Montana Content Standards for Mathematics– Grade 6

Montana Content Standards for Mathematical Practices and Mathematics Content
Adopted November 2011.

Montana Mathematics Content Standards

(Adopted November 2011)

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Standards for Mathematical Practice: Grade 6 Explanations and Examples

Standards	Explanations and Examples
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
6.MP.1. Make sense of problems and persevere in solving them.	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
6.MP.2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
6.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
6.MP.4. Model with mathematics.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
6.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6.MP.6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.
6.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions

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	(i.e. 6 + 2x = 2 (3 + x) by distributive property) and solve equations (i.e. 2c + 3 = 15, 2c = 12 by subtraction property of equality; c=6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
6.MP.8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Adapted from Explanations and Examples Grade 6 produced by the Arizona Department of Education Standards and Assessment Division

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Sixth Grade Overview

Domains	Ratios & Proportional Relationships	The Number System	Expressions and Equations	Geometry	Statistics and Probability
Clusters	• Understand ratio concepts and use ratio reasoning to solve problems	•Apply and extend previous understandings of multiplication and division to divide fractions by fractions •Compute fluently with multi-digit numbers and find common factors and multiples •Apply and extend previous understandings of numbers to the system of rational numbers	• Apply and extend previous understandings of arithmetic to algebraic expressions • Reason about and solve one-variable equations and inequalities • Represent and analyze quantitative relationships between dependent and independent variables	• Solve real-world and mathematical problems involving area, surface area, and volume	• Develop understanding of statistical variability • Summarize and describe distributions
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

In Grade 6, instructional time should focus on four critical areas:

1. ***Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems***
   - Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. ***Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers***
   - Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. ***Writing, interpreting, and using expressions and equations***
   - Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea

of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. ***Developing understanding of statistical thinking***

- Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

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GRADE 6 STANDARDS

Ratios and Proportional Relationship (RP)

Understand ratio concepts and use ratio reasoning to solve problems.

- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." (6.RP.1)
- Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." [Expectations for unit rates in this grade are limited to non-complex fractions.] (6.RP.2)
- Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours?
  - Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  - Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (6.RP.3)

The Number System (NS)

Apply and extend previous understandings of multiplication and division to divide factions by fractions.

- Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because 3/4 of 8/9 is 2/3. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? (6.NS.1)

Compute fluently with multi-digit numbers and find common factors and multiples.

- Fluently divide multi-digit numbers using the standard algorithm. (6.NS.2)
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (6.NS.3)

- Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2). (6.NS.4)

Apply and extend previous understandings of numbers to the system of rational numbers.

- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (6.NS.5)
- Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
  - Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., - (-3) = 3; that 0 is its own opposite.
  - Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane and recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
  - Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (6.NS.6)
- Understand ordering and absolute value of rational numbers.
  - Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.
  - Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3° C > -7° C to express the fact that -3° C is warmer than -7° C.
  - Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.
  - Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars. (6.NS.7)
- Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)

Expressions and Equations (EE)

Apply and extend previous understandings of arithmetic to algebraic expressions.

- Write and evaluate numerical expressions involving whole-number exponents. (6.EE.1)
- Write, read, and evaluate expressions in which letters stand for numbers.
  - Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "subtract *y* from 5" as $5 - y$.

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- o Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
- o Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6\,s^2$ to find the volume and surface area of a cube with sides of length s = ½. (6.EE.2)

- Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. (6.EE.3)
- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. (6.EE.4)

Reason about and solve one-variable equations and inequalities.

- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.5)
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (6.EE.6)
- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. (6.EE.7)
- Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (6.EE.8)

Represent and analyze quantitative relationships between dependent and independent variables.

- Use variables to represent two quantities in a real-world problem from a variety of cultural contexts, including those of Montana American Indians, that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time (6.EE.9)

Geometry (G)

Solve real-world and mathematical problems involving area, surface area, and volume.

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems within

cultural contexts, including those of Montana American Indians. For example, use Montana American Indian designs to decompose shapes and find the area. (6.G.1)

- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (6.G.2)
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.3)
- Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians. (6.G.4)

Statistics and Probability (SP)

Develop understanding of statistical variability.

- Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages. (6.SP.1)
- Understand that a set of data collected (including Montana American Indian demographic data) to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.2)
- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (6.SP.3)

Summarize and describe distributions.

- Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (6.SP.4)
- Summarize numerical data sets in relation to their context, such as by:
  - Reporting the number of observations.
  - Describing the nature of the attribute under investigation, including how it was measured and its units of measurement
  - giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
  - Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.5)