles in the Model Geometry course, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

3. Synthesize and generalize functions and extend understanding of exponential functions to logarithmic functions

- Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this Model Algebra II course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

4. Relate data display and summary statistics to probability and explore a variety of data collection methods

- Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Model Traditional Pathway: Algebra II Content Standards

Number and Quantity

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers.

- Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. (N-CN.1)
- Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (N-CN.2)

Use complex numbers in polynomial identities and equations.

- Solve quadratic equations with real coefficients that have complex solutions. (N-CN.7)
- (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. (N-CN.8)
- (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (N-CN.9)

Vector and Matrix Quantities (N-VM)

Represent and model with vector quantities.

- (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v). (N-VM.1)
- (+) Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors. (N-VM.3)

Perform operations on matrices and use matrices in applications.

- (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (N-VM.6)
- (+) Add, subtract, and multiply matrices of appropriate dimensions. (N-VM.8)
- (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. (N-VM.12)

Seeing Structure in Expressions (A-SSE)

Interpret the structure of if exponential, polynomial, and rational expressions.

- Interpret expressions that represent a quantity in terms of its context.★
  a. Interpret parts of an expression, such as terms, factors, and coefficients.
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* (A-SSE.1)
- Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. (A-SSE.2)

Write expressions in equivalent forms to solve problems.

- Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★ (A-SSE.4)

Arithmetic with Polynomials and Rational Expressions (A-APR)

Perform arithmetic operations on polynomials.

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.1)

Understand the relationship between zeros and factors of polynomials.

- Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$. (A-APR.2)
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (A-APR.3)

Use polynomial identities to solve problems.

- Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. (A-APR.4)
- (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. [The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.] (A-APR.5)

Rewrite rational expressions.

- Rewrite simple rational expressions in different forms; write ${}^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (A-APR.6)
- (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. (A-APR.7)

Creating Equations (A-CED)

Create equations that describe numbers or relationships.

- Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1)
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2)
- Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A-CED.3)

Reasoning with Equations and Inequalities (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning.

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (A-REI.2)

Represent and solve equations and inequalities graphically.

- Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ (A-REI.11)

Functions

Interpreting Functions (F-IF)

Interpret functions that arise in applications in terms of context (polynomial, rational, square root, cube root, trigonometric, and logarithmic functions).

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ (F-IF.4)
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h*(*n*) gives the number of person-hours it takes to assemble *n* engines in a factory, then the positive integers would be an appropriate domain for the function.★ (F-IF.5)
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ (F-IF.6)

Analyze functions using different representations.

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
  - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (F-IF.7)
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (F-IF.8)

- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F-IF.9)

Building Functions (F-BF)

Build a function that models a relationship between two quantities.

- Write a function (simple rational, radical, logarithmic, and trigonometric functions) that describes a relationship between two quantities.★
  b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (F-BF.1)

Build new functions form existing functions.

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3)
- Find inverse functions.
  a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) =2\,x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$. (F-BF.4)

Linear, Quadratic, and Exponential Models (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

- For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (F-LE.4)

Trigonometric Functions (F-TF)

Extend the domain of trigonometric functions using the unit circle.

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (F-TF.1)
- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (F-TF.2)

Model periodic phenomena with trigonometric functions.

- Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline.★ (F-TF.5)

Prove and apply trigonometric identities.

- Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. (F-TF.8)

Statistics and Probability

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable.

- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, tables, and Montana American Indian data sources to estimate areas under the normal curve. (S-ID.4)

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments.

- Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (S-IC.1)
- Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (S-IC.2)

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

- Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (S-IC.3)
- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. (S-IC.4)
- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (S-IC.5)
- Evaluate reports based on data. (S-IC.6)

Using Probability to Make Decisions (S-MD)

Use probability to evaluate outcomes of decisions.

- (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (S-MD.6)
- (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (S-MD.7)

Montana Mathematics Content Standards

(Adopted November 2011)

Model Integrated Pathway: Model Mathematics I Overview

Domains	Number and Quantity	Algebra	Functions	Geometry	Statistics and Probability
	• Reason quantitatively and use units to solve problems.	• Interpret the structure of linear and exponential expressions with integer exponents. • Create equations that describe numbers or relationships. • Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Solve systems of equations. • Represent and solve equations and inequalities graphically.	• Understand the concept of a function and use function notation. • Interpret linear and exponential functions with integer exponents that arise in applications in terms of the context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Make geometric constructions. • Use coordinates to prove simple geometric theorems algebraically.	• Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. • Summarize, represent, and interpret data on two categorical and quantitative variables. • Interpret linear models.
	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.		5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

The fundamental purpose of the Model Mathematics I course is to formalize and extend the mathematics that students learned in the middle grades. For the high school Model Mathematics I course, instructional time should focus on six critical areas, each of which is described in more detail below:

1. **Extend understanding of numerical manipulation to algebraic manipulation.**
   - By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. Students become facile with algebraic manipulation in much the same way that they are facile with numerical manipulation. Algebraic facility includes rearranging and collecting terms, factoring, identifying and canceling common factors in rational expressions, and applying properties of exponents. Students continue this work by using quantities to model and analyze situations, to interpret expressions, and to create equations to describe situations.

2. **Synthesize understanding of function.**
    - In earlier grades, students define, evaluate, and compare functions, and use them to model relationships among quantities. Students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
3. **Deepen and extend understanding of linear relationships.**
    - By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Building on these earlier experiences, students analyze and explain the process of solving an equation, and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships among them.
4. **Apply linear models to data that exhibit a linear trend.**
    - Students' prior experiences with data are the basis for the more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships among quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.
5. **Establish criteria for congruence based on rigid motions.**
    - In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations, and have used these to develop notions about what it means for two objects to be congruent. Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.
6. **Apply the Pythagorean Theorem to the coordinate plane.**
    - Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Montana Mathematics Content Standards

(Adopted November 2011)

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Model Integrated Pathway: Model Mathematics I Content Standards

Number and Quantity

Quantities (N-Q)

Reason quantitatively and use units to solve problems.

- Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (N-Q.1)
- Define appropriate quantities for the purpose of descriptive modeling. (N-Q.2)
- Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (N-Q.3)

Algebra

Seeing Structure in Expressions (A-SSE)

Interpret the structure of linear and exponential expressions with integer exponents.

- Interpret expressions that represent a quantity in terms of its context.★
  a. Interpret parts of an expression, such as terms, factors, and coefficients.
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. (A-SSE.1)

Creating Equations (A-CED)

Create equations that describe numbers or relationships.

- Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1)
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2)
- Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A-CED.3)
- Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R. (A-CED.4)

Reasoning with Equations and Inequalities (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning.

- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A-REI.1)

Solve equations and inequalities in one variable.

- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A-REI.3)

Solve systems of equations.

- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (A-REI.5)
- Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A-REI.6)

Represent and solve equations and inequalities graphically.

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A-REI.10)
- Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ (A-REI.11)
- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A-REI.12)

Functions

Interpreting Functions (F-IF)

Understand the concept of a function and use function notation.

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (F-IF.1)
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F-IF.2)
- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. (F-IF.3)

Interpret functions that arise in applications in terms of context.

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts;

intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ (F-IF.4)

- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ (F-IF.5)
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ (F-IF.6)

Analyze functions using different representations.

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (F-IF.7)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (F-IF.9)

Building Functions (F-BF)

Build a function that models a relationship between two quantities.

- Write linear and exponential functions that describes a relationship between two quantities.★
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (F-BF.1)
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.★ (F-BF.2)

Build new functions form existing functions.

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3)

Linear, Quadratic, and Exponential Models (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

- Distinguish between situations that can be modeled with linear functions and with exponential functions.

   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (F-LE.1)
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (F-LE.2)
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (F-LE.3)

Interpret expressions for functions in terms of the situation they model.

- Interpret the parameters in a linear or exponential function in terms of a context. (F-LE.5)

Geometry

Congruence (G-CO)

Experiment with transformations in the plane.

- Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G-CO.1)
- Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (G-CO.2)
- Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (G-CO.3)
- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (G-CO.4)
- Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G-CO.5)

Understand congruence in terms of rigid motions.

- Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G-CO.6)
- Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (G-CO.7)
- Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (G-CO.8)

Make geometric constructions.

- Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G-CO.12)

- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (G-CO.13)

Expressing Geometric Properties with Equations (G-GPE)

Use coordinates to prove simple geometric theorems algebraically.

- Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (G-GPE.5)
- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ (G-GPE.7)

Statistics and Probability

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable.

- Represent data with plots on the real number line (dot plots, histograms, and box plots). (S-ID.1)
- Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (S-ID.2)
- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S-ID.3)

Summarize, represent, and interpret data on two categorical and quantitative variables.

- Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S-ID.5)
- Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
    b. Informally assess the fit of a function by plotting and analyzing residuals.
    c. Fit a linear function for a scatter plot that suggests a linear association. (S-ID.6)

Interpret linear models.

- Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S-ID.7)
- Compute (using technology) and interpret the correlation coefficient of a linear fit. (S-ID.8)
- Distinguish between correlation and causation. (S-ID.9)

Montana Mathematics Content Standards

(Adopted November 2011)

Model Integrated Pathway: Model Mathematics II Overview

Domains	Number and Quantity	Algebra	Functions	Geometry	Statistics and Probability
	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers. • Reason quantitatively and use units to solve problems. • Perform arithmetic operations with complex numbers. • Use complex numbers in polynomial identities and equations.	• Interpret the structure of quadratic and exponential expressions. • Write quadratic and exponential expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Create equations that describe numbers or relationships. • Solve equations and inequalities in one variable. • Solve systems of equations.	• Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic and exponential models and solve problems.	• Prove geometric theorems, and when appropriate, the converse of theorems. • Understand similarity in terms of similarity transformations. • Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. • Define trigonometric ratios and solve problems involving right triangles. • Understand and apply theorems about circles. • Find arc lengths and areas of sectors of circles. • Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically. • Explain volume formulas and use them to solve problems.	• Understand independence and conditional probability and use them to interpret data from simulations or experiments. • Use the rules of probability to compute probabilities of compound events in a uniform probability model.
	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.		5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

The focus of the Model Mathematics II course is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Model Mathematics I.

For the high school Model Mathematics II course, instructional time should focus on five critical areas:

1. **Extend the laws of exponents to rational exponents.**

- Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions, the number system must be extended so that solutions exist; analogous to the way in which extending the whole numbers to the negative numbers allows $x + 1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system

2. **Compare key characteristics of quadratic functions with those of linear and exponential functions.**

- Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

3. **Create and solve equations and inequalities involving linear, exponential, and quadratic expressions.**

- Students begin by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

4. **Extend work with probability.**

- Building on probability concepts that began in the middle grades, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

5. **Establish criteria for similarity of triangles based on dilations and proportional reasoning.**

- Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Model Integrated Pathway: Model Mathematics II Content Standards

Number and Quantity

The Real Number System (N-RN)

Extend the properties of exponents to rational exponents.

- Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. (N-RN.1)

- Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N-RN.2)

Use properties of rational and irrational numbers.

- Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. (N-RN.3)

Quantities (N-Q)

Reason quantitatively and use units to solve problems.

- Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (N-Q.3)

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers.

- Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. (N-CN.1)
- Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (N-CN.2)

Use complex numbers in polynomial identities and equations.

- Solve quadratic equations with real coefficients that have complex solutions. (N-CN.7)

Algebra

Seeing Structure in Expressions (A-SSE)

Interpret the structure of quadratic and exponential expressions.

- Interpret expressions that represent a quantity in terms of its context.★
  a. Interpret parts of an expression, such as terms, factors, and coefficients.
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. (A-SSE.1)
- Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (A-SSE.2)

Write quadratic and exponential expressions in equivalent forms to solve problems.

- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
  a. Factor a quadratic expression to reveal the zeros of the function it defines.
  b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (A-SEE.3)

Arithmetic with Polynomials and Rational Expressions (A-APR)

Perform arithmetic operations on polynomials.

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.1)

Creating Equations (A-CED)

Create equations that describe numbers or relationships.

- Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1)
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2)
- Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R. (A-CED.4)

Reasoning with Equations and Inequalities (A-REI)

Solve equations and inequalities in one variable.

- Solve quadratic equations in one variable.
  a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. (A-REI.4)

Solve systems of equations.

- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$. (A-REI.7)

Functions

Interpreting Functions (F-IF)

Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of context.

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ (F-IF.4)

- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ (F-IF.5)
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ (F-IF.6)

Analyze functions using different representations.

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (F-IF.7)
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* (F-IF.8)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (F-IF.9)

Building Functions (F-BF)

Build a function that models a relationship between two quantities.

- Write linear, quadratic, and exponential functions that describes a relationship between two quantities.★
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (F-BF.1)

Build new functions form existing functions.

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3)
- Find inverse functions.
  a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) =2 x^3$ *or* $f(x) = (x+1)/(x–1)$ *for* $x \neq 1$. (F-BF.4)

Linear, Quadratic, and Exponential Models (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (F-LE.3)

Geometry

Congruence (G-CO)

Prove geometric theorems.

- Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G-CO.9)
- Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G-CO.10)
- Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G-CO.11)

Similarity, Right Triangles, and Trigonometry (G-SRT)

Understand similarity in terms of similarity transformations.

- Verify experimentally the properties of dilations given by a center and a scale factor:
  a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G-SRT.1)
- Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G-SRT.2)
- Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (G-SRT.3)

Prove theorems involving similarity.

- Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G-SRT.4)
- Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G-SRT.5)

Define trigonometric ratios and solve problems involving right triangles.

- Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G-SRT.6)
- Explain and use the relationship between the sine and cosine of complementary angles. (G-SRT.7)
- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (G-SRT.8)

Circles (G-C)

Understand and apply theorems about circles.

- Prove that all circles are similar. (G-C.1)
- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G-C.2)
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (G-C.3)
- (+) Construct a tangent line from a point outside a given circle to the circle. (G-C.4)

Find arc lengths and area of sectors of circles.

- Derive using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G-C.5)

Expressing Geometric Properties with Equations (G-GPE)

Translate between the geometric description and the equation for a conic section.

- Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G-GPE.1)
- Derive the equation of a parabola given a focus and directrix. (G-GPE.2)

Use coordinates to prove simple geometric theorems algebraically.

- Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). (G-GPE.4)
- Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G-GPE.6)

Geometric Measurement and Dimension (G-GMD)

Explain volume formulas and use them to solve problems.

- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G-GMD.1)
- (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. (G-GMD.2)
- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ (G-GMD.3)

Statistics and Probability

Conditional Probability and the Rules of Probability (S-CP)

Understand independence and conditional probability and use them to interpret data from simulations or experiments.

- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (S-CP.1)
- Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (S-CP.2)
- Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*) and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. (S-CP.3)
- Construct and interpret two-way frequency tables of data, including information from Montana American Indian data sources, when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S-CP.4)
- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S-CP.5)

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

- Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. (S-CP.6)
- Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model. (S-CP.7)
- (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. (S-CP.8)
- (+) Use permutations and combinations to compute probabilities of compound events and solve problems. (S-CP.9)

Montana Mathematics Content Standards

(Adopted November 2011)

Model Integrated Pathway: Model Mathematics III Overview

Domains	Number and Quantity	Algebra	Functions	Geometry	Statistics and Probability
	• Use complex numbers in polynomial identities and equations. • Represent and model with vector quantities. • Perform operations	• Interpret the structure polynomial and rational expressions. • Write expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions. • Create equations that describe numbers or relationships. • Understand solving equations as a process of reasoning and explain the reasoning. • Represent and solve equations and inequalities graphically.	• Interpret functions that arise in applications in terms of the context (rational, polynomial, square root, cube root, trigonometric, logarithmic). • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic and exponential models and solve problems. • Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.	• Apply trigonometry to general triangles. • Visualize relationships between two-dimensional and three-dimensional objects. • Apply geometric concepts in modeling situations.	• Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. • Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies. • Use probability to evaluate outcomes of decisions.
	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.		5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

It is in the Model Mathematics III course that students integrate and apply the mathematics they have learned from their earlier courses. For the high school Model Mathematics III course, instructional time should focus on four critical areas:

1. **Apply methods from probability and statistics to draw inferences and conclusions from data.**

- Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the roles that randomness and careful design play in the conclusions that can be drawn.

2. **Expand understanding of functions to include polynomial, rational, and radical functions.**
   - The structural similarities between the system of polynomials and the system of integers are developed. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. Rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of the Model Mathematics III course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. This critical area also includes exploration of the Fundamental Theorem of Algebra.
3. **Expand right triangle trigonometry to include general triangles.**
   - Students derive the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.
4. **Consolidate functions and geometry to create models and solve contextual problems.**
   - Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of underlying function. They identify appropriate types of functions to model a situation; they adjust parameters to improve the model; and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this Model Mathematics III course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Model Integrated Pathway: Model Mathematics III Content Standards

Number and Quantity

The Complex Number System (N-CN)

Use complex numbers in polynomial identities and equations.

- (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* (N-CN.8)
- (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (N-CN.9)

Vector and Matrix Quantities (N-VM)

Represent and model with vector quantities.

- (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*). (N-VM.1)
- (+) Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors. (N-VM.3)

Perform operations on matrices and use matrices in applications.

- (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (N-VM.6)
- (+) Add, subtract, and multiply matrices of appropriate dimensions. (N-VM.8)
- (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. (N-VM.12)

Algebra

Seeing Structure in Expressions (A-SEE)

Interpret the structure of polynomial and rational expressions.

- Interpret expressions that represent a quantity in terms of its context.★
  a. Interpret parts of an expression, such as terms, factors, and coefficients.
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* (A-SSE.1)
- Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* (A-SSE.2)

Write expressions in equivalent forms to solve problems.

- Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★ (A-SSE.4)

Montana Mathematics Content Standards

(Adopted November 2011)

Arithmetic with Polynomials and Rational Expressions (A-APR)

Perform arithmetic operations on polynomials.

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.1)

Understand the relationship between zeros and factors of polynomials.

- Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$. (A-APR.2)
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (A-APR.3)

Use polynomial identities to solve problems.

- Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. (A-APR.4)
- (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. [The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.] (A-APR.5)

Rewrite rational expressions.

- Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (A-APR.6)
- (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. (A-APR.7)

Creating Equations (A-CED)

Create equations that describe numbers or relationships.

- Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1)
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2)
- Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A-CED.3)

Reasoning with Equations and Inequalities (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning.

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (A-REI.2)

Represent and solve equations and inequalities graphically.

- Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ (A-REI.11)

Functions

Interpreting Functions (F-IF)

Interpret functions that arise in applications in terms of context (rational, polynomial, square root, cube root, trigonometric, logarithmic).

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ (F-IF.4)
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble *n* engines in a factory, then the positive integers would be an appropriate domain for the function.★ (F-IF.5)
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ (F-IF.6)

Analyze functions using different representations.

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
  - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (F-IF.7)
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (F-IF.8)

- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F-IF.9)

Building Functions (F-BF)

Build a function that models a relationship between two quantities.

- Write simple rational and radical functions, logarithmic, and trigonometric functions that describes a relationship between two quantities.★
  b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (F-BF.1)

Build new functions form existing functions.

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3)
- Find inverse functions.
  a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) =2\,x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$. (F-BF.4)

Linear, Quadratic, and Exponential Models (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

- For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (F-LE.4)

Trigonometric Functions (F-TF)

Extend the domain of trigonometric functions using the unit circle.

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (F-TF.1)
- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (F-TF.2)

Model periodic phenomena with trigonometric functions.

- Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline.★ (F-TF.5)

Prove and apply trigonometric identities.

- Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. (F-TF.8)

Geometry

Similar. Right Triangles, and Trigonometry (G-SRT)

Apply trigonometry to general triangles.

- (+) Derive the formula $A = 1/2\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (G-SRT.9)
- (+) Prove the Laws of Sines and Cosines and use them to solve problems. (G-SRT.10)
- (+) Understand and apply the Laws of Sines and Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (G-SRT.11)

Geometric Measurement and Dimension (G-GMD)

Visualize relationships between two-dimensional and three-dimensional objects.

- Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G-GMD.4)

Modeling with Geometry (G-MG)

Apply geometric concepts in modeling situations.

- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).★ (G-MG.1)
- Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ (G-MG.2)
- Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ (G-MG.3)

Statistics and Probability

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable.

- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, tables, and Montana American Indian data sources to estimate areas under the normal curve. (S-ID.4)

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments

- Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (S-IC.1)

- Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (S-IC.2)

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

- Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (S-IC.3)
- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. (S-IC.4)
- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (S-IC.5)
- Evaluate reports based on data. (S-IC.6)

Using Probability to Make Decisions (S-MD)

Use probability to evaluate outcomes of decisions.

- (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (S-MD.6)
- (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (S-MD.7)